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A FULLY LAGRANGIAN APPROACH FOR MODELING ABRASIVE WEAR

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Abstract. The decrease of efficiency of hydraulic machinery is a result of different mechanisms. The impairment of the performance, e.g. of a pump, caused by abrasive wear is thereby not negligible. Abrasive particles transported by the working fluid can lead to a mechanical damage of the surface of the hydraulic machinery components. In this work, a fully Lagrangian approach for modeling the process of abrasive wear is presented. The damage mechanism is a complex process that should be investigated during simulations with this approach. When analyzing abrasive wear of the components several facts have to be taken into account. For example, one important point that has to be taken into consideration is the shape of the abrasive particles, therefore a suitable model for the wear is applied.

In contrast to classical computational fluid dynamic simulations, here the mesh-less Smoothed Particle Hydrodynamics method is used for the modeling of the fluid. The advantage in comparison to classical mesh-based methods, is the much easier description of the free surface of a fluid and the interface between the fluid and a solid body. The abrasive particles are modeled using the Discrete Element Method. For the modeling of the wear the erosion model of Finnie is applied.

In this work the presented Lagrangian approach is used for the simulation of an impact of a free jet with loading and the analysis of the resulting wear. The boundary geometry is the bucket of a pelton turbine. First the theoretical background of this hybrid simulation approach is described and then the simulation results are discussed.

1 INTRODUCTION

The damage of hydraulic machinery is caused by several mechanisms. One of these mechanisms is abrasive wear due to small stiff particles transported as loading of the working fluid. These particles are called abrasive particles. They are transported by the fluid and can lead to a mechanical damage of the surface of the hydraulic machinery [6] and a decrease of its efficiency. To reduce the decrease of the machinery due to abrasive wear it is necessary to investigate the process in detail.

In this work an approach with Lagrangian simulation methods in combination with an erosion model for the analysis of abrasive wear is presented. With this approach an impact of a free jet with particle loading on a simplified bucket of a pelton wheel is simulated and the results are discussed. The free jet, the abrasive particles and the bucket are shown in Figure 1. The opacity of the fluid is reduced in the visualization so that the abrasive particle can be seen in this figure. In this approach two mesh-less methods are combined. The Smoothed Particle Hydrodynamics (SPH) method is applied for the simulation of the fluid. The SPH method is a mesh-less method and, therefore, it is more suitable for simulating the impact of a free jet than mesh-based methods. The description of the free surface is easier with a mesh-less method. Especially the interaction between the fluid and the boundary or the free surface of the fluid requires, e.g., no complex mesh generation. Another point is the easier treatment of the interface between the fluid and the abrasive particles when they are moving near the boundary geometry, which can be a crucial point for mesh-based methods due to large mesh deformations. The abrasive particles are modeled with the Discrete Element Method (DEM). The dynamic behavior of solid particles can be accurately simulated with the DEM method, whereby the particles are not fixed to a mesh and can move freely in space. The hydraulic machinery is also modeled with the DEM method. In this work triangle particles are used for the geometry of the hydraulic machinery. The mesh is modeled with a 3D computer graphics software and then imported as STL-file. The abrasive wear is simulated with an erosion model. There are several factors which have to be taken into account, e.g., the velocity of the particles or the material properties of the particles and the boundary geometry. Depending on the application the erosion model has to be chosen, in this work the model of Finnie is applied for the analysis.

The work is divided into three parts, theoretical background, simulation and the conclusions.

2 Theoretical Background

First the theoretical background of the SPH and the DEM method is introduced. Then the coupling of the simulation methods is presented.

2.1 Smoothed particle hydrodynamics

The SPH method, a Lagrangian mesh-free computational method, is applied for modeling the fluid. The fluid is described by the Navier-Stokes equations. The SPH method is a suitable approach for describing a fluid with free surfaces and the interface between the fluid and a solid like the boundary geometry or stiff particles, because of its mesh-less character.

At the beginning SPH was applied in [10, 17] to investigate different phenomena in

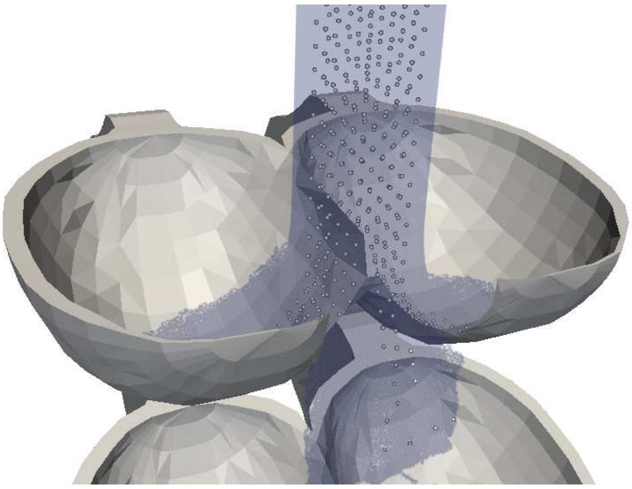

Figure 1: Impact of a free jet with particle loading.

astrophysics. In the past years there are several new fields of applications to which this mesh-free method was applied, like free surfaces [19, 18, 22], fluid-solid interaction [2, 13] and multi-phase flow [3, 11]. Commonly the weakly compressible SPH method or the truly incompressible SPH method is used. The difference is the calculation of the pressure. An overview about the SPH method and its extensions can be found in [16].

The equations for the conservation of mass (1) and for conserving the momentum (2) of the fluid

$$\nabla \cdot \mathbf{v} = 0\,, \tag{1}$$

$$\rho \left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} \right) = -\nabla p + \mu \nabla^2 \mathbf{v} + \mathbf{f} \tag{2}$$

are later discretized with the SPH method. In these equations, $\mathbf{f}$ is the body force acting on the fluid, p is the pressure, $\mathbf{v}$ the velocity, ρ the density and μ the viscosity of the fluid. Basically, there are two steps required to obtain the SPH formulation of the Navier-Stokes equations. The first one is the kernel approximation and the second one is referred to as particle approximation [16]. During these two steps the fluid is discretized into so called particles and at these particles any quantity is described with the functions

$$A\left(\mathbf{r}_a\right) = \sum_b m_b \frac{A_b}{\rho_b} W\left(\mathbf{r}_{ab}, h\right)\,, \tag{3}$$

$$\nabla A(\mathbf{r}_a) = \sum_b m_b \frac{A_b}{\rho_b} \nabla_a W\left(\mathbf{r}_{ab}, h\right)\,. \tag{4}$$

In these functions m_b is the mass, ρ_b the density, $\mathbf{r}_b$ the position of a particle b respectively a, $\mathbf{r}_{ab}$ the distance between particle a and b and h the smoothing length. Depending on the application of the SPH method there are several kernel functions W_{ab} which can be applied in these functions. In this work the classical Gaussian kernel function

$$W(\mathbf{r}_{ab}, h) = \left(\frac{1}{\sqrt{(\pi h^2)^d}}\right) e^{-\mathbf{r}_{ab}^2/h^2} \tag{5}$$

with the dimension d is used for the simulation of the free jet with abrasive particles as loading. The solid abrasive particles within the fluid are not modeled with the SPH method, but with the DEM.

2.2 Discrete element method

The solid particles, here the loading of the free jet, are modeled with the Discrete Element Method [5]. In this method solid particles are interacting. It is also possible to use this method not only for granular but also for bulk material, which consists of free particles which have not a permanent contact like in [8]. In order to model natural failure it is possible to couple several particles with inner-particle bonds. In doing so, a multi-particle body can be formed [9]. The dynamics of the particles can be described with the Newton-Euler equation, [21], which yields

$$m_\mathrm{i}\mathbf{a}_\mathrm{i} = \mathbf{f}_\mathrm{i}\,, \tag{6}$$

$$\mathbf{I}_\mathrm{i} \cdot \dot{\omega}_\mathrm{i} + \omega_\mathrm{i} \times \mathbf{I}_\mathrm{i} \cdot \omega_\mathrm{i} = \mathbf{l}_\mathrm{i}. \tag{7}$$

Here m_i is the mass, $\mathbf{I}_\mathrm{i}$ the inertia tensor and $\mathbf{f}_\mathrm{i}$ and $\mathbf{l}_\mathrm{i}$ are forces and torques. The translational acceleration is $\mathbf{a}_i$ and the angular velocity ω_i. In the presented simulation, particle rotations can be ignored and only the translational part is taken into account.

Several contact models are applied for calculating the force between DEM particles [23]. In this work a Hertzian contact

$$F_\mathrm{ij} = K_\mathrm{ij}\delta_\mathrm{ij}^{\frac{3}{2}} + d\dot{\delta}_\mathrm{ij} \tag{8}$$

is used with

$$K_\mathrm{ij} = \frac{4}{3\pi(h_\mathrm{i} + h_\mathrm{j})}\left(\frac{R_\mathrm{i}R_\mathrm{j}}{R_\mathrm{i} + R_\mathrm{j}}\right)^{\frac{1}{2}},$$

$$h_\mathrm{j} = \frac{1 - \nu_\mathrm{j}^2}{\pi E_\mathrm{j}}.$$

In (8) R_j is the radius of a particle, E_j Young's modulus of the granular material, ν_j the Poisson number, d a damping parameter and δ_ij the overlap of two particles see [14].

2.3 DEM-SPH coupling

There exist different techniques for coupling the DEM and the SPH method, depending on the ratio of size of the SPH particles and the DEM particles. In [12] a coupling technique for small DEM particles is applied and for a much larger DEM particle than SPH particle in [20]. In the presented approach the coupling which is used is similar to the one described in [20].

A neighborhood search is performed every time step in the particle simulation. During the neighborhood search all adjacent particles, which are interacting, are determined. If there is a contact between an SPH and a DEM particle detected, a force for the contact is calculated. Then the contact force is applied to the DEM and the SPH particle. The equation for conserving the momentum of the fluid and the equation of motion for a single DEM particle are

$$m_{\mathrm{i}}\mathbf{a}_{\mathrm{i}} = \mathbf{f}_{\mathrm{i}} + \mathbf{f}_c\,, \tag{9}$$

$$\rho\left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}\cdot\nabla\mathbf{v}\right) = -\nabla p + \mu\nabla^2\mathbf{v} + \mathbf{f} + \mathbf{f}_c\,. \tag{10}$$

In these equations $\mathbf{f}_c$ is the volumetric interaction force between a DEM particle and the fluid.

The force model which is used here, we have already successful applied to other simulations [15]. Similar to [19] we calculate a boundary force in the contact model. The distance and velocity difference with different parameters are taken into account for the force calculation. The same contact model is also applied for the contact of any particle with the boundary geometry.

2.4 Wear

Abrasive wear depends on different properties of the abrasive particles, the fluid and the boundary geometry. One possibility for modeling abrasive wear is the use of an erosion model, often simple models which do not take all properties into account. The system properties which influence abrasive wear can be classified into three categories [1]. The first ones are variables like velocity of the fluid which concern the particle flow, the second ones are particle variables like the shape and the last one are material properties.

There are several models which have different field of application depending on the factors of interest. An overview about several models can be found in [4]. The equation for the removed material derived by Finnie [7] yields

$$W = \begin{cases} \dfrac{mV^2}{\psi pk}\left[\sin(2\alpha) - \dfrac{6}{k}\sin(\alpha)^2\right], & for\,\tan(\alpha) \leq \dfrac{k}{6} \\ \dfrac{mV^2}{\psi pk}\left[\dfrac{6\cos(\alpha)^2}{k}\right], & \tan(\alpha) > \dfrac{k}{6} \end{cases} \tag{11}$$

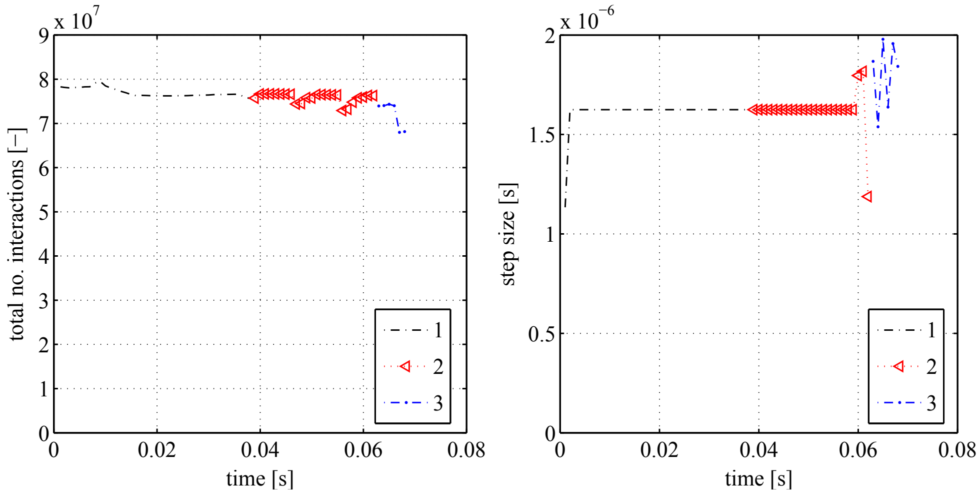

Figure 2: Total number of interactions and step size.

In this equation W is the amount of removed material, m is the mass of the particle, α is the attack angle, k is the ratio of vertical to horizontal force components, ψ is the ratio of the depth of contact to the length of the swarf, V is the absolute velocity of the abrasive particle and p is the flow stress of the component material.

3 Simulation Results

When simulating the impact of a free jet, the simulation of the free surface of the fluid is a crucial point in classical mesh based computational fluid dynamics. Additionally, the interface between the fluid and its loading and the boundary geometry is a challenging point in the presented simulation. Therefore, only mesh-less methods are chosen for this work. In general, the complexity of the geometry is not limited by the approach, as long as there is the mesh data available. Here, for the boundary geometry the simplified bucket of a pelton turbine is used.

Exemplarily in Figure 2 the total number of interactions and the step size of one simulation run are shown. The simulation is divided into three time intervals due to walltime limitation of the supercomputer which is used. For each part a different color is used. It can be seen that the number of interactions is decreasing because the jet is divided by the geometry. The number of interactions between SPH particles is decreasing, and the step size is varying more due to the contact to the boundary.

In Figure 3 the fluid is shown at two different states of the simulation. The loading of the free jet without visualization of the fluid is shown in Figure 4. In these figures only one time instant of the simulation is shown. In this configuration the angle between the fluid and the upper edge of the bucket is zero. The material of the abrasive particles is silicon dioxide in form of quartz. The percentage of the loading to the total volume is 0.15%. The properties of the fluid are those of water at 20° Celsius.

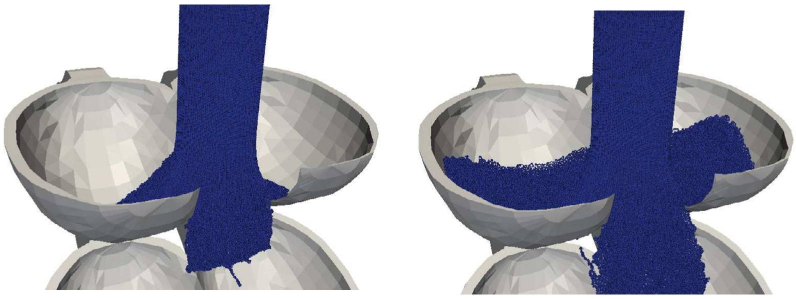

Figure 3: Free jet at two different time instants.

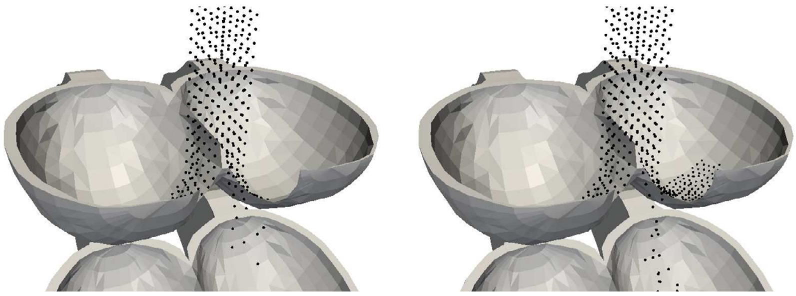

Figure 4: Loading of the free jet at two different time instants.

For the analysis of the wear, are three different configurations used. In the three configurations the angle between the free jet and the upper edge of the bucket is varied about $\pm$ 20°. The angle between the bucket and the jet in these configurations can be seen in Figure 5. The material parameters of the boundary geometry for the wear model are taken from [7]. In Figure 5 the wear for three different angles is shown.

Depending on the angle the wear is varying and the pattern of wear in the bucket is different for each configuration.

4 CONCLUSIONS

The simulation of the impact of a free jet is presented. The modeling of the fluid, the abrasive particles and the boundary geometry with mesh-less methods works fine for this kind of application. The simulation of the free surfaces of the fluid and the interface between the fluid and the particles needs no additional complicated computation. Depending on the number of particles the computational time is rising. For three different configurations the wear is simulated and the results show different wear patterns. Wear is depending on several parameters, which have to be chosen carefully. Also the velocity of the fluid and the proportion of volume is influencing to the results. In this work, the influence of the angle between the fluid jet and the upper edge of the bucket is varied. The next step will be varying other parameters such as the loading ratio. The approach using mesh-less methods in combination with a wear model is well suited for the analysis

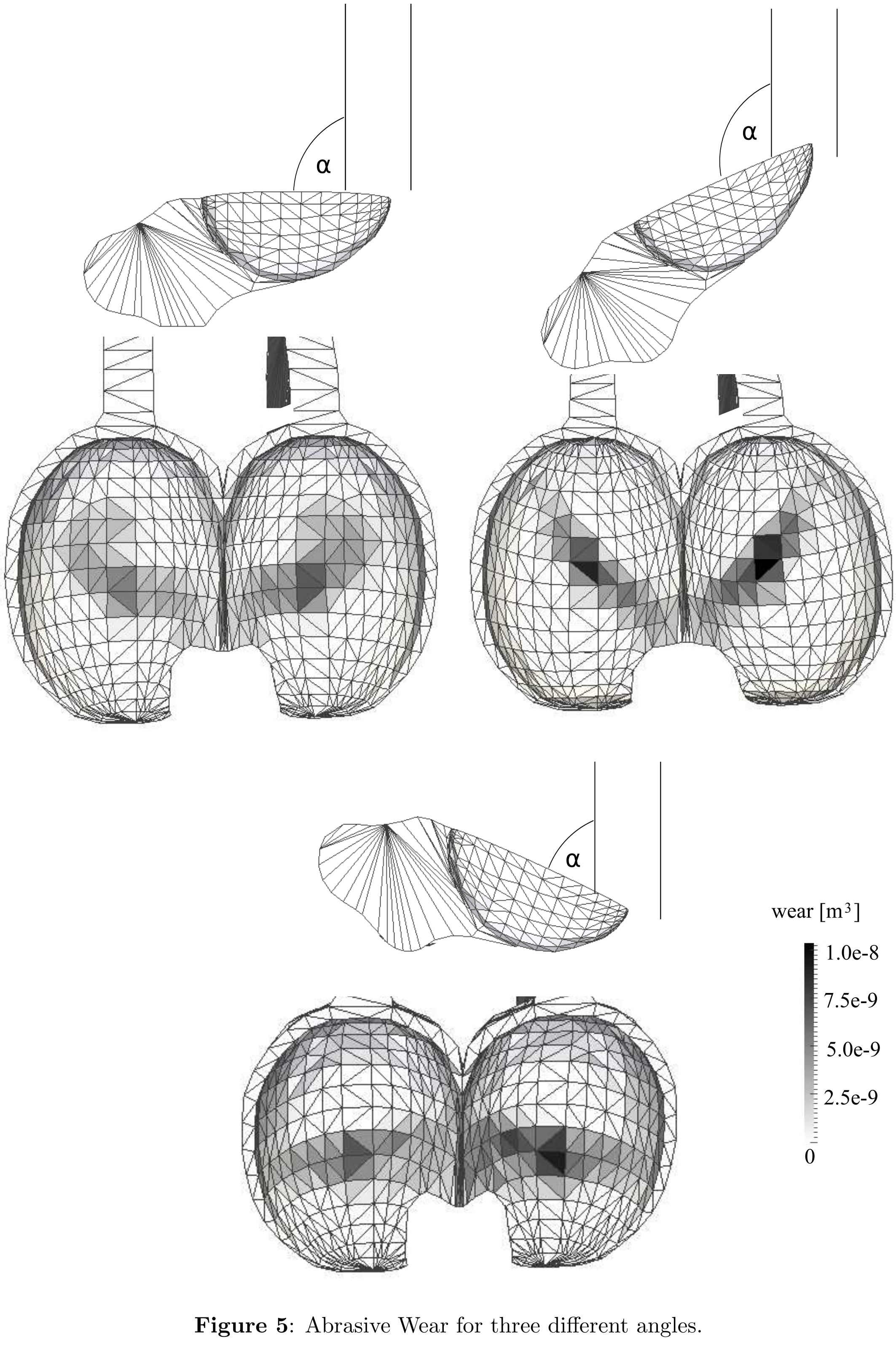

Figure 5: Abrasive Wear for three different angles.

of wear.

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