

INVESTIGATION OF INTERNAL EROSION PROCESSES USING A COUPLED DEM-FLUID METHOD

H. SARI*, B. CHAREYRE *, E. CATALANO*, P. PHILIPPE† AND E. VINCENS ‡

*Grenoble-INP , UJF-Grenoble 1, CNRS UMR 5521, 3SR
Grenoble F-38041, France
e-mail: bruno.chareyre@grenoble-inp.fr

† Cemagref – Geomechanics Group – OHAX.
Aix-en-Provence F-13182 , France

‡ Université de Lyon, LTDS Ecole Centrale de Lyon UMR CNRS 5513
Ecully F-69134, France

Key words: Erosion, Segregation, Seepage, Discrete Element Method, Stokes Flow.

Abstract. The evolution of granular beds subjected to upward seepage flow is investigated using a coupled DEM-fluid model implemented by Catalano et al. in the open-source software Yade-DEM. Firstly, filtration properties of a coarse narrowly graded material are analyzed by simulating the transport of smaller particles from a base layer through the coarse filter by gravitational loading or downward flow with uniform pressure gradient. The results are analysed on the basis of the *constriction size distribution* (CSD) of the filter which describes statistically the sizes of throats between pores in the material. Secondly, we examine the results obtained when, instead of two different layers, the coarse and fine materials are initially mixed in one unique layer and subjected to gravity. Thirdly, this mixture of coarse and fine particles is subjected to both gravity and a non-uniform pressure gradient, by injecting the fluid in one point below the layer, as inspired by previous experiments. Similar channeling patterns are obtained in both experiments and simulations when the boundary condition at the injection point is an imposed flux. This boundary condition results in a recirculation mechanism that remains confined in a finite zone around the injection point as long as the flux is below a threshold value. By simulating an imposed pressure condition, we finally show that instabilities can be triggered by the transport of small particles away from the injection point. This segregation process results in a lower porosity and an increased pressure gradient above the eroded zone, so that the instability-triggering pressure gradient in bi-dispersed mixtures is lower than in mono-dispersed mixtures.

1 INTRODUCTION

The stability of granular soils subjected to seepage flow is a major concern in civil engineering, particularly for the safety of hydraulic structure like dikes or dams. The difficulty to carry out experiments in the laboratory can be overcome thanks to the capability of DEM

simulations to model the process of transport of fine particles through a coarse granular material. In this paper, a DEM-fluid model, previously developed by Catalano et al. [1][2] in the open-source software Yade-DEM [3], is used to analyze the migration of fines particles in different configurations of granular beds (i.e. coarse and fine base particles either in two separate layers or mixed in a unique layer) and with various fluid flow conditions.

2 DEM-PFV COUPLING MODEL

The numerical model is based on the coupling between a discrete element model (DEM) for the solid particles and a flow model for the interstitial fluid. Particles are represented as spheres, and the interaction between particles are of the elastic-plastic type, with normal and tangential stiffness k_n and k_s , and Coulomb friction angle ϕ . Particles motion is governed by Newton's laws of acceleration and more details of the algorithms can be found in [4].

The fluid flow model is based on a pore-scale discretization of Stokes equations for incompressible fluids, where *pores* are defined by tetrahedra resulting from the regular Delaunay tessellation of the sphere packings. Although similar ideas have been proposed by previous authors [15,16], the derivation and implementation of a 3D formulation for deformable spheres packings have only been done recently [2].

The governing equations are averaged at the pore scale. The emerging formulation is of the finite volumes type, referred below as *PFV* (« Pore scale Finite Volumes »). At each time step, the geometry and rate of deformation of each pore is updated on the basis of particles motion. In turn, the fluxes are determined and fluid forces on particles are computed and integrated in the laws of motion for each particle. The only parameter of the flow problem is the viscosity of the fluid η , taken equal to that of pure water in what follows.

DEM-PFV coupling is well suited for studying particles transport induced by seepage flow, as opposed to the more common coupling methods where the resolution of the flow discretization is above the particles scale (DEM-CFD coupling, see e.g. [5]). Indeed, DEM-PFV models define forces on particles that are not collinear with the macroscale pressure but, instead, reflect the local voids geometry and connectivity. On the other hand DEM-CFD couplings can only define average fluid forces on groups of particles, and the direction of forces is imposed by the mean fluid velocity at the macroscale. The definition of the fluid forces in the DEM-PFV model has been validated in [1].

3 FILTRATION TEST WITH TWO LAYERS

3.1 Dry filtration under gravity

The filter, composed of 4000 spherical particles, is created thanks to a sphere growth process which produces a homogeneous sample with a uniform porosity [12]. The filter is generated with a friction angle equal to 10° between the coarse particles and the corresponding porosity is 0.35. The same value is kept in all the simulations. In the filter, all the particles are blocked and its thickness is about $25 \times D_{50}$ in the vertical direction and its width is $22 \times D_{50}$ in the two horizontal directions. In most of simulations, narrowly graded materials are used with a ratio between the maximal and the minimal diameter equal to 3 and 2 for the coarse and fine particles respectively. These ratios are similar to the ones used in previous suffusion experiments by Sail et al. [6] which are considered a benchmark for this

work. The fines particles (4000) initially at rest above the coarse layer, are dropped under gravity on the filter (fig.1). No particles are dropped near the lateral walls ($2 \times D_{\max}$) since at this distance the arrangement of particles within the filter is disturbed due to wall effects.

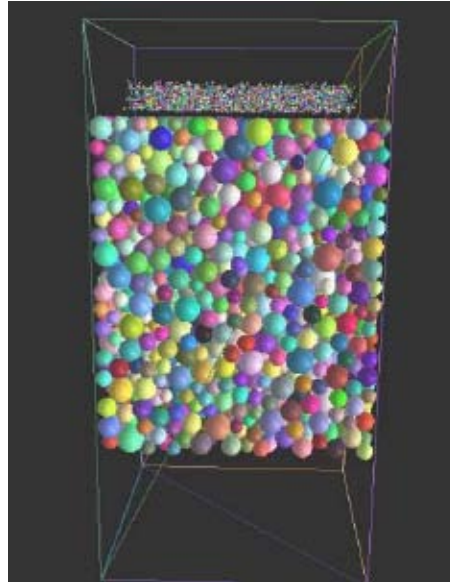


Figure 1: Filter of spherical particles and fine particles on the top surface.

Most of the literature about filter criteria as the well-known work by Sherard et al. [7] proposes to use the ratio D_{15}/d_{85} , which is called the Terzaghi criterion, recommended by ICOLD¹ for the design of filters. Following this approach, the filtration properties will be analyzed as a function of this ratio. The friction between the coarse and the fine particles is another important parameter and four different friction angles will be tested.

Fig.2 gives the percentage of the successful passing particles through the filter. One can note that increasing the local friction between the fine and coarse particles, there are more trapped spheres. This result was also found by Reboul (2008) [8] who considered a local friction equal to 0 for a conservative study of DEM filtration in granular materials. As the local friction angle increases, the fine particles loose kinematic energy more quickly. As a consequence, their ability to pass through a constriction and to continue their movement is more rapidly restricted. The distribution of trapped particles within the filter is compatible with an exponential distribution as can be seen in Fig. 3(a) where the different layers are numbered from the top to the bottom of the filter. Note that such exponential distributions are recovered in experiments, as the ones that have been carried out in Cemagref (Aix-en-Provence, France) for the TRANSOL² project thanks to an index-matching technique. A typical result is presented in Fig.3(b).

¹ International Commission of Large Dams

² Transport of mass through granular soil (2007-2010). National project financially supported by the French Agency of Research (ANR)

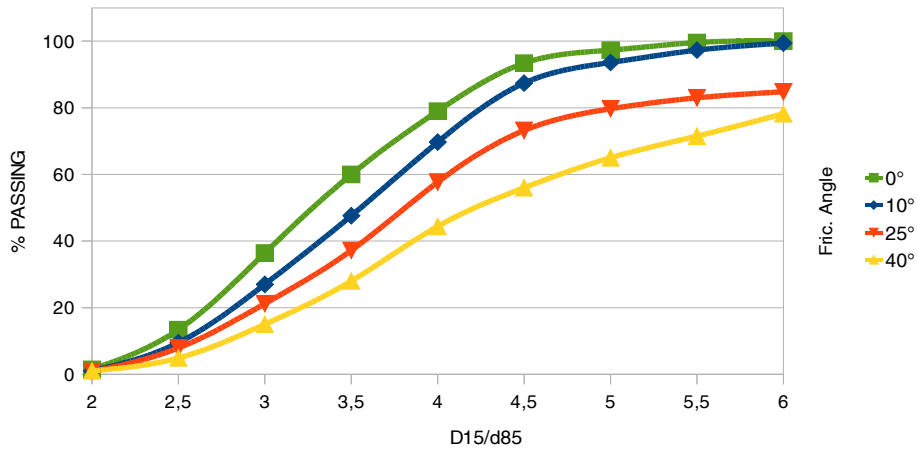


Figure 2: Passing particles according to D_{15}/d_{85} ratio and the friction between the fine and coarse particles.

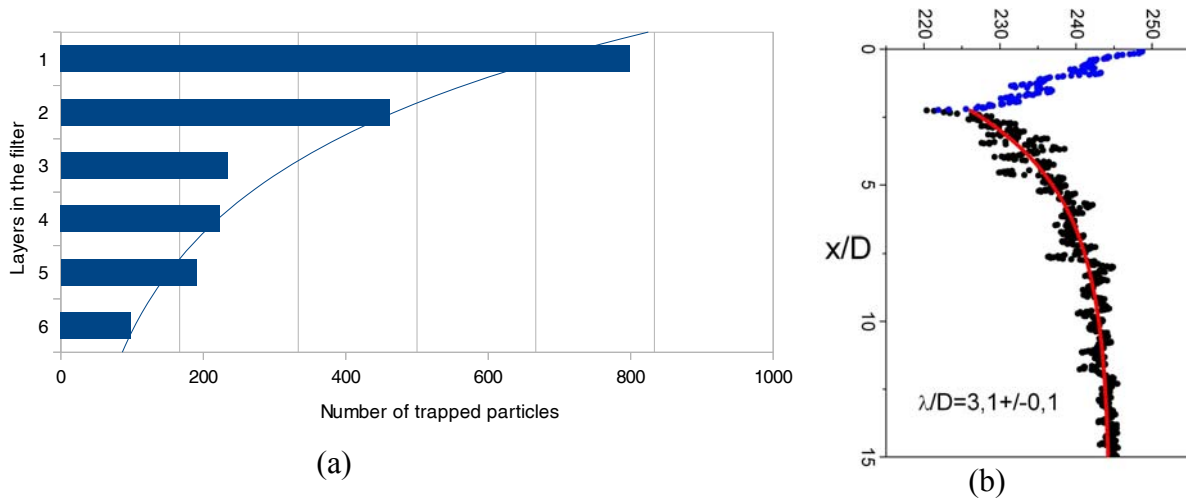


Figure 3: Vertical distribution of trapped particles in the filter; (a): DEM-PFV simulation, (b): experiment on glass beads, after [13].

Fig.4 shows the Constriction Size Distribution (CSD) of the filter and the Particle Size Distribution (PSD) of the base particles in different cases of filtration. The CSD criterion is used for the filtration properties because it is more precise than the Terzaghi criterion and better reflects the physical phenomena involved in filtration tests (Silveira (1965) [9], Witt (1993) [10], among others). Actually, the CSD reflects the geometrical retention capacity of a material since constrictions correspond to the narrowest paths between pores. For the computation of the CSD, a control volume excluding the filter particle which arrangement could be influenced by wall effects has been defined. We can see for example that for the fine's PSD giving 99 % of passing, nearly all the base particles are smaller than the smallest constriction, say the constriction made with the three smallest filter particles in mutual contact. One can also notice that the smaller constrictions which are more numerous in the medium are mainly responsible for trapping particles since only 20% of constrictions are responsible for trapping 50% of particles. For the 1% PSD, the smallest fine particle diameter is greater than the mode of the distribution which gives the value of the largest occurrence of

constriction size in the medium. Since the probability of having pores connected by constriction sizes greater than this mode is low, the probability of a successful transit across the filter decreases dramatically [8].

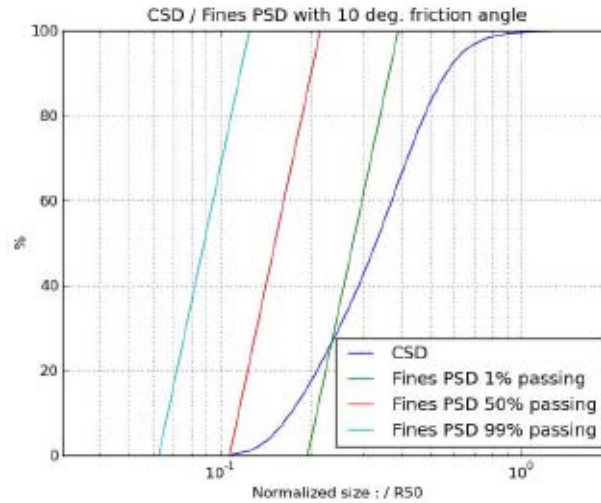


Figure 4: Particle Size Distribution (PSD) of fine particles associated to different passing percentage.

Table 1: Filter efficiency according to the ratio D_{15}/d_{85} .

Material	Cu	D_{15}/d_{85}	Filter	Indraratna et al [11]
Base particles	1,29			
Coarse filter	1,47	15,4	50 % passing	Ineffective
Medium filter	1,49	5,2	1 % passing	Effective

Other simulations were performed with other values of coefficient of uniformity C_u of the filter, i.e. other D_{max}/D_{min} . These are values chosen by Indraratna *et al* [11] (table 1). The thickness of their filter is nearly $200 \times D_{50}$ whereas, the one involved in this study is $25 \times D_{50}$. So, if we take into account this and the fact that the distribution is exponential, a filter which get through 50% of the particles with a thickness of $25 \times D_{50}$ must be ineffective, as found by Indraratna with a thickness of $200 \times D_{50}$. For 1 % of the particles passing by the filter, since the probability of further movement through the filter decreases with the filter thickness, it will be even more effective for a $200 \times D_{50}$ filter thickness.

3.2 Downward flow

In this section, the same kind of simulations is performed but gravity is removed and replaced by a downward hydraulic flow. To limit computational time, a smaller thickness of $8 \times D_{50}$ is used for the filter. The hydraulic flow is induced by imposing a pressure gradient of 150 Pa/m. This new situation is analyzed for two D_{15}/d_{85} ratios equal to 3 and 1.5. The

friction angle during the creation of the sample and between fine and coarse particles is 10° .

Table 2: Percentage of passing particles for dry filtration and involving a water flow.

D_{15}/d_{85}	% of passed particles	
	Gravity	Flow
3	66 %	84 %
1,5	1,3 %	5 %

Table 2 shows that the flow allows passing more numerous particles than with the gravity alone. This result is quite logical and it can be explained by the fact that we have a unique vertical force with the gravity whereas due to the tortuosity of the coarse particles filter, the flow also generates forces in the horizontal axes and allows the reorientation of the paths towards lateral constriction in pores. Figure 5 gives a more detailed view of the results given in Table 2.

Fig. 5(a) and 5(b) show the distributions of trapped particles either with the flow or with gravity and for the two D_{15}/d_{85} ratios. These distributions are obviously very different depending on the size ratio. At a given ratio, there is an almost systematic but very low decrease of trapped particles in each horizontal layer and the form of the dropped particle distribution is nearly the same between gravity and flow.

For $D_{15}/d_{85}=1.5$, as expected, most of particles are trapped within the first layer. The larger particles block some paths but, since they are not that numerous, other large enough paths are still available for the penetration of finer particles within the sample. This simulation reflects the phenomenon of superficial filter clogging. For $D_{15}/d_{85}=3$, the filtration of fine particles takes place more gradually and seems to be more related to the maximum probable path length a particle of a given diameter can flow through a granular filter.

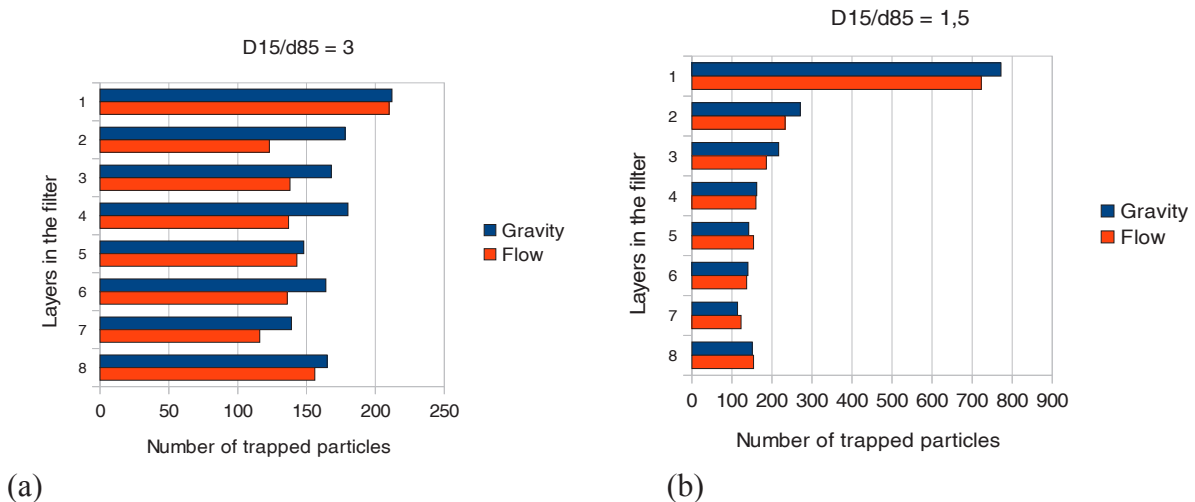


Figure 5: Trapped particles after filtration by gravity or by downward flow for (a) $D_{15}/d_{85}=3$ and (b) $D_{15}/d_{85}=1.5$.

4 FILTER AND FINE PARTICLES MIXED IN ONE LAYER

In this section, we present the results obtained when coarse and fine particles are initially mixed in one unique layer. In the first part we briefly present results obtained with gravitational loading alone, to verify that there is a potential movement of fine particles in such configuration. However, the primary objective here is to investigate the migration of particles in materials subjected simultaneously to both gravitational loading and fluid forces, when the flux is oriented upward (opposed to gravity). We simulate a point injection at the bottom of the box, as inspired by experiments carried out at the Cemagref by Philippe [13-14].

4.1 Simulations with gravity

We present the results obtained with $D_{15}/d_{85}=4$, for different proportions of fine and coarse particles. All particles are free to move, with the exception of coarse particles inside the bottom layer, which have the degrees of freedom blocked along the z-axis, as if they were supported by a coarse sieve. The thickness is about $8 \times d_{50}$ in the three directions.

In table 3, we examine how many fine particles will fall out of the packing as function of the initial fine/coarse distribution. Note that 0% passing is never expected in this configuration, since fines from the bottom boundary will always fall freely (they don't have to cross the filter). The relevant information is the number of fines blocked inside the filter in the final configuration. Note also that when the initial number of fines is high, a settlement of the whole packing is expected while fines are moving down progressively, emptying the gaps between big particles. The results show that $D_{15}/d_{85} = 4$ in a mixed packing allows potential movement of at least a fraction of the small particles.

Table 3: Passing particles according to the percentage of fine particles by weight.

$D_{15}/d_{85} = 4$						
% of fines by weight	0,5%	5%	10%	20%	30%	40%
% of successful passing	49,00%	47,00%	53,00%	48%	77,00%	99,00%

4.2 Local upward fluid injection

Finally, we present the simulations including gravity and an upward fluid flow. The boundary condition defining the influx is an imposed pressure in the first step, and an imposed flux in the second step. The D_{15}/d_{85} ratio is 3, the percentage of fines is 30 % and all particles are free to move. These values have been chosen to highlight the suffusion process. The sample thickness is about $6 \times d_{50}$ and the width about $17 \times d_{50}$ in the horizontal directions.

Suffusion of the fine particles is generally obtained in the simulations as soon as the influx is high enough, both with imposed pressure or imposed flux (Fig.6). Similar effects have been observed in experiments, [e.g 6].

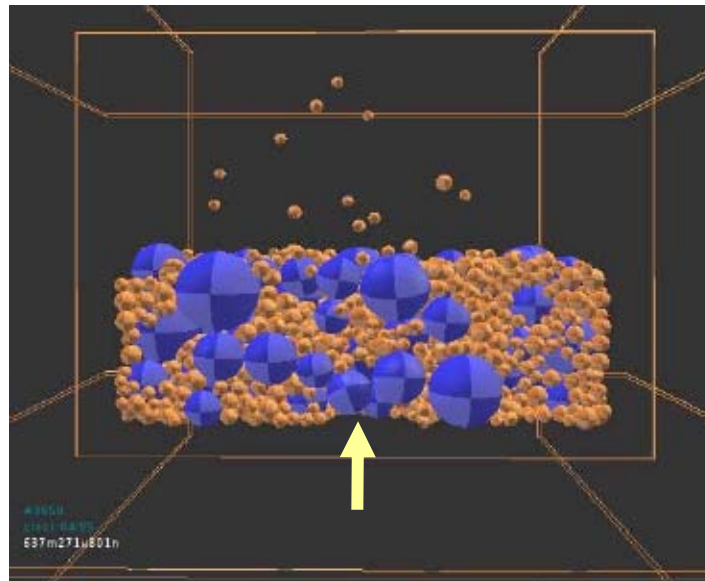


Figure 6: View of a packing subjected to upward flow (vertical cut). The arrow denotes the injection point.

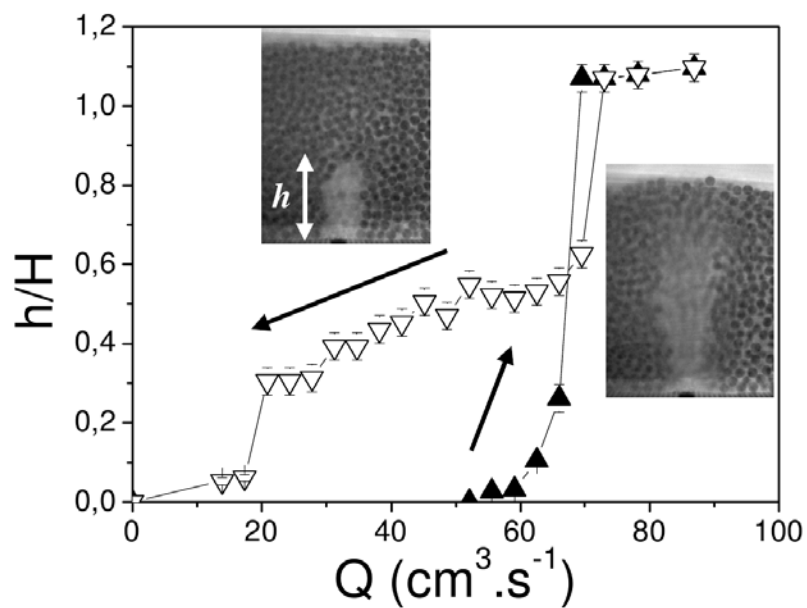


Figure 7: Measurements of the height of the fluidized zone in a granular layer above an injection point after Philippe [13,14].

With an imposed pressure, we observe a threshold value (near 10kPa) beneath which the system is stable and the flow is constant. For a pressure higher than this value, and after enough simulation time, a small cavity appears around the injection point. As soon as this erosion process is initiated, the flux increases rapidly (Fig.8), and the cavity grows larger. This process generally results in a large cylinder-shaped void crossing the packing vertically, together with a recirculation mechanism of fine particles. These effects are qualitatively

similar to what was reported by Philippe [13,14] – see fig. 7. The difference is that the particles size distribution is uniform in Philippe’s work and that is why the suffusion zone is larger in our case, due to the coarse particles. Some of the particles are also moved out of the layer and stay suspended until they fall down and touch the packing again, like in a suspension.

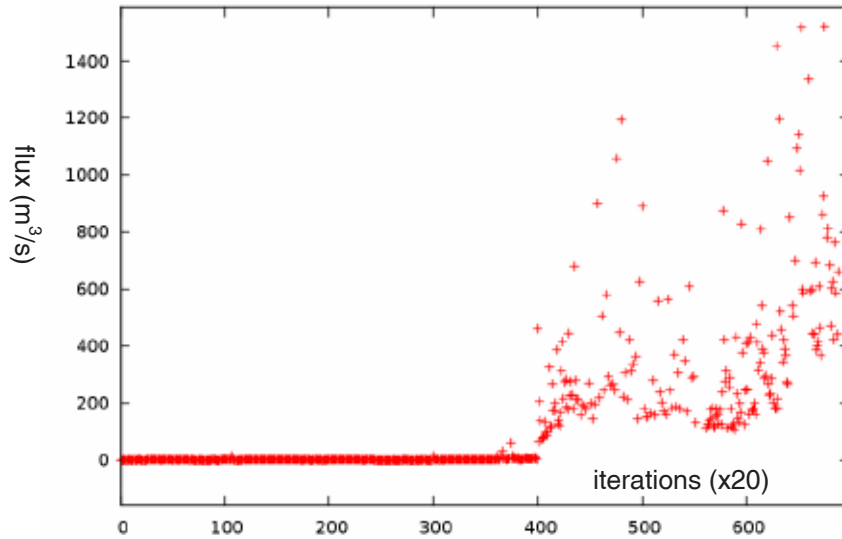


Figure 8: Flux versus time (number of iterations) for an imposed pressure of 10kPa at the injection point.

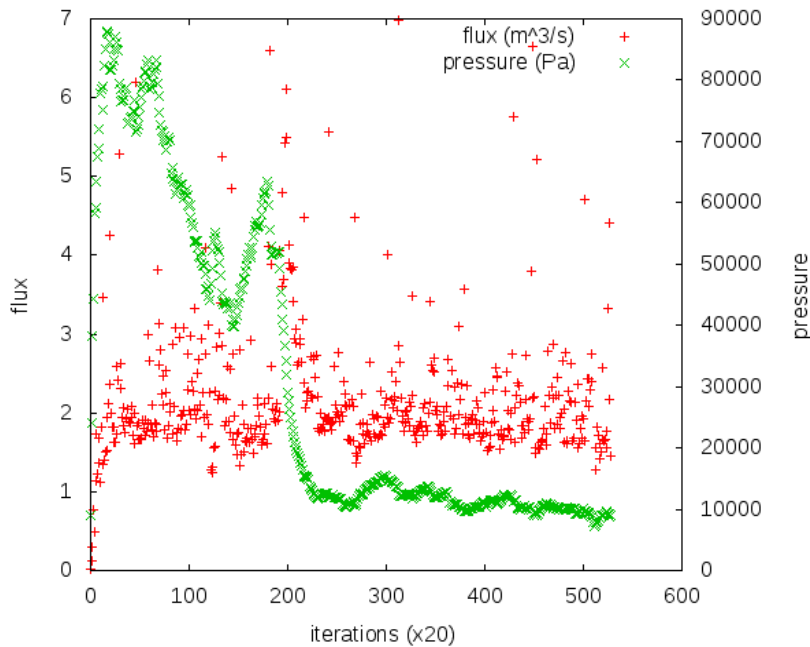


Figure 9: Flux and pressure values with an imposed flux value of 2 m³/s.

With an imposed flux, there is also a minimal value of the flux in order to trigger some movements of particles. For an imposed flux higher than this limit value, some of the small grains migrate away from the injection point (it is noticeable that, unlike fine particles, the coarse part of the filter is stable for the range of fluxes we present here).

The major difference between imposed pressure and imposed flux lies in what happens after the initiation of erosion processes. Fig.9 shows the values of both the flux and pressure for an imposed flux of $2 \text{ m}^3/\text{s}$. We shall note that the value of the flux cannot be imposed directly. Instead, it is needed to adjust the value of the pressure via a servo-control algorithm in order to maintain the flux to a prescribed value. One can notice in fig. 9 that the control does not prevent fluctuations around the prescribed value. This is due to the strong coupling between fluid flow and particles re-arrangements which, in turn, modify the conductivity of the packing. Nevertheless, the average flux is close to $2 \text{ m}^3/\text{s}$ whereas the pressure decreases rapidly while the erosion mechanisms tend to increase the conductivity of the system. As a result, there is no rapid evolution of the system as in the previous case (i.e. imposed pressure). The layer is always compact. The cavity near the injection point reaches a constant size, and no particles are ejected above the packing.

CONCLUSION

Numerical simulations of elementary tests involving filtration and erosion processes have been presented. The preliminary results are generally in agreement with experimental results. Although more investigations are needed to further validate the coupled model, we believe that DEM is relevant for understanding the fundamental mechanisms governing the complex phenomena of particles migration and instabilities in granular materials. In this context the DEM-PFV coupling has the advantage of reflecting mechanisms occurring at the particles scale (movement of fines, but also the effect of local heterogeneities of the solid fraction) with acceptable computation times and memory footprint.

Current work aims at simulating more realistic configurations, like the ones tested in Philippe [13], with larger numbers of particles. Still, the number of simulated particles will not approach the number of particles in most of the experiments. It will raise interesting questions on similarity conditions and scaling laws in internal erosion problems in order to compare simulations and experiments.

REFERENCES

- [1] B. Chareyre, A. Cortis, E. Catalano, E. Barthélémy, *Pore-scale Modeling of Viscous Flow and Induced Forces in Dense Sphere Packings*, (submitted), 2011, available online at <http://arxiv.org/abs/1105.0297v2>.
- [2] E. Catalano, B. Chareyre, A. Cortis, E. Barthélémy, *A pore-scale hydro-mechanical coupled model for geomaterials*, in. Particles 2011 II International Conference on Particle-Based Methods, E. Oñate and D.R.J. Owen (Eds), Barcelona, 2011.
- [3] V. Šmilauer, E. Catalano, B. Chareyre, S. Dorofeenko, J. Duriez, A. Gladky, J. Kozicki, C. Modenese, L. Scholtès, L. Sibille, J. Stránský, K. Thoeni, *Yade Reference Documentation*. In Yade Documentation (V. Šmilauer, ed.), 2010 (<http://yade-dem.org/doc/>)
- [4] V. Šmilauer and B. Chareyre, *Yade DEM Formulation*, in Yade Documentation (V. Šmilauer, ed.), 2010, (<http://yade-dem.org/doc/formulation.html>).
- [5] F. Chen, E. Drumm, G. Guiochon, Coupled discrete element and finite volume solution of two classical soil mechanics problems, *Computers and Geotechnics*, 2011. DOI 10.1016/j.compgeo.2011.03.009
- [6] Y. Sail, D. Marot, L. Sibille, A. Alexis, *Suffusion tests on cohesionless granular matter*, (submitted), 2011
- [7] J. L. Sherard, L.P.Dunnigan and J.R. Talbot *Basic properties of sand and gravel filters*, *Journal of Geotechnical Engineering* Vol. 110, No. 6, June 1984, pp. 684-700
- [8] N. Reboul, *Transport de particules dans les milieux granulaires – Application à l'érosion interne* (in French), PhD thesis, Ecole Centrale de Lyon, 2008.
- [9] A. Silveira. *An analysis of the problem of washing through in protective filters*. 6th International Conference on Soil Mechanics and Foundation Engineering, Montreal, Canada, 2, 1965; 551-555.
- [10] K.J. Witt KJ. *"Reliability study of granular filters"*. *Filters in Geotechnical and Hydraulic Engineering*, Brauns, Heibaum and Schuler (eds), 1993; 35-41
- [11] B. Indraratna, A K. Raut, and H. Khabbaz, *Constriction-Based Retention Criterion for Granular Filter Design*, *J. Geotech. and Geoenviron. Engrg.* 133, 266 (2007).
- [12] B. Chareyre, L. Briancon, P. Villard, *Theoretical versus experimental modeling of the anchorage capacity of geotextiles in trenches*. *Geosynthetic International* 9, pages 97–123, 2002.
- [13] P. Philippe, *Mise au point d'une sonde locale dans un milieu granulaire modèle. Application à la filtration et à la fluidisation localisée*. in ANR-Transol, Final Report, 2010.
- [14] P. Philippe, M. Badiane, *Chimney of Fluidization and « Sandboil » in a Granular Soil*. in *Advances in Bifurcation and Degradation in Geomaterials – Proceedings of the 9th International Workshop on Bifurcation and Degradation in Geomaterials*. Bonelli S., Dascalu C., Nicot F. Editors. Springer Science. Pp. 125-130 (2011).
- [15] M. Hakuno. Simulation of the dynamic liquefaction of sand. In *Earthq. geotechnical engineering*, pages 857–862. Balkema, 1995.
- [16] R. R. O. Bonilla. Numerical simulation of undrained granular media. PhD thesis, University of Waterloo, 2004.