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UNIFIED DESCRIPTION OF DRY AND FLUID FRICTIONS BY SUBLOADING-OVERSTRESS FRICTION MODEL

TOSHIYUKI OZAKI* AND KOICHI HASHIGUCHI†

* Design of Overhead Transmission line Dept.
Kyushu Electric Engineering Consultants Inc.
Kiyokawa 2-13-6, Fukuoka 810-0005, Japan
e-mail: ozaki@dengi.co.jp

†MSC Software, Ltd.
Shinjuku First West 8F,
1-23-7, Nishishinjuku, Shinjuku-ku, Tokyo 160-0023, Japan
e-mail: e-mail: hashikoi87@gmail.com

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Abstract. The subloading-overstress friction model is formulated for the unified description of the dry and the fluid frictions which exhibit the negative and the positive rate dependences, i.e. the decrease and the increase, respectively, of friction resistance. The validity of this model will be verified by the comparisons with test data in this article.

1 INTRODUCTION

The dry friction without a lubrication exhibits the decrease of friction resistance with the sliding velocity, which is called the *negative rate-sensitivity*. On the other hand, the fluid friction exhibits the increase of friction resistance with the sliding velocity, which is called the *positive rate-sensitivity*. The generalized friction model, called the *subloading-overstress friction model* [1], is formulated based on the subloading surface model [2] [3] [4], which is capable of describing the dry and the fluid frictions by the unified formulation.

The validity of the subloading-overstress friction model is examined by the comparisons with test data in this article. Then, the capability of describing both the dry and the fluid frictions is verified by these comparisons.

2 SLIDING DISPLACEMENT AND CONTACT TRACTION VECTORS

The sliding displacement vector $\bar{\mathbf{u}}$, which is defined as the sliding displacement of the counter (slave) body to the main (master) body, is orthogonally decomposed into the normal sliding displacement vector $\bar{\mathbf{u}}_n$ and the tangential sliding displacement vector $\bar{\mathbf{u}}_t$ to the contact surface as follows:

$$\bar{\mathbf{u}} = \bar{\mathbf{u}}_n + \bar{\mathbf{u}}_t \tag{1}$$

where

$$\begin{cases} \bar{\mathbf{u}}_n = (\bar{\mathbf{u}} \bullet \mathbf{n})\mathbf{n} = (\mathbf{n} \otimes \mathbf{n})\bar{\mathbf{u}} = -\bar{u}_n \mathbf{n} \\ \bar{\mathbf{u}}_t = \bar{\mathbf{u}} - \bar{\mathbf{u}}_n = (\mathbf{I} - \mathbf{n} \otimes \mathbf{n})\bar{\mathbf{u}} \end{cases} \tag{2}$$

$\mathbf{n}$ being the unit outward-normal vector of the surface of main body and

$$\bar{u}_n \equiv -\mathbf{n} \bullet \bar{\mathbf{u}}_n = -\mathbf{n} \bullet \bar{\mathbf{u}} \tag{3}$$

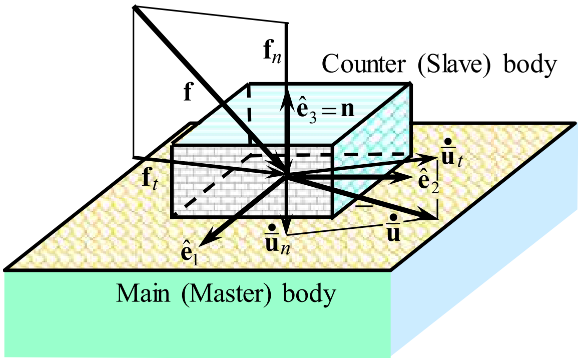

Figure1 Contact traction and sliding velocity.

The sliding displacement vector $\bar{\mathbf{u}}$ can be exactly decomposed into the elastic sliding displacement $\bar{\mathbf{u}}^e$ and the viscoplastic sliding displacement $\bar{\mathbf{u}}^{vp}$ in the additive form even for the finite sliding displacement, i.e.

$$\bar{\mathbf{u}} = \bar{\mathbf{u}}^e + \bar{\mathbf{u}}^{vp} \tag{4}$$

$$\begin{cases} \bar{\mathbf{u}}^e = \bar{\mathbf{u}}_n^e + \bar{\mathbf{u}}_t^e \\ \bar{\mathbf{u}}^{vp} = \bar{\mathbf{u}}_n^{vp} + \bar{\mathbf{u}}_t^{vp} \end{cases} \tag{5}$$

where

$$\begin{cases} \bar{\mathbf{u}}_n^e = (\bar{\mathbf{u}}^e \bullet \mathbf{n})\mathbf{n} = (\mathbf{n} \otimes \mathbf{n})\bar{\mathbf{u}}^e = -\bar{u}_n^e \mathbf{n} \\ \bar{\mathbf{u}}_t^e = \bar{\mathbf{u}}^e - \bar{\mathbf{u}}_n^e = (\mathbf{I} - \mathbf{n} \otimes \mathbf{n})\bar{\mathbf{u}}^e \end{cases} \tag{6}$$

$$\begin{cases} \bar{\mathbf{u}}_n^{vp} = (\bar{\mathbf{u}}^{vp} \bullet \mathbf{n})\mathbf{n} = (\mathbf{n} \otimes \mathbf{n})\bar{\mathbf{u}}^{vp} \\ \bar{\mathbf{u}}_t^{vp} = \bar{\mathbf{u}}^{vp} - \bar{\mathbf{u}}_n^{vp} = (\mathbf{I} - \mathbf{n} \otimes \mathbf{n})\bar{\mathbf{u}}^{vp} \end{cases} \tag{7}$$

setting

$$\bar{u}_n^e \equiv -\mathbf{n} \bullet \bar{\mathbf{u}}_n^e = -\mathbf{n} \bullet \bar{\mathbf{u}}^e \tag{8}$$

The minus sign is added for $\bar{u}_n^e$ to be positive when the counter body approaches the main body. The plastic sliding displacement $\bar{\mathbf{u}}^{vp}$ is derived by the unloading to the contact traction-free state along the hyperelastic constitutive equation which will be formulated in Section 3. The viscoplastic sliding flow rule will be formulated to fulfill $\bar{\mathbf{u}}_n^{vp} = \mathbf{0}$ in Section 4.

The *contact traction vector* $\mathbf{f}$ acting on the main body is additively decomposed into the *normal traction vector* $\mathbf{f}_n$ and the *tangential traction vector* $\mathbf{f}_t$ as follows (see Figure 2):

$$\mathbf{f} = \mathbf{f}_n + \mathbf{f}_t = -f_n \mathbf{n} + f_t \mathbf{t}_f \tag{9}$$

where

$$\begin{cases} \mathbf{f}_n \equiv (\mathbf{n} \bullet \mathbf{f})\mathbf{n} = (\mathbf{n} \otimes \mathbf{n})\mathbf{f} = - f_n \mathbf{n} \\ \mathbf{f}_t \equiv \mathbf{f} - \mathbf{f}_n = (\mathbf{I} - \mathbf{n} \otimes \mathbf{n})\mathbf{f} = f_t \mathbf{t}_f \end{cases} \tag{10}$$

$$\begin{cases} f_n \equiv - \mathbf{n} \bullet \mathbf{f} \\ f_t \equiv \mathbf{t}_f \bullet \mathbf{f} = \|\mathbf{f}_t\|, \ \mathbf{t}_f \equiv \dfrac{(\mathbf{I} - \mathbf{n} \otimes \mathbf{n})\mathbf{f}}{\|(\mathbf{I} - \mathbf{n} \otimes \mathbf{n})\mathbf{f}\|} = - \ (\mathbf{n} \bullet \mathbf{t}_f = 0, \ \|\mathbf{t}_f\| = 1) \end{cases} \tag{11}$$

The minus sign is added for f_n to be positive when the compression is applied to the main body by the counter body.

The contact traction vector $\mathbf{f}$, $\mathbf{f}_n$ and $\mathbf{f}_t$ are calculated from the Cauchy stress $\boldsymbol{\sigma}$ applied to the contact surface by virtue of the Cauchy's fundamental theorem as follows:

$$\begin{cases} \mathbf{f} = \boldsymbol{\sigma}\mathbf{n} \\ \mathbf{f}_n = (\mathbf{n} \bullet \boldsymbol{\sigma}\mathbf{n})\mathbf{n} = (\mathbf{n} \otimes \mathbf{n})\boldsymbol{\sigma}\mathbf{n} \\ \mathbf{f}_t = (\mathbf{I} - \mathbf{n} \otimes \mathbf{n})\boldsymbol{\sigma}\mathbf{n} \end{cases} \tag{12}$$

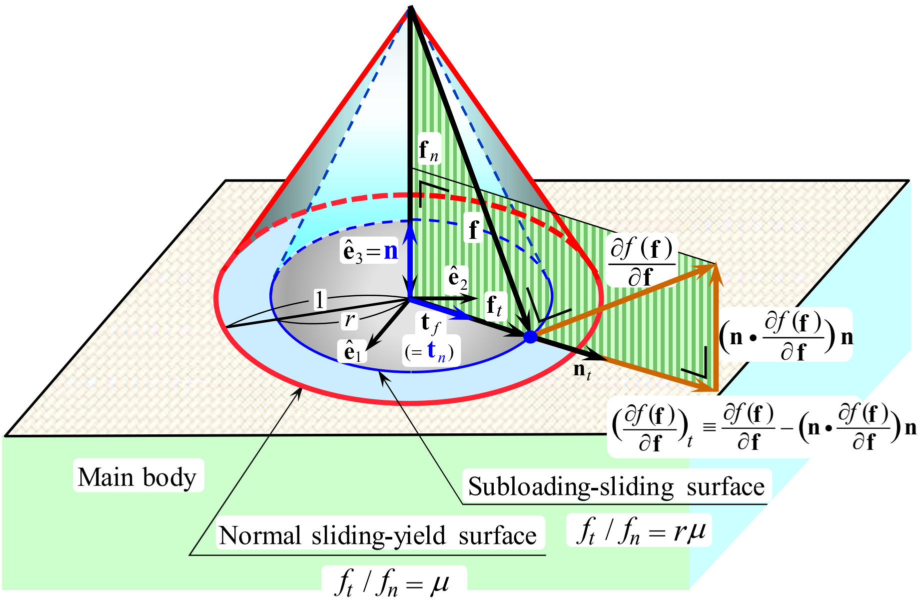

Figure 2 Coulomb-type normal- and subloading-sliding surfaces

3 HYPERELASTIC SLIDING EQUATION

Let the contact traction vector $\mathbf{f}$ be given by the hyperelastic relation with the elastic sliding displacement energy function $\varphi(\bar{\mathbf{u}}^e)$ as follows:

$$\mathbf{f} = \frac{\partial \varphi(\bar{\mathbf{u}}^e)}{\partial \bar{\mathbf{u}}^e} \tag{13}$$

The simplest function $\partial \varphi(\bar{\mathbf{u}}^e)$ is given by the quadratic form:

$$\partial \varphi(\bar{\mathbf{u}}^e) = \bar{\mathbf{u}}^e \bullet \bar{\mathbf{E}} \bar{\mathbf{u}}^e / 2 \tag{14}$$

where the second-order symmetric tensor $\overline{\mathbf{E}}$ is the elastic contact tangent modulus tensor fulfilling the symmetry $\overline{\mathbf{E}} = \overline{\mathbf{E}}^T$. The substitution of Eq. (14) into Eq. (13) leads to

$$\mathbf{f} = \overline{\mathbf{E}}\overline{\mathbf{u}}^e \tag{15}$$

The inverse relation of Eq. (15) is given by

$$\overline{\mathbf{u}}^e = \overline{\mathbf{E}}^{-1}\mathbf{f} \tag{16}$$

The elastic contact tangent modulus tensor $\overline{\mathbf{E}}$ is given for the isotropy on the contact surface as follows:

$$\begin{cases} \overline{\mathbf{E}} = \alpha_n \mathbf{n}\otimes\mathbf{n} + \alpha_t(\mathbf{I} - \mathbf{n}\otimes\mathbf{n}) \\ \overline{\mathbf{E}}^{-1} = \dfrac{1}{\alpha_n}\mathbf{n}\otimes\mathbf{n} + \dfrac{1}{\alpha_t}(\mathbf{I} - \mathbf{n}\otimes\mathbf{n}) \end{cases} \tag{17}$$

where α_n and α_t are the normal and tangential *contact elastic moduli*, respectively. Their values are quite large usually as $10^2 - 10^5$ GPa/mm^3 for metals because the elastic sliding is caused by elastic deformations of the surface asperities. Equations (15) and (16) with Eq. (17) leads to

$$\begin{cases} \mathbf{f} = \alpha_t \overline{\mathbf{u}}_t^e + \alpha_n \overline{\mathbf{u}}_n^e \\ \overline{\mathbf{u}}^e = \dfrac{1}{\alpha_t}\mathbf{f}_t + \dfrac{1}{\alpha_n}\mathbf{f}_n \end{cases} \tag{18}$$

Now, introduce the normalized rectangular coordinate system $(\hat{\mathbf{e}}_1,\ \hat{\mathbf{e}}_2,\ \hat{\mathbf{e}}_3) = (\hat{\mathbf{e}}_1,\ \hat{\mathbf{e}}_2,\ \mathbf{n})$ fixed to the contact surface, which changes with the rotation of the contact surface. The elastic sliding displacement and the contact traction are described as follows:

$$\begin{cases} \mathbf{f} = f_1\hat{\mathbf{e}}_1 + f_2\hat{\mathbf{e}}_2 + f_n\mathbf{n} \\ \overline{\mathbf{u}}^e = \overline{u}_1^e\hat{\mathbf{e}}_1 + \overline{u}_2^e\hat{\mathbf{e}}_2 + \overline{u}_n^e\mathbf{n} \end{cases} \tag{19}$$

Hence, Eq. (15) is described in the simple form as follows:

$$\begin{Bmatrix} f_1 \\ f_2 \\ f_n \end{Bmatrix} = \begin{bmatrix} \alpha_t & 0 & 0 \\ 0 & \alpha_t & 0 \\ 0 & 0 & \alpha_n \end{bmatrix} \begin{Bmatrix} \overline{u}_1^e \\ \overline{u}_2^e \\ \overline{u}_n^e \end{Bmatrix}, \quad \begin{Bmatrix} \overline{u}_1^e \\ \overline{u}_2^e \\ \overline{u}_n^e \end{Bmatrix} = \begin{bmatrix} 1/\alpha_t & 0 & 0 \\ 0 & 1/\alpha_t & 0 \\ 0 & 0 & 1/\alpha_n \end{bmatrix} \begin{Bmatrix} f_1 \\ f_2 \\ f_n \end{Bmatrix} \tag{20}$$

The sliding velocity vector $\dot{\overline{\mathbf{u}}}$ is the objective vector, since it is not an absolute velocity vector but the mutual velocity vector between surface points on the master and the counter bodies. Therefore, it is not necessary to use a corotational velocity vector but we only have to use the time derivative for the sliding velocity vector. Further, note that one does not need to adopt a corotational rate but one has only to use the time derivative for the contact traction vector $\mathbf{f}$ by the fact: The contact traction $\mathbf{f}$ is calculated from the hyperelastic equation with the substitution of the elastic displacement $\overline{\mathbf{u}}^e$ which is obtained by subtracting the plastic displacement vector $\overline{\mathbf{u}}^{vp}$ from the displacement vector $\overline{\mathbf{u}}$.

4 NORMAL-SLIDING YIELD AND SUBLOADING-SLIDING SURFACES

Assume the following sliding-yield surface with the isotropic hardening/softening, which describes the sliding-yield condition.

$$f(\mathbf{f}) = \mu \tag{21}$$

μ is the isotropic hardening/softening function denoting the variation of the size of the sliding-yield surface. The friction-yield stress function $f(\mathbf{f})$ for the Coulomb friction law is given by

$$f(\mathbf{f}) = f_t / f_n \tag{22}$$

for which μ specifies the coefficient of friction.

Then, in order to introduce the measure of approaching degree to the sliding-yield surface, renamed the *normal sliding-yield surface*, let the following *subloading-sliding surface* passing through the current contact stress and maintaining a similarity to the normal sliding-yield surface be introduced, which plays the general measure of approaching degree of the contact stress to the normal sliding-yield surface (see Figure 2).

$$f(\mathbf{f}) = r\mu \tag{23}$$

where $r\ (0 \le r \le 1)$ is the ratio of the size of the subloading surface to that of the normal sliding-yield surface and called the *normal sliding-yield ratio*, playing the role of the measure of the approaching degree of the contact stress to the normal sliding-yield surface.

The evolution rule of the isotropic hardening/softening function μ in Eq. (21) is extended as follows:

$$\overset{\bullet}{\mu} = \underbrace{-\kappa\Big(\frac{\mu}{\mu_k}-1\Big)\|\overset{\bullet}{\mathbf{u}}{}^{vp}\|}_{\text{Negative}} + \underbrace{\xi\Big(1-\frac{\mu}{\mu_s}\Big)}_{\text{Positive}} \tag{24}$$

The viscoplastic sliding rate is given as follows:

$$\overset{\bullet}{\mathbf{u}}{}^{vp} = \bar{\Gamma}\mathbf{n}_t \ \ (\bar{\Gamma} \ge 0) \tag{25}$$

where $\bar{\Gamma}$ and $\mathbf{n}_t$ are the magnitude and direction, respectively, of the plastic sliding velocity as shown below.

$$\mathbf{n}_t \equiv \Big(\frac{\partial f(\mathbf{f})}{\partial \mathbf{f}}\Big)_t \Big/ \Big\|\Big(\frac{\partial f(\mathbf{f})}{\partial \mathbf{f}}\Big)_t\Big\| \quad (\|\mathbf{n}_t\|=1,\ \mathbf{n}\bullet\mathbf{n}_t = 0) \tag{0.26}$$

with

$$\Big(\frac{\partial f(\mathbf{f})}{\partial \mathbf{f}}\Big)_t \equiv \frac{\partial f(\mathbf{f})}{\partial \mathbf{f}} - \Big(\mathbf{n}\bullet\frac{\partial f(\mathbf{f})}{\partial \mathbf{f}}\Big)\mathbf{n} = (\mathbf{I}-\mathbf{n}\otimes\mathbf{n})\frac{\partial f(\mathbf{f})}{\partial \mathbf{f}} \tag{0.27}$$

$$\bar{\Gamma} \equiv \frac{1}{\bar{\mu}_v}\frac{\langle r - r_s\rangle^n}{r_m - r}\begin{cases} > 0 & \text{for } r<r_m \\ \to \infty & \text{for } r \to r_m \end{cases} \tag{28}$$

or

$$\bar{\Gamma} \equiv \frac{1}{\bar{\mu}_v}\frac{\langle \exp[n(r - r_s)]-1\rangle}{r_m - r}\begin{cases} > 0 & \text{for } r<r_m \\ \to \infty & \text{for } r \to r_m \end{cases} \tag{29}$$

where $\bar{\mu}_v$, $n\ (\geq 1)$ and $r_m\ (\geq 1)$ are the material parameters, while r_m is the maximum value of r, and

$$\dot{r}_s = \begin{cases} \bar{U}(r_s)\|\dot{\bar{\mathbf{u}}}^{vp}\| \text{ under } \dot{\bar{\mathbf{u}}}^{vp} \neq \mathbf{0} \text{ for } r_s < r \\ \left.\begin{array}{l} = 0 \text{ for } \dot{\bar{\mathbf{u}}}^{e} = \mathbf{0} \\ < 0 \text{ for } \dot{\bar{\mathbf{u}}}^{e} \neq \mathbf{0} \end{array}\right\} \text{ under } \dot{\bar{\mathbf{u}}}^{vp} = \mathbf{0} \text{ for } r_s = r\ (\dot{r}_s = \dot{r}) \end{cases} \tag{30}$$

with

$$\bar{U}(r_s) = \tilde{u}\cot\left(\frac{\pi}{2} r_s\right). \tag{31}$$

Here, $r\ (0 \leq r < r_m)$ in Eq. (23) is renamed as the *dynamic sliding-loading ratio* and $r_s\ (0 \leq r_s \leq r \leq 1)$ is called the *static normal sliding-yield ratio* because it designates the normal sliding-yield ratio which evolves under the virtual quasi-static elastoplastic sliding process. The viscoplastic sliding velocity is induced by the overstress $f(\mathbf{f}) - r_s\mu$ from the subloading friction surface:

$$f(\mathbf{f}) = r_s\mu, \text{ i.e. } r = r_s \tag{32}$$

so that a smooth elastic–viscoplastic transition is described.

The sliding rate and its inverse relation are given by Eqs. (4), (15) and (25) as follows:

$$\begin{cases} \dot{\bar{\mathbf{u}}} = \bar{\mathbf{E}}^{-1}\dot{\mathbf{f}} + \bar{\Gamma}\mathbf{n}_t \\ \dot{\mathbf{f}} = \bar{\mathbf{E}}\dot{\bar{\mathbf{u}}} - \bar{\Gamma}\bar{\mathbf{E}}\mathbf{n}_t \end{cases} \tag{33}$$

which is represented in the incremental form as follows:

$$\begin{cases} d\bar{\mathbf{u}} = \bar{\mathbf{E}}^{-1}d\mathbf{f} + \bar{\Gamma}\mathbf{n}_t dt \\ d\mathbf{f} = \bar{\mathbf{E}}d\bar{\mathbf{u}} - \bar{\Gamma}\bar{\mathbf{E}}\mathbf{n}_t dt \end{cases} \tag{34}$$

5 CALCULATION PROCEDURE

The calculation by the subloading-overstress friction model may be performed by the following procedure.

1. $d\bar{\mathbf{u}}^{vp}$ is calculated by the input of dt into Eq. (25), and then its accumulation leads to $\bar{\mathbf{u}}^{vp}$.
2. $d\mu$ and dr_s are calculated by inputs of $d\bar{\mathbf{u}}^{vp}$ into Eqs. (24) and (30), and their accumulations lead to μ and r_s.
3. The elastic sliding displacement $\bar{\mathbf{u}}^{e}$ is calculated by $\bar{\mathbf{u}}^{e} = \bar{\mathbf{u}} - \bar{\mathbf{u}}^{vp}$.
4. $\mathbf{f}$ is calculated by $\mathbf{f} = \bar{\mathbf{E}}\bar{\mathbf{u}}^{e}$.
5. r is calculated by $r = (f_t / f_n)/\mu$.

These calculation processes are repeated for the further sliding.

6 COMPARISON WITH TEST DATA

The comparison of the simulation by the subloading-overstress friction model with the test data [5] for the dry friction is shown in Figure 3. The test curve for sliding between roughly polished steel surfaces under the quite low sliding velocity $\dot{\bar{u}}_t \leq 0.0002\,\mathrm{mm/s}$ is simulated well enough by the present model, where the material parameters are selected as follows:

$$\mu_s = 0.58,\ \mu_k = 0.38, \kappa = 35\mathrm{mm}^{-1},\ \ \xi = 0.001/\mathrm{s}$$

$$\tilde{u} = 1500\mathrm{mm}^{-1}, \mu_v = 150,\ r_m = 2.0, n = 3$$

$$\alpha_n = \alpha_t = 10000\mathrm{N/mm}^3$$

under the condition

$$f_n = 10\mathrm{MPa},\ \dot{\bar{u}}_t = 0.0002\mathrm{mm/s}$$

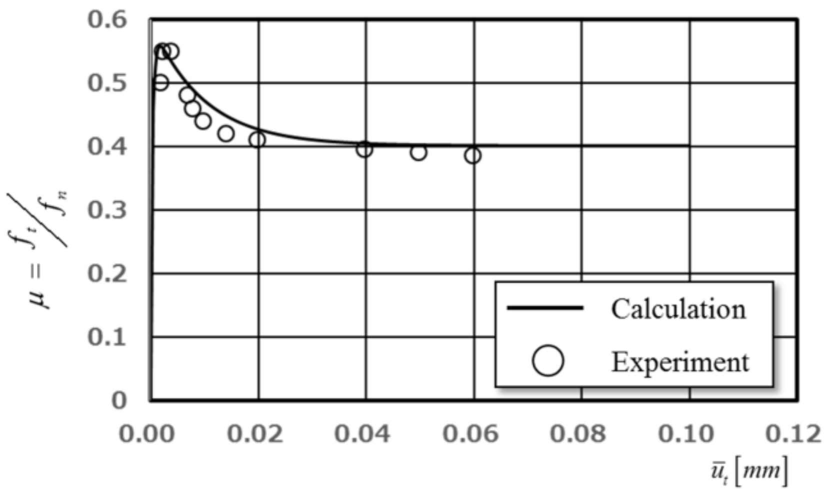

Figure 3 Comparison with test data [5] for dry friction.

The comparison of the simulation by the present model with the test data [6] for the fluid friction is shown in Figure 4. The test plate is the galvannealed steel sheet which is sandwiched by the steel SKD-11 plates. The normal contact stress is 5.56 MPa. The friction surfaces were coated with the anti-rust oil prior to the tests. The drawing velocity of the test plate is set at the five levels 1, 10, 50, 100, 200 mm/min. The simulation of the test result using the exponential function in Eq. (29) is shown by the solid lines, using the following values for the material constants.

$$\mu_s = 0.1,\ \mu_k = 0.09, \kappa = 0.2\mathrm{mm}^{-1},\ \ \xi = 0.0009\,/\,\mathrm{s}$$
$$\tilde{u} = 80\mathrm{mm}^{-1}, \mu_v = 10,\ r_m = 1.4, n = 8$$
$$\alpha_n = \alpha_t = 1000\mathrm{MPa}\,/\,\mathrm{mm}$$

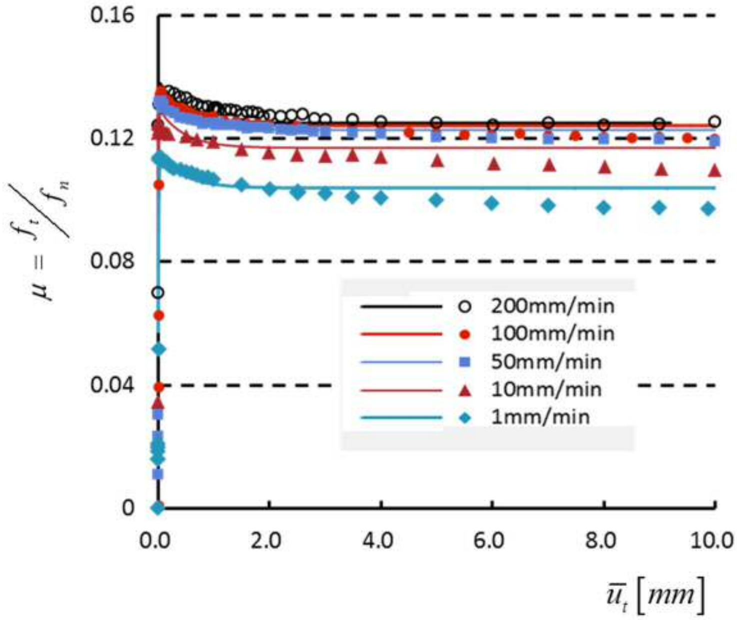

Figure 4 Comparison with test data for fluid friction.

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