A PARAMETRIC TRANSFER FUNCTION FOR REAL-TIME SIMULATION OF COUPLED COMPLEX PROBLEMS

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Abstract. Industrial production lines often involve multistage manufacturing processes with coupled boundary conditions. The output of a process is the input of another processing stage. The end product of such production line is complicated to optimize since its simulation includes countless number of parameters and degrees of freedom. Therefore, incorporating all the end product parameters as extra coordinates of the problem is still an intractable approach, despite the recent advances in computation power and model order reduction techniques.

In this work, we explore an alternative approach using a physically based mechanical transfer function method, which expresses all the physics of the problem in a single function. All part external effects, including boundary conditions for example, become an input of such function. The output result of the proposed function is a real-time simulation of the consider product, for any possible input set of parameters.

1 INTRODUCTION

Numerical simulation is gaining its place at a fast pace in the industrial field [6]. The aim of accurate and optimized manufacturing processes is pushing the manufacturer to adopt the numerical simulation as an uncircumventable numerical tool in their design applications. However, multiple industrial production lines involve multistage manufacturing processes with coupled boundary conditions. The output of a process is the input of another one. Such simulations are still intractable with the current simulation techniques, even with the current progress in computing power. In fact, the solution of a second process in a production line may live in an extremely large dimensionality space, since the first process output value at each node can be considered as an extra parameter of the problem.

Model order reduction techniques are an appealing solution in this situation, since the problem's number of degrees of freedom scales linearly with respect to the problem dimensionality [5, 11, 2]. The model order reduction techniques were successfully used in different multidimensional problems [7, 8, 4]. However, even with the current model reduction techniques of high dimensionality space, the aforementioned problems with coupled boundary conditions are still intractable. In fact, the dimensionality of the problems explode rapidly, and the model order reduction techniques face « saturation » and thus converge slowly or fail. For example, a multistage metal forming process should consider the residual stresses at each point when coming out of stage i, as an extra coordinate of the deformation problem faced at stage $i + 1$, to cover all the possible deformations after stage $i+1$. Such approach is prohibitive and involves countless dimensions of the problem. Previous works tried to circumvent this problem by considering the relevant coordinates in a subspace with implicit coordinate system [9, 10]. However, the implicit coordinate system is not trivial to construct and may not exist for every situation. Moreover, identifying these coordinates requires a deep knowledge of the process at hand. Other attempts where do to parametrize the output of process i using the input parameters of the same process i [1].

In this work, we aim at circumventing the complexity of the problem and the dimensionality explosion, by creating a transfer function approach for each manufacturing stage. The transfer function approach shall model the physics inside the material, however all external effects like the loads, boundary conditions... will become an input to the transfer functions. Classical works on transfer functions use the Laplace transformation to find the solution of differential problems for relatively simple cases [3]. In this work, we create a transfer function approach based on the discretized problem stiffness and/or mass matrix, that results in real-time simulation of the depicted problem.

2 Transfer function approach for fixed boundary conditions

In this section, for sake of simplicity, we consider the illustrative example of a 1D steady-state heat transfer problem :

$$-k\frac{\partial^2 T}{\partial x^2} = F \tag{1}$$

where k the thermal conductivity and T is the temperature field in a domain $x \in [0; L]$. The discretized form of the problem is obtained such as :

$$k\mathcal{A}\mathbf{T} = \mathbf{B} \tag{2}$$

Where $\mathcal{A}$ is the discretized matrix of the Laplacian illustrated in equation (1), $\mathbf{T}$ the vector of discretized temperature field values and $\mathbf{B}$ the vector of the discretized second hand side of the equation. Classical model reduction approaches tend to solve the differential equation of the problem using the parameters of the problem as extra coordinates, for instance if F is a polynomial function of degree 2

$$F = a + bx + cx^2 \tag{3}$$

each coefficient of the polynomial terms a, b and c becomes an extra coordinate of the problem. Considering the conductivity as extra coordinate, one have to solve a 5D problem, inside chosen intervals for each parameter, such as :

$$T = f(x, a, b, c, k) \tag{4}$$

Now Considering the following boundary conditions :

$$\begin{cases} T(0) = 0 \\ T(L) = 0 \end{cases} \tag{5}$$

One can find $\mathcal{R}$, the reduced matrix form of $\mathcal{A}$. Using $\mathcal{R}$, One can write :

$$\mathbf{T} = \frac{1}{k} \cdot \mathcal{R}^{-1} \cdot B_r \tag{6}$$

where B_r is the reduced form of the second hand size of the discretized problem form B. Since all the parameters appears explicitly in equation (6), it can be used in real time to solve the problem for any chosen value of the parameters k, a, b or c. This approach does not require a parameter intervals predefinition for k, a, b and c. Thus, the solution can be literally found in real-time for any value of the chosen parameters. Moreover, the physical properties of the problem are now incorporated into $\mathcal{R}^{-1}$, while all the external effects are fixed on the fly.
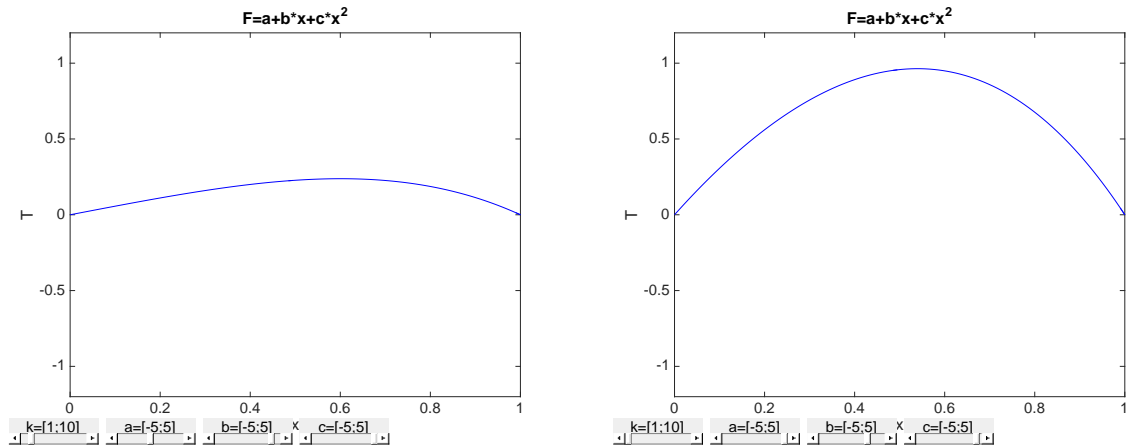
Figure 1: Figure illustrating a Matlab GUI slider showing the results of equation (1) in real time for any variation of the input parameters
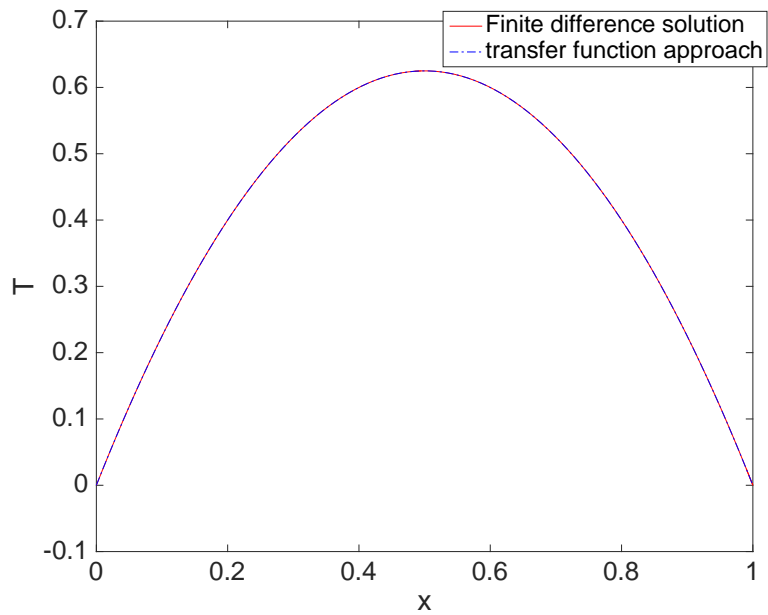

Figure 2: Figure comparing the classical finite difference solution to the illustrated transfer function approach solution

Figure 1 illustrates a Matlab GUI plugin which uses sliders to change the values of the input parameters and illustrates in real time the solution of the problem. Figure 2 compares the solution of the problem illustrated by the given approach, with the finite differences classical solution.

The disadvantage of this approach is the impossibility of inversing the stiffness matrix $\mathcal{A}$ without prescribing the boundary conditions. Boundary conditions are external parameters and one may wish to list them as an input to the transfer function. In such case, no reduced matrix $\mathcal{R}$ can be computed and the matrix $\mathcal{A}$, being singular, can't be inversed. Thus, a different approach shall be considered.

3 Transfer function approach with Dirichelet boundary conditions as function input

In this section we illustrate a novel approach to solve the problem while considering the Dirichlet boundary conditions as extra coordinates of the problem. We consider the heat transfer problem depicted in equation (1), with the prescribed Dirichelet boundary conditions :

$$\begin{cases} T(0) = T_0 \\ T(L) = T_L \end{cases} \tag{7}$$

For sake of simplicity, we consider the conductivity $k = 1$ in the following example. The depicted boundary conditions are extra coordinates of the problem and thus shall not be prescribed to the problem before inversing the matrix. The original discretized system is written as :

$$\mathcal{A} \cdot \begin{pmatrix} T_0 \\ T_1 \\ \vdots \\ T_L \end{pmatrix} = \mathbf{B} + \begin{pmatrix} -f_0 \\ 0 \\ \vdots \\ f_L \end{pmatrix} \tag{8}$$

where f_0 and f_L are the prescribed heat flux on each of the two sides on the domain, $x = 0$ and $x = L$ respectively. f_0 and f_L are unknowns « a priori » and can be found as a post-processing of the discrete problem. However, f_0 and f_L should satisfy the first and last equation of the linear system :

$$\begin{cases} \sum_{i=1}^{\mathbb{N}} \mathcal{A}_{1i} \mathbf{T}_i = \mathbf{B}_1 - f_0 \\ \sum_{i=1}^{\mathbb{N}} \mathcal{A}_{\mathbb{N}i} \mathbf{T}_i = \mathbf{B}_{\mathbb{N}} + f_L \end{cases} \tag{9}$$

$\mathbb{N}$ being the number of nodes in the domain. To define the solution of the problem, we define the matrix $\mathcal{M}$ as the pseudo-inverse of the matrix $\mathcal{A}$. Then we define the preliminary solution of the problem $\mathbf{P}$ by :

$$\mathbf{P} = \mathcal{M} \cdot (\mathbf{B} + \mathbf{f}) \tag{10}$$

where :

$$\mathbf{f} = \begin{pmatrix} -f_0 \\ 0 \\ \vdots \\ f_L \end{pmatrix} \tag{11}$$

Starting with a first random guess for f_0 and f_L, and defining the solution of the problem $\mathbf{T}$ as :

$$\mathbf{T} = \mathbf{P} + d_1 \cdot x + d_2 \tag{12}$$

where d_1 and d_2 two constants defining the potential linear transformation of the preliminary solution. We may define now a system of 4 equations with 4 unknowns :

$$\begin{cases} \sum_{i=1}^{\mathbb{N}} \mathcal{A}_{1i} \mathbf{T}_i = \mathbf{B}_1 - f_0 \\ \sum_{i=1}^{\mathbb{N}} \mathcal{A}_{\mathbb{N}i} \mathbf{T}_i = \mathbf{B}_{\mathbb{N}} + f_L \\ \mathbf{P}_1 + d_1 \cdot 0 + d_2 = T_0 \\ \mathbf{P}_2 + d_1 \cdot L + d_2 = T_L \end{cases} \tag{13}$$

where the unknowns are d_1, d_2, f_0 and f_L. Since discretized problems are linear in general, the system depicted in equation (13) is solved with only one iteration using Newton's algorithm. The solution is obtained almost instantly, for any prescribed boundary condition.

Figure 3 illustrates the solution for a system with $T_0 = 1$ and $T_L = 10$, with a constant heat generation term $F = 100$ in all the domain. The solution is compared to the finite difference one, the relative error does not exceed 1.26%. The solution is obtained within $31.7ms$ on a portable PC and the system depicted in equation (13) was solved with only one Newton's algorithm iteration.

To solve the illustrated problem online, in real-time, only the matrices $\mathcal{A}$ and its pseudo-inverse $\mathcal{M}$ should be stored on the simulation platform. All the other parameters, being external to the intrinsic properties of the material, can be considered as an input to the illustrated transfer functions method.

4 Conclusion

In this work we proved the possibility of using a transfer function approach to simulate in real time a problem where coupled boundary conditions are present, or when the boundary conditions of a problem are the result of another one's solution.
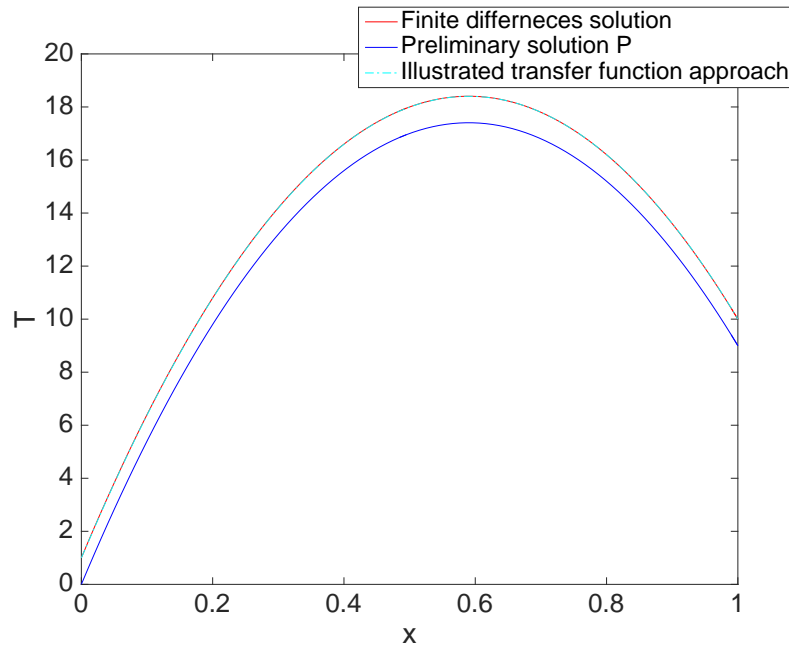
Figure 3: An example illustrating the boundary conditions as extra parameters of the problem, using the transfer function appraoch, and comparing the obtained solution to classical finite differences

The presented method uses the pseudo-inverse of the discretized stiffness matrix of the problem to derive the solution of the problem. The method starts with the identification of the heat flux prescribed on the Dirichelet boundary conditions, as well as the linear solution contribution with respect to a preliminary one, obtained directly from the pseudo-inverse matrix. The illustrated method offers the possibility to solve in real-time any problem whose inputs are coupled to another one's output, or when the inputs are unknown « a priori ». The described method has an advantage over classical model reduction formulations, especially when the number of degrees of freedom increases dramatically.

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