GENERAL CASE OF MOVEMENT OF SOLID SYSTEM WITH TWO MASSIVE ECCENTRICS ON A ROUGH PLANE

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Abstract. The mechanism consisting of the tripod with two rotating eccentrics as internal movers is investigated. The tripod moves with dry friction on horizontal plane. Rotating massive eccentrics enable the tripod to slide and spin. In mathematical model general equations of motion are considered for sliding and spinning of the tripod.

1 INTRODUCTION

In two previous papers [1-2], a solid system with two massive eccentrics, standing on a rough surface as a tripod, was considered in two special cases: purely translational motion without rotation and purely rotational motion with one fixed support point. In this paper a general case is considered where the system can move in a plane without restriction on the type of motion.

The main result of this work is that the equations of motion for the general case of motion of a tripod with two eccentrics are obtained.

The main feature of this study is that there is an experimental stand that allows you to study the behaviour of the mechanical system in full-scale tests.

The significance of the work is that the direction of research – movement due to internal movers and systems with three points of contact with surface – is quite interesting for many researchers [3-9].

2 MECHANISM

The solid system with two massive eccentrics on a rough plane is presented on the Fig. 1. It is the mechanism that consists of mechanical and electronical parts. Mechanical parts and motors are taken from Makeblock. Microcontroller and motor driver are Arduino compatible.

3 MATHEMATICAL MODEL

For consideration of our system it is convenient to divide the system into three components interacted with each other – a tripod and two pendulums. View from above is presented on the Fig. 2. The tripod has three points of contact with horizontal surface in points A, B, C. Pendulum OP has cylindrical hinge in point O and rotates around ξ-axis (Fig. 2, 3). Pendulum SQ has cylindrical hinge in point S and rotates around η-axis (Fig. 2, 4).

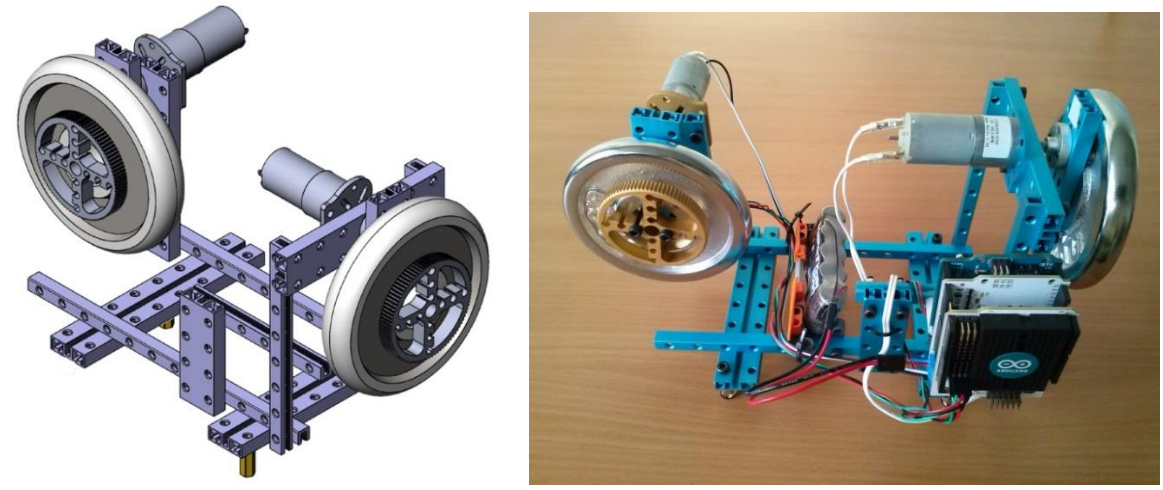

Figure 1: CAD model of the mechanism and the assembled mechanism

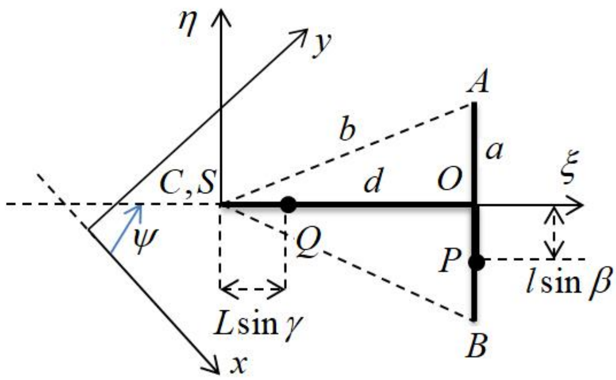

Figure 2: Tripod with two pendulums. View from above

System of axes $\xi\eta\zeta$ is connected to the tripod and moves with it. System of axes xyz is connected to the surface and does not move. Hence, $\xi\eta\zeta$ is non-inertial, xyz is inertial. Cylindrical hinges O and S are above the surface at the heights h and H, respectively. Lengths of pendulums OP and SQ are l and L, respectively. Geometrical parameters of the tripod in horizontal plane are described by a, b, d.

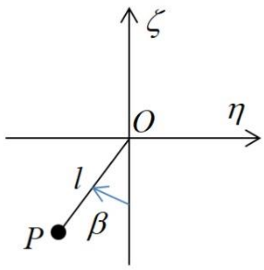

Figure 3: Pendulum P in cylindrical hinge O. View from ξ-axis

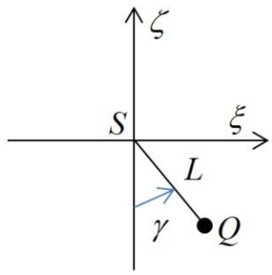

Figure 4: Pendulum Q in cylindrical hinge S. View from η-axis

The movement in horizontal plane of point C of the tripod is determined by variables x, y. Rotation of the tripod is determined by variable Ψ. Rotation of pendulums OP and SQ is determined by β and γ, respectively.

Now let us consider the general case of mechanism's movement on the rough surface when the pendulums rotate and the tripod moves in plane with sliding and spinning and without jumping above the surface.

For the tripod we can write equation of forces in the inertial reference system:

$$\overline{F}_A + \overline{F}_B + \overline{F}_C + \overline{N}_A + \overline{N}_B + \overline{N}_C + M\overline{g} + \overline{R}_O + \overline{R}_S = M\overline{W}_C, \tag{1}$$

where $\overline{F}_A, \overline{F}_B, \overline{F}_C$ – friction forces in points A, B, C; $\overline{N}_A, \overline{N}_B, \overline{N}_C$ – normal reactions in points A, B, C; $M\overline{g}$ – gravity force; $\overline{R}_O$ – reaction in point O, where the tripod is connected with the pendulum OP through a cylindrical hinge O; $\overline{R}_S$ – reaction in point S, where the tripod is connected with the pendulum SQ through a cylindrical hinge S; C – center of inertia of the tripod (for simplicity); $\overline{W}_C$ – acceleration of point C.

In projections on axes $\xi\eta\zeta$ equation (1) looks like:

$$\begin{bmatrix} F_{A\xi} + F_{B\xi} + F_{C\xi} - R_{O\xi} - R_{S\xi} \\ F_{A\eta} + F_{B\eta} + F_{C\eta} - R_{O\eta} - R_{S\eta} \\ N_A + N_B + N_C - R_{O\zeta} - R_{S\zeta} - Mg \end{bmatrix} = \begin{bmatrix} MW_{C\xi} \\ MW_{C\eta} \\ 0 \end{bmatrix}. \tag{2}$$

Now let us write for an angular momentum of the tripod relative to its center of inertia:

$$\frac{d\overline{K}_C^{tripod}}{dt} = \overline{CA} \times \left(\overline{N}_A + \overline{F}_A\right) + \overline{CB} \times \left(\overline{N}_B + \overline{F}_B\right) + \overline{CO} \times \overline{R}_O + \overline{CS} \times \overline{R}_S + \overline{\mu}_O + \overline{\mu}_S, \tag{3}$$

where $\overline{K}_C^{tripod} = J\dot{\psi}\overline{e}_\zeta$ – angular momentum of tripod with respect to the point C; $\overline{\mu}_O, \overline{\mu}_S$ – moments of reaction to the electric motors, rotating pendulums in hinges O, S.

For the vectors in (3) we have:

$$\overline{CA} = \begin{bmatrix} d & a & 0 \end{bmatrix}^T, \ \overline{CB} = \begin{bmatrix} d & -a & 0 \end{bmatrix}^T,$$
$$\overline{CO} = \begin{bmatrix} d & 0 & h \end{bmatrix}^T, \ \overline{CS} = \begin{bmatrix} 0 & 0 & H \end{bmatrix}^T,$$
$$\overline{\mu}_O = \begin{bmatrix} \mu_O & 0 & 0 \end{bmatrix}^T, \ \overline{\mu}_S = \begin{bmatrix} 0 & \mu_S & 0 \end{bmatrix}^T.$$

After calculating vector products in (3) we get:

$$\begin{bmatrix} a\left(N_A - N_B\right) + hR_{O\eta} + HR_{S\eta} + \mu_O \\ -d\left(R_{O\zeta} - N_A - N_B\right) - hR_{O\xi} - HR_{S\xi} + \mu_S \\ d\left(F_{A\eta} + F_{B\eta} - R_{O\eta}\right) + a\left(F_{B\xi} - F_{A\xi}\right) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ J\ddot{\Psi} \end{bmatrix}. \tag{4}$$

For the pendulum P in cylindrical hinge O we can write equation of forces in the non-inertial reference system:

$$\frac{d\overline{p}_{rel}^{pend\_P}}{dt} = \overline{R}^{ext\_P} + \overline{R}^{in1\_P} + \overline{R}^{in2\_P}, \tag{5}$$

where $\overline{p}_{rel}^{pend\_P}$ – momentum of pendulum P in non-inertial frame; $\overline{R}^{ext\_P}$ – the sum of the external forces acting on the pendulum P, $\overline{R}^{in1\_P}, \overline{R}^{in2\_P}$ – forces of inertia.

For the relative momentum of pendulum P with the mass m_P and the length l we have:

$$\overline{p}_{rel}^{pend\_P} = m_P \overline{V}_P^{rel} = m_P \overline{\omega}_{OP}^{rel} \times \overline{OP} = m_P \begin{bmatrix} 0 \\ -l\dot{\beta}\cos\beta \\ l\dot{\beta}\sin\beta \end{bmatrix},$$

where β – angle between the OP and the vertical ζ.

For external forces in (5):

$$\overline{R}^{ext\_P} = \overline{R}_O + m_P \overline{g} = \begin{bmatrix} R_{O\xi} \\ R_{O\eta} \\ R_{O\zeta} - m_P g \end{bmatrix},$$

where $\overline{R}_O$ here for pendulum P is opposite to the $\overline{R}_O$ in the equations for tripod.

For inertia force $\overline{R}^{in1\_P}$ we have:

$$\overline{R}^{in1\_P} = -m_P\left(\overline{W}_C + \ddot{\psi}\overline{e}_\zeta \times \overline{CP} + \dot{\psi}\overline{e}_\zeta \times \left(\dot{\psi}\overline{e}_\zeta \times \overline{CP}\right)\right) = -m_P \begin{bmatrix} W_{C\xi} - \dot{\psi}^2 d + \ddot{\psi} l \sin\beta \\ W_{C\eta} + \dot{\psi}^2 l \sin\beta + \ddot{\psi} d \\ 0 \end{bmatrix},$$

and for inertia force $\overline{R}^{in2\_P}$:

$$\overline{R}^{in2\_P} = -2m_P \dot{\psi}\overline{e}_\zeta \times \overline{V}_P^{rel} = -m_P \begin{bmatrix} 2\dot{\psi}\dot{\beta} l \cos\beta \\ 0 \\ 0 \end{bmatrix}.$$

So finally for the relative momentum of pendulum P after calculating derivatives:

$$m_P \begin{bmatrix} 0 \\ -\ddot{\beta} l \cos\beta + \dot{\beta}^2 l \sin\beta \\ \ddot{\beta} l \sin\beta + \dot{\beta}^2 l \cos\beta \end{bmatrix} = \begin{bmatrix} R_{O\xi} - m_P\left(W_{C\xi} - \dot{\psi}^2 d + \ddot{\psi} l \sin\beta + 2\dot{\psi}\dot{\beta} l \cos\beta\right) \\ R_{O\eta} - m_P\left(W_{C\eta} + \dot{\psi}^2 l \sin\beta + \ddot{\psi} d\right) \\ R_{O\zeta} - m_P g \end{bmatrix}.$$

From here we can express reaction in hinge O:

$$\begin{bmatrix} R_{O\xi} \\ R_{O\eta} \\ R_{O\zeta} \end{bmatrix} = \begin{bmatrix} m_P\left(W_{C\xi} - \dot{\psi}^2 d + \ddot{\psi} l \sin\beta + 2\dot{\psi}\dot{\beta} l \cos\beta\right) \\ m_P\left(W_{C\eta} - \ddot{\beta} l \cos\beta + \dot{\beta}^2 l \sin\beta + \dot{\psi}^2 l \sin\beta + \ddot{\psi} d\right) \\ m_P\left(\ddot{\beta} l \sin\beta + \dot{\beta}^2 l \cos\beta + g\right) \end{bmatrix}. \tag{6}$$

For the pendulum Q in cylindrical hinge S we can write equation of forces in the non-inertial reference system:

$$\frac{d\bar{p}_{rel}^{pend\_Q}}{dt} = \bar{R}^{ext\_Q} + \bar{R}^{in1\_Q} + \bar{R}^{in2\_Q}, \tag{7}$$

where $\bar{p}_{rel}^{pend\_Q}$ – momentum of pendulum Q in non-inertial frame; $\bar{R}^{ext\_Q}$ – the sum of the external forces acting on the pendulum Q, $\bar{R}^{in1\_Q}, \bar{R}^{in2\_Q}$ – forces of inertia.

For the relative momentum of pendulum Q with the mass m_Q and the length L we have:

$$\bar{p}_{rel}^{pend\_Q} = m_Q \bar{V}_Q^{rel} = m_Q \bar{\omega}_{SQ}^{rel} \times \overline{SQ} = m_Q \begin{bmatrix} L\dot{\gamma}\cos\gamma \\ 0 \\ L\dot{\gamma}\sin\gamma \end{bmatrix},$$

where γ – angle between the SQ and the vertical ζ.

For external forces in (7):

$$\bar{R}^{ext\_Q} = \bar{R}_S + m_Q \bar{g} = \begin{bmatrix} R_{S\xi} \\ R_{S\eta} \\ R_{S\zeta} - m_Q g \end{bmatrix},$$

where $\bar{R}_S$ here for pendulum Q is opposite to the $\bar{R}_S$ in the equations for tripod.

For inertia force $\bar{R}^{in1\_Q}$ we have:

$$\bar{R}^{in1\_Q} = -m_Q\left(\bar{W}_C + \ddot{\psi}\bar{e}_\zeta \times \overline{CQ} + \dot{\psi}\bar{e}_\zeta \times \left(\dot{\psi}\bar{e}_\zeta \times \overline{CQ}\right)\right) = -m_Q \begin{bmatrix} W_{C\xi} - \dot{\psi}^2 L \sin\gamma \\ W_{C\eta} + \ddot{\psi} L \sin\gamma \\ 0 \end{bmatrix}$$

and for inertia force $\bar{R}^{in2\_Q}$:

$$\bar{R}^{in2\_Q} = -2m_Q \dot{\psi}\bar{e}_\zeta \times \bar{V}_Q^{rel} = -m_Q \begin{bmatrix} 0 \\ 2\dot{\psi}\dot{\gamma} l \cos\gamma \\ 0 \end{bmatrix}.$$

So finally for the relative momentum of pendulum Q after calculating derivatives:

$$m_Q \begin{bmatrix} \ddot{\gamma} L \cos\gamma - \dot{\gamma}^2 L \sin\gamma \\ 0 \\ \ddot{\gamma} L \sin\gamma + \dot{\gamma}^2 L \cos\gamma \end{bmatrix} = \begin{bmatrix} R_{S\xi} - m_Q\left(W_{C\xi} - \dot{\psi}^2 L \sin\gamma\right) \\ R_{S\eta} - m_Q\left(W_{C\eta} + \ddot{\psi} L \sin\gamma + 2\dot{\psi}\dot{\gamma} L \cos\gamma\right) \\ R_{S\zeta} - m_S g \end{bmatrix}.$$

From here we can express reaction in hinge S:

$$\begin{bmatrix} R_{S\xi} \\ R_{S\eta} \\ R_{S\zeta} \end{bmatrix} = \begin{bmatrix} m_Q\left(\ddot{\gamma}L\cos\gamma - \dot{\gamma}^2 L\sin\gamma + W_{C\xi} - \dot{\psi}^2 L\sin\gamma\right) \\ m_Q\left(W_{C\eta} + \ddot{\psi}L\sin\gamma + 2\dot{\psi}\dot{\gamma}L\cos\gamma\right) \\ m_Q\left(\ddot{\gamma}L\sin\gamma + \dot{\gamma}^2 L\cos\gamma + g\right) \end{bmatrix}. \tag{8}$$

Let's return to the equation (4). From the first and the second line in (4) we have the system for normal reactions in points A, B:

$$\begin{cases} N_B - N_A = \dfrac{1}{a}\left(hR_{O\eta} + HR_{S\eta} + \mu_O\right) \\ N_B + N_A = R_{O\zeta} + \dfrac{1}{d}\left(\mu_S - hR_{O\xi} - HR_{S\xi}\right) \end{cases}$$

Solving this system we obtain:

$$\begin{aligned} N_A &= \frac{1}{2}\left(R_{O\zeta} + \frac{1}{d}\left(\mu_S - hR_{O\xi} - HR_{S\xi}\right) - \frac{1}{a}\left(hR_{O\eta} + HR_{S\eta} + \mu_O\right)\right), \\ N_B &= \frac{1}{2}\left(\frac{1}{a}\left(hR_{O\eta} + HR_{S\eta} + \mu_O\right) + R_{O\zeta} + \frac{1}{d}\left(\mu_S - hR_{O\xi} - HR_{S\xi}\right)\right). \end{aligned} \tag{9}$$

These normal reactions we can substitute in the third line of (2) where:

$$N_C = Mg + R_{O\zeta} + R_{S\zeta} - N_A - N_B. \tag{10}$$

Let us write equation for angular momentum of the pendulum P in non-inertial reference frame $O\xi\eta\zeta$:

$$\frac{d\bar{K}_{O\,rel}^{pend\_P}}{dt} = \bar{M}_O^{ext\_P} + \bar{M}_O^{in1\_P} + \bar{M}_O^{in2\_P}, \tag{11}$$

where $\bar{K}_{O\,rel}^{pend\_P} = -m_P l^2 \dot{\beta}\bar{e}_\xi$ – relative angular momentum of the pendulum; $\bar{M}_O^{ext\_P}$ – moment of external forces; $\bar{M}_O^{in1\_P}, \bar{M}_O^{in2\_P}$ – moments of inertia forces. All these moments are taken with respect to the point O.

For the derivative in left we have:

$$\frac{d\bar{K}_{O\,rel}^{pend\_P}}{dt} = -m_P l^2 \ddot{\beta}\bar{e}_\xi.$$

For $\bar{M}_O^{ext\_P}$ we have:

$$\bar{M}_O^{ext\_P} = \overline{OP} \times m_P\bar{g} + \bar{\mu}_O = \left(m_P gl\sin\beta - \mu_O\right)\bar{e}_\xi,$$

where $\bar{\mu}_O$ – moment of the electric motor rotating pendulum P – here it is opposite to the $\bar{\mu}_O$ in equation for tripod (3).

For $\bar{M}_O^{in1\_P}$ we have:

$$\bar{M}_O^{in1\_P} = \overline{OP} \times \bar{R}^{in1\_P} = m_P l \begin{bmatrix} -\cos\beta\left(W_{C\eta} + \dot{\psi}^2 l \sin\beta + \ddot{\psi} d\right) \\ \cos\beta\left(W_{C\xi} - \dot{\psi}^2 d + \ddot{\psi} l \sin\beta\right) \\ -\sin\beta\left(W_{C\xi} - \dot{\psi}^2 d + \ddot{\psi} l \sin\varphi\right) \end{bmatrix}.$$

For $\bar{M}_O^{in2\_P}$ we have:

$$\bar{M}_O^{in2\_P} = \overline{OP} \times \bar{R}^{in2\_P} = m_P l^2 \begin{bmatrix} 0 \\ 2\dot{\psi}\dot{\beta}\cos^2\beta \\ -2\dot{\psi}\dot{\beta}\sin\beta\cos\beta \end{bmatrix}.$$

Substituting these expressions into equation (11), we obtain:

$$\begin{bmatrix} -m_P l^2 \ddot{\beta} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} m_P g l \sin\beta - \mu_O - m_P l \cos\beta\left(W_{C\eta} + \dot{\psi}^2 l \sin\beta + \ddot{\psi} d\right) \\ m_P l \cos\beta\left(W_{C\xi} - \dot{\psi}^2 d + \ddot{\psi} l \sin\beta + 2l\dot{\psi}\dot{\beta}\cos\beta\right) \\ m_P l \sin\beta\left(W_{C\xi} - \dot{\psi}^2 d + \ddot{\psi} l \sin\beta + 2l\dot{\psi}\dot{\beta}\cos\beta\right) \end{bmatrix}. \tag{12}$$

Let us write equation for angular momentum of the pendulum Q in non-inertial reference frame $S\xi\eta\zeta$:

$$\frac{d\bar{K}_{S\,rel}^{pend\_Q}}{dt} = \bar{M}_S^{ext\_Q} + \bar{M}_S^{in1\_Q} + \bar{M}_S^{in2\_Q}, \tag{13}$$

where $\bar{K}_{S\,rel}^{pend\_Q} = -m_Q L^2 \dot{\gamma} \bar{e}_\eta$ – relative angular momentum of the pendulum; $\bar{M}_S^{ext\_Q}$ – moment of external forces; $\bar{M}_S^{in1\_Q}, \bar{M}_S^{in2\_Q}$ – moments of inertia forces. All these moments are taken with respect to the point S.

For the derivative in left we have:

$$\frac{d\bar{K}_{S\,rel}^{pend\_Q}}{dt} = -m_Q L^2 \ddot{\gamma} \bar{e}_\eta.$$

For $\bar{M}_S^{ext\_Q}$ we have:

$$\bar{M}_S^{ext\_Q} = \overline{SQ} \times m_Q \bar{g} + \bar{\mu}_S = \left(m_Q g L \sin\gamma - \mu_S\right)\bar{e}_\eta,$$

where $\bar{\mu}_S$ – moment of the electric motor rotating pendulum Q – here it is opposite to the $\bar{\mu}_S$ in equation for the tripod (3).

For $\bar{M}_S^{in1\_Q}$ we have:

$$\bar{M}_S^{in1\_Q} = \overline{SQ} \times \bar{R}^{in1\_Q} = m_Q L \begin{bmatrix} -\cos\gamma\left(W_{C\eta} + \ddot{\psi} L \sin\gamma\right) \\ \cos\gamma\left(W_{C\xi} - \dot{\psi}^2 L \sin\gamma\right) \\ -\sin\gamma\left(W_{C\eta} + \ddot{\psi} L \sin\gamma\right) \end{bmatrix}.$$

For $\bar{M}_S^{in2\_Q}$ we have:

$$\bar{M}_S^{in2\_Q} = \overline{SQ} \times \bar{R}^{in2\_Q} = m_Q L^2 \begin{bmatrix} -2\dot{\psi}\dot{\gamma}\cos^2\gamma \\ 0 \\ -2\dot{\psi}\dot{\gamma}\sin\gamma\cos\gamma \end{bmatrix}.$$

Substituting these expressions into equation (13), we obtain:

$$\begin{bmatrix} 0 \\ -m_Q L^2 \ddot{\gamma} \\ 0 \end{bmatrix} = \begin{bmatrix} -m_Q L\cos\gamma\left(W_{C\eta} + \ddot{\psi} L\sin\gamma + 2L\dot{\psi}\dot{\gamma}\cos\gamma\right) \\ m_Q g L\sin\gamma - \mu_S + m_Q L\cos\gamma\left(W_{C\xi} - \dot{\psi}^2 L\sin\gamma\right) \\ m_Q L\sin\gamma\left(W_{C\eta} + \ddot{\psi} L\sin\gamma + 2L\dot{\psi}\dot{\gamma}\cos\gamma\right) \end{bmatrix}. \tag{14}$$

The dynamics of the electric motors can be described by the equations:

$$\begin{cases} \tau_O \dfrac{d\mu_O}{dt} + \mu_O = b_{O1} U_O - b_{O2}\dot{\beta} \\ \tau_S \dfrac{d\mu_S}{dt} + \mu_S = b_{S1} U_O - b_{S2}\dot{\gamma} \end{cases} \tag{15}$$

where μ_O, μ_S – torque of electric motors in hinges O, S; $\tau_O, b_{O1}, b_{O2}, \tau_S, b_{S1}, b_{S2},$ – electric motors constants; U_O, U_S – voltage applied to the motors; and also:

$$\tau_O, b_{O1}, b_{O2}, \tau_S, b_{S1}, b_{S2}, U_O, U_S, \dot{\beta}, \dot{\gamma} > 0.$$

4 CONCLUSIONS

- General equations of motion are obtained for the mechanism consisting of the tripod and two pendulums as internal movers.
- General case of motion of the tripod is considered on a rough horizontal plane – the tripod moves not only translationally and not only rotationally but with sliding and spinning at the same time.

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