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Transportation Research Procedia 47 (2020) 353–360

Transportation Research Procedia

www.elsevier.com/locate/procedia

22nd EURO Working Group on Transportation Meeting, EWGT 2019, 18-20 September 2019, Barcelona, Spain

Modeling public transportation networks for a circular city: the role of urban subcenters and mobility density

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Abstract

The concentration of both employment and services in a specific area of a town generates positive effects, but also impacts (congestion, transit issues, and others). Urban subcenters seek to approach economic activities to residents in peripheral urban spaces. The objective of this research is to evaluate the contribution to the mobility of implementing urban subcenters in a city. The model has a total cost function (users and agency costs) on a circular city (ring and radial routes) formulated using the continuous approximation method. The model solution addresses with mathematical optimization. The model evaluates a BRT network applied to scenarios of urban subcenters. The results of the modeling show that the implementation of subcenters obtains savings of 3.5% in rush hour. Thus, this strategy of urban planning generates improvements in the functioning of a public transportation system. Moreover, the maximum benefits are obtained in medium-sized subcenters in comparison to the CBD, which allows balancing user and agency costs. Therefore, the outcomes may be better with an urban pattern with subcenters, and a transit scheme adapted to the demand needs.

Keywords: Public transportation network design; Urban subcenter; Continuous approximation method.

1. Introduction

Nowadays, contemporary cities face several problems. One of them is the extreme concentration of urban services in a specific area called the central business district (CBD) and in a specific period of a day (peak period). In some cases, a transportation strategy may solve these issues, i.e., demand and transportation systems management (TDM, TSM). In other cases, we should apply urban planning tools such as the implementation of urban subcenters. Urban subcenters aim to bring economic activities (e.g., services, employment) closer to people who live in

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10.1016/j.trpro.2020.03.109

peripheral urban spaces. The implementation of new subcenters seeks to balance attractiveness among all zones in a city in order to reach a balance between trip attraction and generation. Thus, new areas in a city will have an urban dynamic more efficient. Unfortunately, there is not a quantitative evaluation of results that could have this policy.

Russo and Musolino (2012) proposed a unifying modeling framework to model interactions between spatial economic and transport systems simultaneously. Rodrigue et al. (2017) identified three basic network structures: distributed, centralized, and decentralized networks. The first case has no subcenters. The second case has a highly demanded CBD. The third case also has a CBD, but the city also contains subcenters. Transportation network design depends on the level of service required and its urban structure (Rodrigue et al., 2017). Several works have analyzed BRT networks. It highlights papers such as Daganzo (2010) and Tirachini et al. (2010). The former proposed a hybrid grid transit on a square region with a uniform demand. If a city is neither big nor has low demand, then BRT system can compete with cars. The latter compared transit modes (bus, tram, and metro) considering a radial system in a circular city with a CBD. In several scenarios, the results show that the BRT system is the best alternative.

Ouyang et al. (2014) formulated a method based on continuum approximations that design bus networks for an urban grid system with non-homogeneous demand. Unfortunately, the non-homogeneous demand functions are identical for generated and attracted trips; therefore, this does not allow us to analyze the behavior of subcenters and their impacts. We formulate a mathematical model using the continuous approximation (CA) method, which was initially proposed by G.F. Newell. It will be useful regardless of whether the information inputs are perfect. In this case, information of user arriving, traffic conditions, and others are not accurate; therefore, it is valid to model with smooth and continuous functions. These conditions are common in a macroscopic problem of network design. The CA method assumes that inputs slowly vary over the domain; the total cost can be the aggregation of small sub-region costs, which depend on local characteristics (Daganzo, 2005). This method has applications in solving transit problems (e.g., Medina-Tapia et al., 2013), logistics problems (e.g., Pulido et al., 2015), and private transportation issues as well (e.g., Medina-Tapia and Robusté, 2019, 2018).

The objective of the investigation is to evaluate the impact of the implementation of urban subcenters for the improvement of a BRT network, although the model can also apply to other transit modes (e.g., tramway, subway). A CA model is formulated considering the spatial design as well as the operation design of a transit system. The analysis considers a set of urban subcenter scenarios to evaluate the improvements in mobility. The theoretical model is applied to specific cases of urban subcenters.

In the next section, we will expose some previous papers. After that, we will present the main attributes of the model and their results. Finally, we will end with conclusions and further research.

2. Methodology

The region of analysis is a circular city of radius R [km], which has ring and radial transit services. The model implemented in polar coordinates assumes that passengers can travel on these two types of transit routes at four directions. Users can travel clockwise/anticlockwise direction ($L_c = \{l_c\colon clockwise, l_a\colon anticlockwise\}$) on circular routes, or users can go to the center or come out from this ($L_r = \{l_i\colon inside, l_o\colon outside\}$) on radial routes. Users are continuously distributed over the city considering three scenarios with a heterogeneous pattern of demand. Thus, the demand $D(r_f,\theta_f,r_t,\theta_t)$ in polar coordinates represents the density of trips from an origin (r_f,θ_f) to a destination (r_t,θ_t) (Vaughan, 1986). Therefore, the demand may be imperfectly distributed over the territory. This approach also allows obtaining adapted transit schemes to demand when city planners implement new subcenters. From this, we obtain the generated demand function $\lambda(r,\theta)$ at a point (r,θ) in [user/km^2·h] (Eq. 1). The mobility pattern supposes that passengers travel using the shortest route through circular, radial, or both types of transit services. The period of the analysis is the rush hour of a town (P_m) because the peak period defines the required infrastructure.

$$\lambda(r,\theta)=\int_0^{2\pi}\int_0^{R}D(r,\theta,r_t,\theta_t)\ r_t\ dr_t\ d\theta_t \qquad (1)$$

The mathematical formulation has two components, user (T_T^u in [user·h/P_m]) and agency (C_T^a in [\$/P_m]) costs, and three decision variables, two spatial variables and one operational variable (e.g., Estrada et al., 2011).

Nomenclature

Decision variables:

$d^c(r,\theta)$	distance between circular transportation routes at a point (r,θ) [km/route]
$\Phi^r(r,\theta)$	angle between radial routes at a point (r,θ) [radian/route], which $d^r(r,\theta) = \Phi^r(r,\theta)\cdot r$, where $d^r(r,\theta)$ is the distance between radial corridor routes at a point (r,θ)
H	headway between vehicles at the system [h/veh]

Parameters:

T	duration of the peak period or rush hour [h/P_m]
$\alpha,\beta,\gamma,\delta$	perception of access (α), waiting (β), travel (γ), and transfer (δ) time by users [dimensionless]
$v^a(r)$	average access speed of accessing or egressing on a route at radius r [km/h·route]
v^w	average walking speed to transfer between stations [km/h]
v^t	cruising speed of a transit vehicle [km/h]
τ^s	average time lost at a station [h/station]
t^f	positioning time at a terminal by a transit vehicle [h]
η^d	number of workers on a vehicle [worker/veh]
χ^T	average walking distance between two stops [km]
K^v	capacity of a vehicle [user/veh]
K^h	minimum time between consecutive vehicles [h/veh]
K^d	minimum separation between two transit routes [km/route]

Unitary cost parameters:

μ	travel time value by average user [\$/user·h]
φ^k	unitary cost per a vehicle [\$/veh·P_m]
φ^g	driver's wage per hour [\$/worker·h]
φ^o	operating cost per kilometer traveled on a cruising speed [\$/veh·km·route]
φ^p	linear infrastructure cost [\$/km·route·P_m]
φ^s	nodal (stop or station) infrastructure cost [\$/station·route·P_m]

2.1. User Costs

The total time of users (T_T^u in [user·h/P_m], Eq. 2) has four elements: access, waiting, trip, and transfer time.

$$T_T^u = \int_0^{2\pi}\int_0^{R}\left(f^A(r,\theta)\cdot T\cdot t^A(r,\theta) + \sum_{l\in\{L_c,L_r\}}\left(f_l^W(r,\theta)\cdot T\cdot t^W + f_l^V(r,\theta)\cdot T\cdot t^V(r,\theta) + f_l^T(r,\theta)\cdot T\cdot t^T \right)\right)\cdot r\, dr\, d\theta \tag{2}$$

- **Access/egress time**. In Eq. 2, the access time depends on the demand and the average access time per user. The demand that access in rush hour ($f^A(r,\theta)\cdot T$ in [user/km^2]) is the density of users that board and alight at a station. The average access time ($t^A(r,\theta) = \alpha\cdot(d^c(r,\theta) + \Phi^r(r,\theta)\cdot r)/4\cdot v^a(r)$) depends on the time perception α (TRB, 2013), the average distance between the origin/destination and the closest bus-stop (a quarter of the maximum walking distance, $(d^c(r,\theta) + \Phi^r(r,\theta)\cdot r)/4$), and the access speed ($v^a(r)$).
- **Waiting time**. In Eq. 2, the user density $f_l^W(r,\theta)\cdot T$ ([user/km^2]) at the direction $l = \{L_c, L_r\}$ waits $1/2$ headway, which depends on the user perception ($t^W = \beta\cdot H/2$). The model assumes that the headways are deterministic and perfectly regular (e.g., Medina-Tapia et al., 2013).
- **In-vehicle travel time**. The transit is a BRT system that does not share infrastructure; therefore, congestion will not affect the system. In Eq. 2, the local travel time per kilometer depends on two components: the user density on a bus in rush hour ($f_l^V(r,\theta)\cdot T$ at the direction l in [user/km]), and the travel time per kilometer ($t_l^V(r,\theta)$ in [h/user·km]) incurred by a user on a vehicle considering a factor γ. The user density for radial routes includes a factor because of the corridor width increases to the periphery.

- **Transfer time**. In Eq. 2, the transfer depends on the user density that transfers at a point ($f_l^T(r,\theta)\cdot T$ in [user/km^2]), and the average transfer time ($t^T = \delta\cdot(\chi^T/v^w + H/2)$). The demand considers that the corridor width increases to the periphery.

2.2. Agency Costs

The agency cost has three components (C_T^a in [$/$P_m$]): capital, operational, and infrastructure cost (Eq. 3).

$$C_T^a = \int_0^{2\pi}\int_0^{R} \sum_{l\in\{L_c,L_r\}} \left(\left(\frac{t_l^C(r,\theta)\cdot\varphi^k}{H} + \frac{t_l^C(r,\theta)\cdot\eta^d\cdot T\cdot\varphi^g}{H} + \frac{2\cdot T\cdot\varphi^o}{H} + 2\cdot\varphi^p \right)\cdot\frac{1}{d^l(r,\theta)} \right) + \left(\frac{4\cdot\varphi^s}{\Phi^r(r,\theta)\cdot r\cdot d^c(r,\theta)} \right)\cdot r\,dr\,d\theta \quad (3)$$

- **Capital cost**. In Eq. 3, it depends on the fleet (t_l^c/H in [veh] in which t_l^c is the cycle time at route $l\in\{L_c,L_r\}$), and the cost per vehicle (φ^k). Three elements compose the cycle time at a route l: the time at cruising speed, the positioning time at terminals (t^f/ι_l where $\iota_l = 2\pi r$ if $l = L_c$; $\iota_l = R$ if $l = L_r$), and the lost time by a bus-stop (τ^s).
- **Operational cost**. It has two components: the on-vehicle crew cost and operation cost of a vehicle. The total salary ($t_l^c\cdot\eta^d\cdot T\cdot\varphi^g/H$ at route l, Eq. 3) is in proportion to the fleet, the number of workers on a vehicle (η^d), and the salary in the rush hour ($T\cdot\varphi^g$). The total operating cost depends on two components: the number of vehicles that run during the rush hour ($2\cdot T/H$), and the operation cost per unit of distance traveled (φ^o).
- **Infrastructure cost**. Firstly, the road cost also has two components related to each type of corridor: the corridor density ($1/d^c(r,\theta)$ or $1/\Phi^r(r,\theta)\cdot r$), and the unitary cost (φ^p) in Eq. 3. Secondly, the local station cost is composed of the density of stations ($1/\Phi^r(r,\theta)\cdot r\cdot d^c(r,\theta)$) and the unitary cost ($\varphi^s$) in Eq. 3.

2.3. Mathematical formulation

The mathematical problem ($TC = \mu\cdot T_T^u + C_T^a$) is a nonlinear system with inequality constraints (Eq. 4).

$$\begin{aligned}
&Min\ TC = \mu\cdot T_T^u + C_T^a && (a)\\
&s.t. && \\
&\max_{(r,\theta),l\in L_c}\left(f_l^V(r,\theta)\cdot d^c(r,\theta)\cdot H\right)\le K^v && (b)\\
&\max_{(r,\theta),l\in L_r}\left(f_l^V(r,\theta)\cdot\Phi^r(r,\theta)\cdot(R+r)/2\cdot H\right)\le K^v && (c)\\
&d^c(r,\theta)\ge K^d \quad \forall(r,\theta) && (d)\\
&\Phi^r(r,\theta)\cdot r\ge K^d \quad \forall(r,\theta) && (e)\\
&H\ge K^h && (f)
\end{aligned} \quad (4)$$

The objective function has two components: user ($\mu\cdot T_T^u$ in which μ is the travel time value, and T_T^u is the user time function, Eq. 2) and agency cost (C_T^a, Eq. 3). Moreover, the problem has three sets of constraints. The first set (Eq. 4(b) and (c)) ensures that the demand does not exceed the capacity of a vehicle (K^v). The second set (Eq. 4(d) and (e)) makes sure a minimum distance between stations to reach the cruising speed before arriving at the next station (K^d). The third constraint (Eq. 4(f)) makes sure that the optimum headway has a minimum separation (time) between vehicles (K^h) (TRB, 2013).

Karush-Kuhn-Tucker (KKT) conditions allow finding the necessary conditions. The first-order optimization conditions enable obtaining optimized analytical functions, and it will permit getting the optimal value on each city point. The necessary conditions are not sufficient for optimality, but all local minimum satisfies these conditions. To find the optimal solution is required to analyze the convexity of the problem. In the CA method, the minimized function is $tc(r,\theta)$ that is the local cost at the point (r,θ), in which, the problem has three continuous decision variables ($d^c(r,\theta), \Phi^r(r,\theta), H$). From the continuous solution, the authors applied three sequential steps for obtaining the discrete location of routes. First, the inverse of optimal continuous variables allows obtaining density functions of routes in [route/km] or [route/rad], i.e., the area below of these curves is the total number of routes (*N*). Second, the total area can divide into *N* sub-areas, which each one represents a route corridor, i.e., the value of the integral over each area will be about one. Third, the solution of this sub-problem is through a system of inequalities on a differential radius for ring routes and a differential angular for obtaining radial routes. Therefore, the system of inequalities defines the location of routes and the border of each route corridor.

3. Analyzed scenarios

We analyze a sequence of eleven scenarios of concentric cities with a radius of 15 [km], which generates 500 [user/km^2·h]. This sequential set presents from a city without subcenters to a city with a subcenter 1.25 times the CBD. Figure 1 represents this sequence through four cases: (a) a mono-centric city without subcenters (A), although the CBD attracts many users (it is not a trip-generating zone); (b) a city with a CBD and a subcenter that generates and attracts users, which is about 60% of the CBD (0.625xCBD) (B); (c) a city with a CBD and a subcenter that has the same size as the CBD (1.0xCBD) (C); and (d) a city with a subcenter 25% higher than the CBD (1.25xCBD) (D). Graphs in the row (i) represent generated trips, and graphs in the row (ii) are attracted trips. Table 1 presents the parameters used in the modeling of a BRT system. Several of them come from an exhaustive literature review.

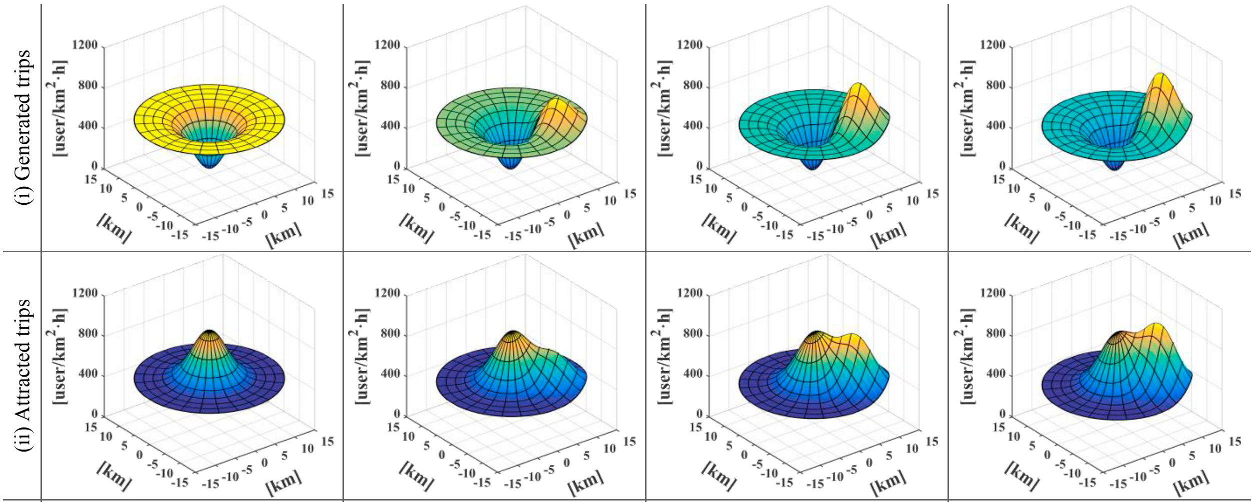


Figure 1. Scenarios with heterogeneous demand: 500 [user/km^2·h].

Table 1. Parameters used in the modeling of a BRT system.

Parameter	Value	Parameter	Value	Parameter	Value
T	1.5 [h/P_m]	t^f	5 [min]	μ	1.48 [\$/user·h] (4)
$\alpha, \beta, \gamma, \delta$	2.2, 2.1, 1.0, 2.5 (1)	η^d	1 [worker/veh]	φ^k	37.9 [\$/veh·P_m] (3)
$v^a(r)$	$3 + 1.4\bar{6} \cdot r$ [km/h·route]	K^v	101 [user/veh] (3)	φ^g	23 [\$/worker·h] (3)
v^t	90 [km/h]	K^d	0.079 [km/route] (3)	φ^o	1.6 [\$/veh·km·route] (3)
v^w	3.0 [km/h]	K^h	60 [seg/veh] (3)	φ^p	62.1 [\$/km·route·P_m] (3)
τ^s	49 [s/station] (2)			φ^s	6.2 [\$/station·route·P_m] (3)

Source: (1) TRB (2013); (2) Adapted from TRB (2013); (3) Adapted from ATC (2006); (4) Jara-Díaz et al. (2017).

4. Results and discussion

The model gets two optimal results: headway and continuous density functions of the distance between transit routes. The headway reaches the minimum value. Figure 2 shows optimal route densities (the inverse of the distance between routes) for ring and radial routes (i-ii), and each urban scenario: (a-d). Figure 2(iii) contains the optimal objective function values (OFV). Consequently, we obtain a discrete solution from continuous functions for each type of route. Figure 2(iv) presents the optimal network scheme and includes both transit routes.

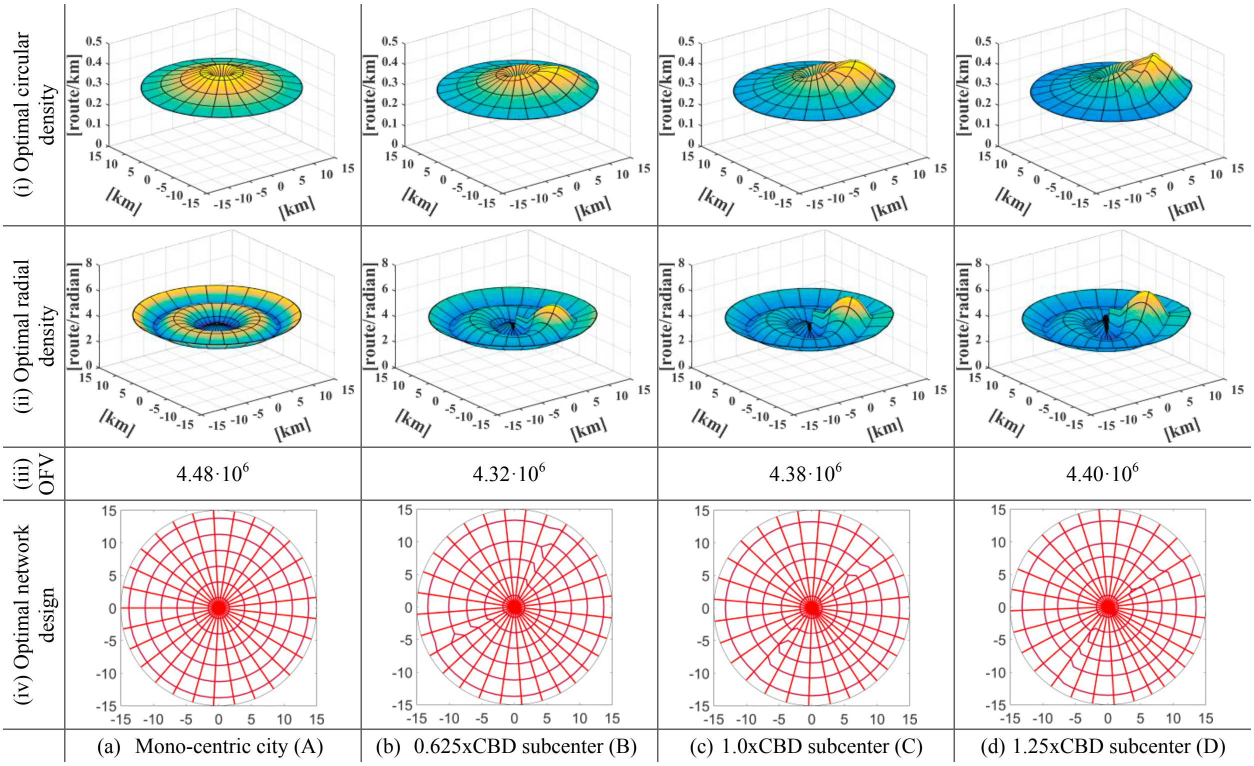
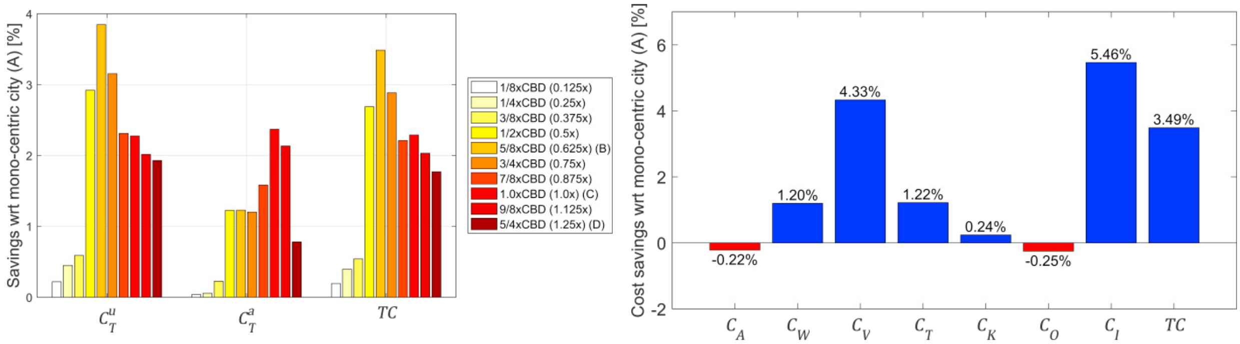


Figure 2. Optimal transit network design: BRT system.

In scenario A, the constraints are not active over the urban region for ring roads (Figure 2(i)-(a)). The optimal density is higher around 3 km from the center, but this decreases abruptly toward the center and slowly toward the periphery. For radial roads, the buses do not travel full during the first kilometers of a radial corridor (≈ 4 km). After that, the demand dominates the optimal density of radial routes (Figure 2(ii)-(a)). In the other scenarios, the effect of the subcenter is progressively observed from Figure 2(i-ii)-(b-d). The result of discretization (Figure 2(iv)) shows the effect of the implementation of subcenters. In scenarios A, B, and C, the city needs 29 radial routes, although these are more concentrated in the subcenter area when its density increases. If the subcenter is bigger than the CBD, the city needs 30 radial routes (scenario D). Concerning ring routes, all scenarios require six routes, although the infrastructure can be shared in the area without subcenters for scenarios B, C, and D. These save infrastructure cost but increase the access cost.

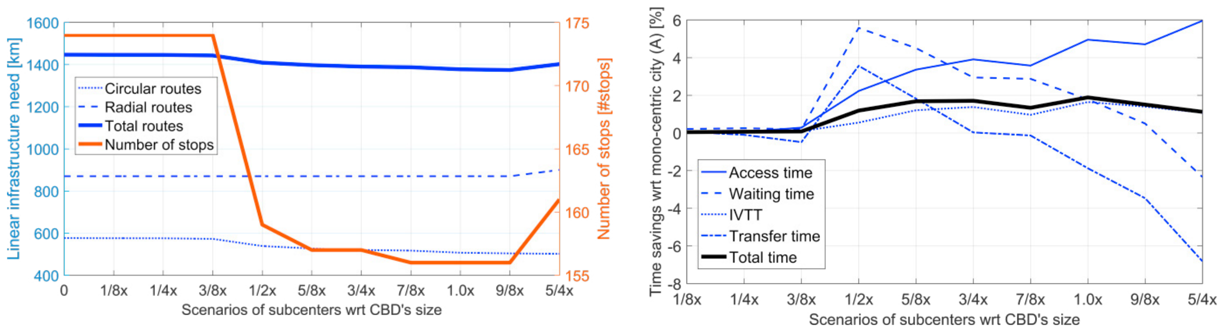
Figure 3 shows a comparison of costs and travel time between urban scenarios. Figure 3(a) shows the evolution of user, agency, and total savings concerning scenario A. The scenarios consider a sequential increment of the subcenter size from 0.125 (1/8xCBD) to 1.25 times the CBD (5/4xCBD, scenario D). The implementation of subcenters presents savings at all scenarios. For users, the maximum savings are observed with medium-size subcenters (0.5x to 0.75x). For the agency, more significant savings are observed when the subcenters are more prominent and the system reduces the infrastructure in areas with low demand. For user and agency components, the maximum savings are obtained in scenario B in comparison to scenario A (3.49%). In Figure 3(b), we compare savings between scenarios A and B. All stages of a trip have savings except access (0.22%) and operating cost (0.25%). The most significant savings are in the infrastructure, in which some routes share infrastructure on the opposite side of the subcenter.



(a) Cost savings wrt the mono-centric city (only CBD) (b) Comparison of costs between (A) and (B) scenarios

Figure 3. Cost savings considering a set of urban scenarios for a BRT system.

Figure 4(a) represents the infrastructure needs considering the sequential set of urban scenarios. The first four scenarios maintain the same infrastructure needs. After that, the BRT system can merge services at the same corridor around areas with low demand, which reduces linear infrastructures and BRT stops. The minimum number of stops is between 0.875x and 1.125x of the CBD. In the last scenario (1.25xCBD), transit requires a new radial corridor. Figure 4(b) represents time savings regarding scenario A. The savings are mainly obtained from the scenario 3/8xCBD. Access and travel time savings strongly influence these savings. Waiting and transfer times increase (in percentage) although they are smaller (in magnitude) than the access and travel time.



(a) Infrastructure needs (b) Comparison of travel time

Figure 4. Infrastructure needs and travel time considering a set of urban scenarios for the Subcenter policy.

5. Conclusions and future research

The proposed methodology based on the CA method allows us to analyze improvements for the mobility of a city or quantify the required primary infrastructure. Moreover, the results of the model can contribute to the development of spatial planning instruments, which require several studies to define an objective image of a city (future urban planning scheme) that can include urban subcenters. The results show how radial corridors allow massive trips of users from the periphery to central areas of a city, although buses only circulate full during 2/3 of a route. On the other hand, circular routes bring passengers closer to their destinations, e.g., CBD or urban subcenters. However, ring routes also have a significant role in the city periphery. The role of subcenters makes it possible to improve the mobility of inhabitants in scattered, large cities. Also, subcenters create an opportunity to optimize transit systems. The results show that this strategy of urban planning allows for obtaining a savings of 3.5% in rush hour. These savings are about $40MM in a year considering a subcenter in just morning's peak periods. Another relevant aspect is the size of the subcenters. The results show that subcenters as large as the CBD (or even larger) do not benefit a public transportation system. The maximum benefits are obtained in medium-sized subcenters in comparison to the CBD because there must be a balance between user and agency costs. In future works, the application to a specific case will improve the analysis considering real cities in current and future mobility scenarios. The application of this model is macroscopic, which can be applied to an entire city or a specific zone. The model allows for determining the density of transit services necessary to satisfy demand with a balance of adequate total costs. In addition, the analytical formulation of the model allows identifying key elements that could generate problems or conflicts between a transportation system and an urban system.

Acknowledgements

The authors are members of BIT-Barcelona Innovative Transportation research group at BarcelonaTech. The work of the first author was also supported by CONICYT PFCHA/BCH 72160291 scholarship. This study had support from the Project 061312MT of the Departamento de Investigaciones Científicas y Tecnológicas (DICYT) of the Vicerrectoría de Investigación, Desarrollo e Innovación of the Universidad de Santiago de Chile (USACH).

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