

# INFLUENCE OF POROSITY PERCOLATION ON MECHANICAL PROPERTIES OF CERAMIC MATERIALS. 3D SIMULATION USING MOVABLE CELLULAR AUTOMATA

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**Key words:** Porous Materials, Mechanical Properties, Computer Simulation, Movable Cellular Automata

**Abstract.** 3D computer simulation of mechanical behavior of a brittle porous material under uniaxial compression is considered. The movable cellular automaton method, which is a representative of particle methods in solid mechanics, is used for computation. In an initial structure the automata are positioned in FCC packing. The pores are set up explicitly by removing single automata from the initial structure. The computational results show that the curves of dependence of strength and elastic properties of the modeled specimens on porosity have a break at the porosity value about 20%, i.e. percolation threshold. The obtained results are in close agreement with available experimental data.

## 1 INTRODUCTION

It is well known that porosity dependence of strength and elastic properties of porous materials is determined by the pore morphology. If porosity is small, each pore is closed and isolated from the others. With porosity increasing the number of pores becomes greater and the distance among pores decreases. When porosity reaches percolation threshold the pores are not closed anymore and consequently form a new morphology. Thus, dependence of strength and elastic properties on porosity should be changed when percolation threshold is exceeded.

Note, it is very difficult to study the problem experimentally, because of stochastic nature of the pore morphology in real materials. For example, it is impossible to fabricate specimens with various porosity value and the same porous structure. At the same time potentialities of the modern computational mechanics allow to study in detail not only deformation of complex heterogeneous materials but also fracture of such materials.

## 2 COMPUTATIONAL MODEL FOR POROUS CERAMICS

For 3D computer simulation of mechanical behavior of a porous material under uniaxial compression we used the movable cellular automaton method (MCA) [1]. MCA is a method in discrete computational mechanics which allows modeling deformation and fracture of heterogeneous material with taking into account the structure of the material explicitly. Modeled material is represented as a set of elements of finite size (automata) interacting with each another according to some pre-determined rules. Mechanical interaction among automata are defined by so called response function, which can be interpreted as a local stress-strain diagram. Equation of translation motion and rotation of the automata are written in many-body approach. This means that interacting force between a couple of automata depends not only on position and state of these automata but also on position and state of their neighboring automata. This allows modeling as solid also as granular material behavior within the framework of one MCA method.

Response function of automata used in this study corresponded to  $ZrO_2$  ceramics with average size of pores commensurable with the grain size of the material. According to the pore distribution diagram of this material [2] the automaton size in the computations was  $1\ \mu\text{m}$ . All the modeled specimens where bricks with a square base. The loading was applied by assignment of vertical velocity for automata of the top layer of a specimen. At the same time the substrate automata were rigidly fixed. At the initial stage of the loading velocity of the upper automata increased by a sinusoidal law from 0 to 1 cm/s and then was constant.

Pores were generated by removing single automata from the basic structure. The porosity values were varied from 0 to 50%. Note, that the maximal porosity on the assumption of pore disconnection in closed packing of elements for 2D is  $1/2 = 50\%$  (Fig. 1,a) while for 3D it is  $1/4 = 25\%$  (in Fig. 1,b one closed packing plane is shown for FCC). In this

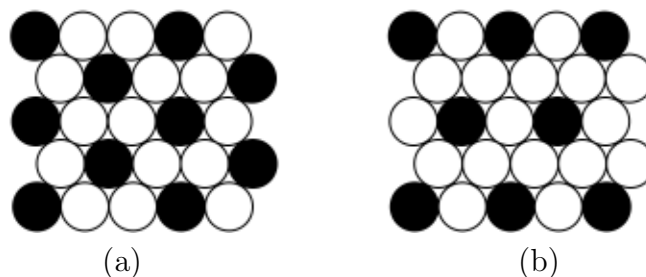


Figure 1: Periodic pores (black) in closed packing

case the pores are situated regularly.

The maximal value of closed porosity for random removing of single automata from FCC packing is 17.6 %. The percolation limit for 3D FCC packing is 19.8 %. Consequently, all the specimens used in this study with porosity greater than 20 % contained clusters of interconnecting pores. For the specimens with porosity greater than 25 % their porosity was interconnected.

In [3] it is shown that porosity structure defines not only elastic, but also strength properties of a material. The clusters of interconnecting pores present a new element of structure in addition to single closed pores. Thus, with porosity increasing the changes of elastic and strength properties of a material are expected after transition of the percolation limit.

### 3 DETERMINING THE REPRESENTATIVE VOLUME

At the first stage it is required to determine the representative volume of the modeled material. For material without pores the representative volume was determined based on four solid specimens. For this purpose the convergence of effective elastic modulus of the specimen  $E_{\text{eff}}$  to the automaton elastic modulus  $E_0$  with increasing the specimen size was analyzed.

The convergence was assumed to be satisfied if  $E_{\text{eff}}$  differed from  $E_0$  not greater than 3 %. The computations showed the representative volume was equal  $10 \mu\text{m}$  of the brick base size. The fracture pattern (cracks) in this specimen is shown in Fig. 2. The cracks are generated from the stress concentrators and propagate along maximum tangential stress direction.

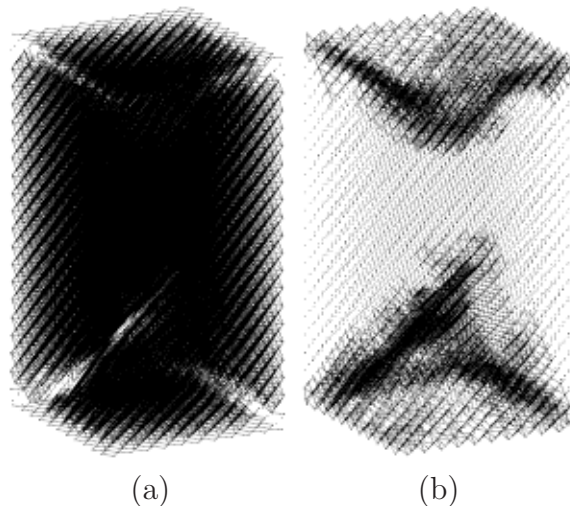
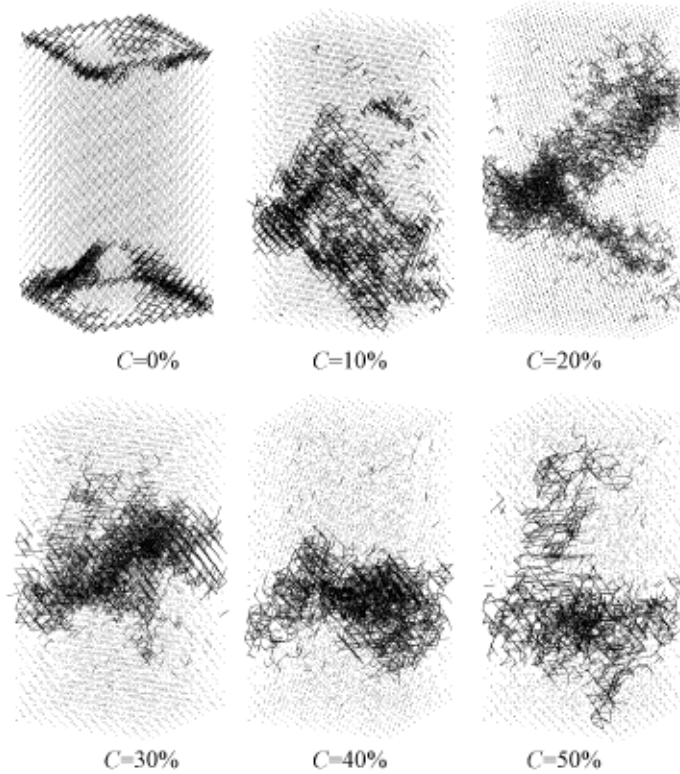


Figure 2: Fracture pattern of a solid ceramic specimen under compression: a) lines connect linked automata, b) lines connect unlinked automata

If a material contains pores the cracks may initiate not in the corners but in the bulk

region of the highest local porosity. The direction of cracks propagation in such material is defined by the porous topology. Fig. 3 shows the first cracks in porous specimens with various values of porosity. One can see that the failure behavior of a porous material with large porosity and the failure of a solid material differs dramatically. In particular, the path of the crack propagation in porous material is rather crinkly.



**Figure 3:** First cracks in specimens with various values of porosity  $C$  under compression

The fracture pattern of the modeled brittle porous 3D specimens qualitative corresponds to 2D simulation results [3]. The quantitative difference is defined by the fact that porosity in 2D specimen is “interconnected” in the normal direction to the computational plane.

To analyze the dependence of elastic and strength properties of the modeled material on its porosity it is necessary to determine the size of representative specimen (volume). It is obvious that the representative volume for a porous material is different for different values of porosity. The simulations conducted show that for porosity value less than 15 % the representative volume is equal  $20 \mu\text{m}$  (Fig. 4). For porosity value from 15 up to 35 % the size of representative volume is found to be  $30 \mu\text{m}$  (Fig. 4,b). For porosity varied from 35 up to 50 % —  $40 \mu\text{m}$  (Fig. 4,c).

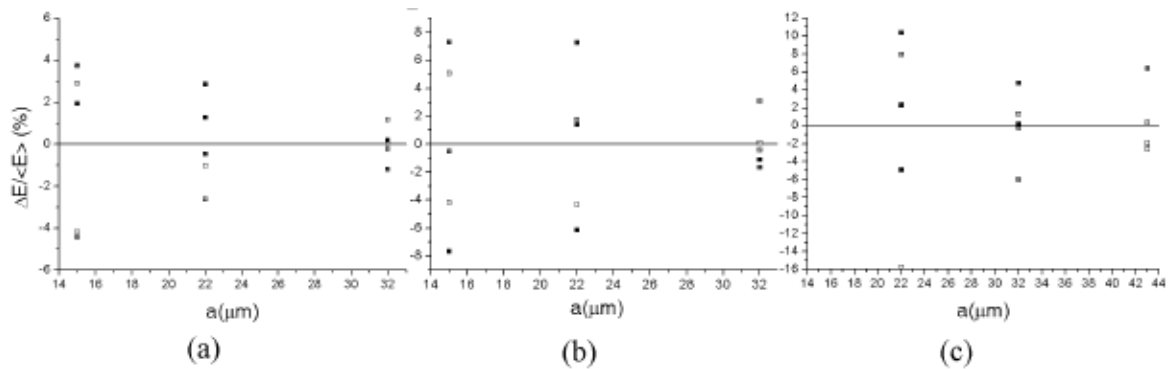


Figure 4: Convergence of effective elastic moduli of the modeled specimens on their basis size for various porosity: a) 10 %, b) 25 %, c) 50 %

#### 4 STUDYING ELASTIC AND STRENGTH PROPERTIES OF POROUS CERAMICS

The loading diagrams of 3D modeled specimens are shown in Fig. 5. They are in close qualitative agreement with 2D simulation results [3, 4]. A quantitative difference between 2D and 3D results could be explained by the different influence of porosity on effective porous characteristics in 2D and 3D tasks.

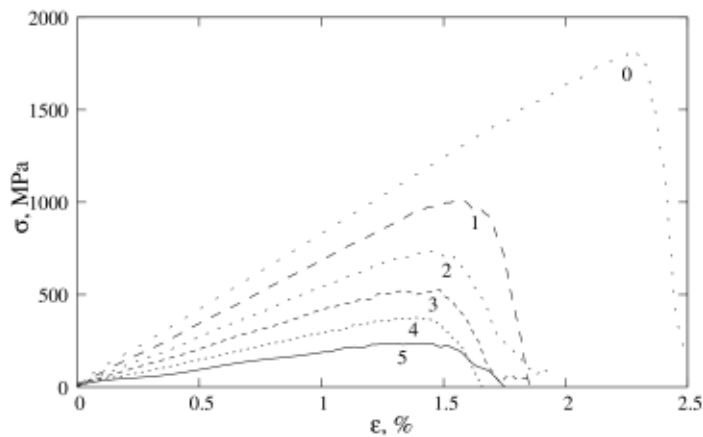


Figure 5: Loading diagrams for ceramics with various porosity  $C = n \cdot 10\%$

First, it has to be noted that 3D specimens with porosity stochastically distributed in space can demonstrate quasi-viscous regime of fracture like 2D models [4]. To be able to failure in this regime the porosity of 3D specimens has to be greater than 40 % (Fig. 5).

Let us consider dependence of the effective elastic modulus of 3D specimens on its porosity. In Fig. 6,a each square represents the value averaged on five representative specimens with various pore distribution in space. This plot obviously can be divided into two characteristic parts connected with porous structure: the first corresponds to closed

pores (5...20%), the second corresponds to clusters of interconnected pores (20...50%). The experimental data taken from [2] are presented in Fig. 6,b for comparison. One can see the computational results are in close qualitative agreement with the experimental data. However, the inclinations of approximating lines showing degree of the influence of porosity on elastic characteristics of a material within the intervals are little bit different for the experimental and computational data. It can be explained by stochastic choice of elements for pore generation in the modeled specimens, and hence by the orientations of clusters of interconnected pores, which leads to reduction of porous structure influence on mechanical properties of a material. Besides, according to [2] percolation transitions in ceramics  $ZrO_2$  cause microstructure change. In particular, internal stresses in ceramics with continuous porous structure restrain grain growth. Thus, simulation of uniaxial compression of brittle porous 3D specimens by the movable cellular automaton method showed that percolation transition from the closed pores to interconnected pores in a porous material leads to change in the dependence of its elastic properties on porosity.

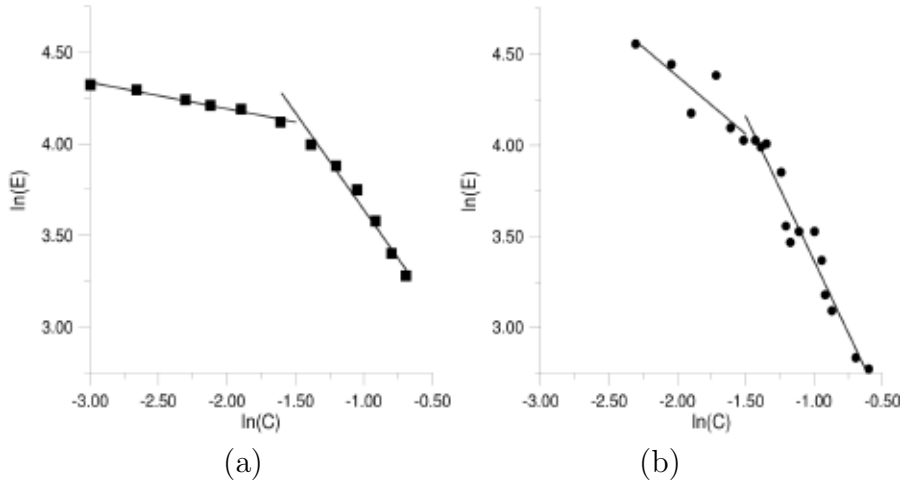


Figure 6: Dependences of elastic modulus of ceramics on its porosity in logarithmic coordinates: a) simulation; b) experimental data

It is necessary to notice that this result could be obtained only in three-dimensional simulation because two-dimensional specimens with interconnected porosity are not topologically connected and do not resist to the mechanical loading. Besides, the porosity value at which formation of interconnected clusters occurs in a two-dimensional case (22.4%) is considerably less than the corresponding percolation limit (69.6%).

Fig. 7 shows the simulated and experimental dependencies of ceramics strength on its porosity. Again it can be divided into two characteristic parts connected with porous structure.

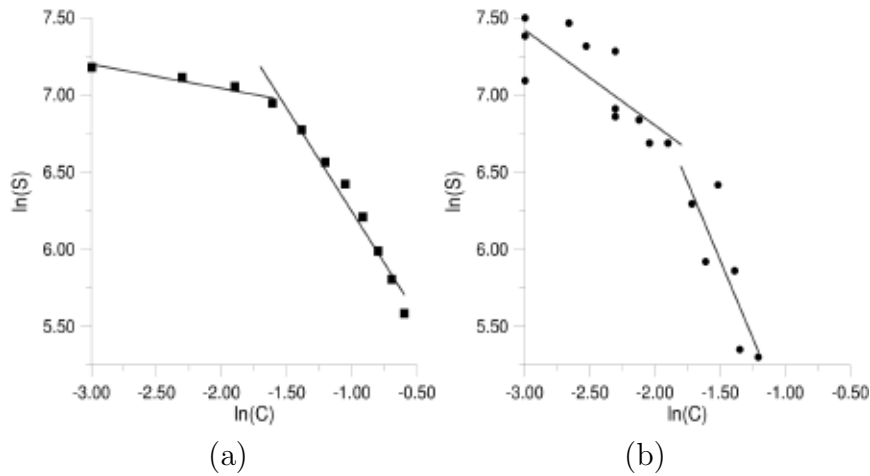


Figure 7: Dependences of ceramics strength on porosity in logarithmic coordinates: a) simulation, b) experimental data

## 5 CONCLUSIONS

The simulations performed by the movable cellular automaton method show that the curves of dependence of strength and elastic properties of the brittle porous specimens on porosity have a break at the porosity value about 20 %, which corresponds to percolation threshold. The obtained results are in close agreement with available experimental data.

In summary it is necessary to notice that for the purpose of the further application of the proposed 3D MCA model of porous ceramics it is necessary to extend the model with possibility of describing nonlinear properties of materials (degradation of elastic properties, relaxation at high-speed loading, etc.)

## ACKNOWLEDGEMENTS

This work was supported by the interdisciplinary integration projects No 32 and No 113 of the Siberian Branch of the Russian Academy of Sciences.

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