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NUMERICAL SIMULATION OF LANDSLIDE-RESERVOIR INTERACTION USING A PFEM APPROACH

M. CREMONESI[1] AND U. PEREGO[2]

[1] Department of Civil and Environmental Engineering - Politecnico di Milano
Piazza Leonardo da Vinci, 32, 20133 Milano, Italy
massimiliano.cremonesi@polimi.it

[2] Department of Civil and Environmental Engineering - Politecnico di Milano
Piazza Leonardo da Vinci, 32, 20133 Milano, Italy
umberto.perego@polimi.it

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Abstract. A Particle Finite Element Method is here applied to the simulation of landslide-water interaction. An elastic-visco-plastic non-Newtonian, Bingham-like constitutive model has been used to describe the landslide material. Two examples are presented to show the potential of the approach.

1 INTRODUCTION

Landslides are extreme natural phenomena that occur frequently and can cause a large number of casualties and extensive damage to infrastructures. One of the most critical situations is a landslide caused by a seismic shaking or by heavy rainfalls impinging into the reservoir of a dam. This landslide could generate a wave whose dimensions can endanger the safety of the structure and of the surrounding area, a situation which occurred with tragic consequences in Italy in 1963 as a consequence of the landslide of Mount Toc that led to the Vajont tragedy.

Recent developments in the simulation techniques of coupled problems have led to efficient analysis procedures allowing for the simulation of landslide-reservoir interactions (see e.g. [1, 2]). For the simulation of these phenomena, a numerical approach must be capable of tracking interfaces, free surfaces undergoing large displacements and fast propagating waves. A recently developed Lagrangian finite element approach formulated in the spirit of the Particle Finite Element Method [3, 4, 5] , is here reconsidered and adapted to the specific case of landslide-reservoir interaction. Due to its ability to automatically track free-surfaces and interfaces, the method is particularly suited for these types of problems.

The Navier-Stokes equations are used to model both the landslide and the reservoir motion. A classical Newtonian law is used to describe the basin water. Conversely, the constitutive behaviour of the landslide is described using an elastic-visco-plastic law based on a non-Newtonian, Bingham-like fluid. This model allows to consider both the triggering of the landslide (for example due to seismic shaking) and its propagation along a slope. The interaction between the landslide and the reservoir water is entirely described using the PFEM, without introducing any other algorithm.

The proposed approach has been validated against benchmarks taken from the literature, showing a good agreement with the expected results.

2 BALANCE EQUATIONS

In a moving reference domain Ω_t, the equations of motion of both water and landslide can be written in the ALE form [6]:

$$\rho\left(\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{c}\cdot\nabla)\mathbf{u}\right) = \nabla\cdot\boldsymbol{\sigma} + \rho\mathbf{b} \quad \text{in } \Omega_t\times(0,T) \tag{1}$$

$$\nabla\cdot\mathbf{u} = 0 \quad \text{in } \Omega_t\times(0,T) \tag{2}$$

where ρ is the density of the fluid, $\mathbf{u}$ is the velocity, σ is the Cauchy stress tensor and the spatial operator ∇ is defined in the reference configuration Ω_t. The convective velocity $\mathbf{c}$ is defined as:

$$\mathbf{c} = \mathbf{u} - \mathbf{v} \tag{3}$$

where $\mathbf{v}$ represents the mesh velocity . In general an equation governing the evolution of the mesh $\mathbf{v}$ is needed [6]. The standard Eulerian description of the equation of motion can be recovered selecting $\mathbf{v} = 0$ (i.e. $\mathbf{c} = \mathbf{u}$), so imposing that the mesh is fixed. Conversely, the Lagrangian description is obtained imposing $\mathbf{v} = \mathbf{u}$ (i.e. $\mathbf{c} = 0$), so that the mesh moves with the velocity of the fluid.

Equations (1) - (2) must be supplemented with proper initial and boundary conditions. The boundary $\partial\Omega_t$ is assumed to be partitioned in three non-overlapping subsets $\partial\Omega_t = \partial_1\Omega_t \cup \partial_2\Omega_t \cup \partial_3\Omega_t$. On $\partial_1\Omega_t$ standard Dirichlet boundary conditions are imposed, on $\partial_2\Omega_t$ surface forces are applied while on $\partial_3\Omega_t$ slip boundary conditions are considered. Slip boundary conditions have been introduced to better represent the behaviour of the interface between the slope and the landslide. On this interface, the fluid tangential velocity can be written as:

$$u_t^{slip} = \beta(\tau_{nt} - \tau_0) \tag{4}$$

where τ_{nt} is the tangential component of the traction acting on the surface of normal $\mathbf{n}$, β a parameter defining the amount of slip and τ_0 is a threshold stress. Condition 4 states that the slip is resisted by a tangential force proportional to the relative velocity. For $\beta = 0$ the no-slip boundary condition is recovered, while $\beta \to \infty$ represents the stress free boundary condition.

Equations (1) - (2) are solved in a Lagrangian framework in all the domain except on the boundary where slip conditions are imposed. In fact the Lagrangian nature of the equations allows the use of the PFEM in its original form, avoiding the introduction of a new equation for the mesh motion. Moreover using the Lagrangian approach it is not necessary to stabilize the convective terms typical of the Eulerian framework. On the contrary, to impose slip boundary conditions on the slope surface without moving the mesh nodes, the convective velocity **c** has been defined to be equal to the velocity of the fluid, keeping fixed the position of the boundary mesh nodes.

3 CONSTITUTIVE LAW

Both the landslide and the reservoir water have been modeled as viscous fluids. The Cauchy stress tensor $\boldsymbol{\sigma} = \boldsymbol{\sigma}(\mathbf{x}, t)$ is decomposed into its hydrostatic p and deviatoric $\boldsymbol{\tau}$ components as $\boldsymbol{\sigma} = -p\mathbf{I} + \boldsymbol{\tau}$ where $\mathbf{I}$ is the identity tensor.

Water is assumed to be a Newtonian isotropic incompressible fluid. Focusing on the one-dimensional case, the constitutive law can be expressed as:

$$\tau = \mu\dot{\gamma} \tag{5}$$

where μ is the dynamic viscosity and $\dot{\gamma}$ is the one-dimensional shear rate.

The landslide material is assumed to obey an elastic-visco-plastic non-Newtonian, Bingham-like constitutive model, whose rheological model is sketched in figure 1. The

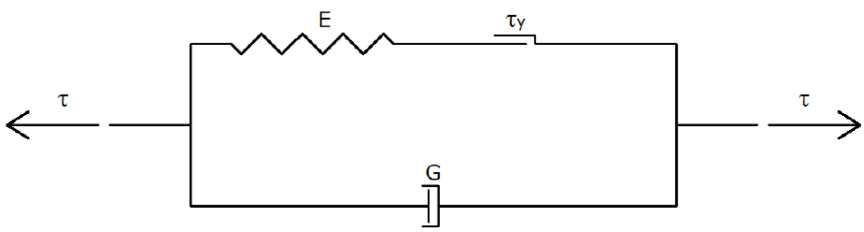


Figure 1: Rheological model for the landslide constitutive law.

presence of an elastic term makes the model able to consider also the initial phase of static equilibrium which precedes the activation of the landslide motion.

In the initial static equilibrium phase small elastic strains can take place. Moreover the velocities are small and the deviatoric effective stress is below the yield limit so that the viscous strains are also small. When external actions trigger the landslide motion and the elastic limit is exceeded, large viscoplastic deformations take place, so that the elastic part of the strain can be neglected. Then, the running landslide behaves as a viscoplastic Bingham fluid.

The introduction of the static phase changes the balance equation (1) introducing an elastic internal force contribution, in addition to the standard viscous term. As usual, the primary variables are nodal velocities and pressures, but the displacements should be computed through time integration to allow for the computation of the stiffness contribution in the static phase.

In the assumed model, the deviatoric stress τ can be expressed in one dimension as:

$$\tau = \begin{cases} \mu\dot{\gamma} + G\gamma^e & \text{for} \quad \tau < \tau_y \\ \tau_y \frac{\dot{\gamma}}{|\dot{\gamma}|} + \mu\dot{\gamma} & \text{for} \quad \tau \geq \tau_y \end{cases} \tag{6}$$

where γ^e is the elastic part of the deviatoric strain, $\dot{\gamma} = \dot{\gamma}^e + \dot{\gamma}^p$ is the deviatoric strain rate and τ_y a yield shear stress. When $\tau < \tau_y$ the behaviour is viscoelastic and dominated by the elastic term $G\gamma^e$ since $\dot{\gamma} \simeq 0$, conversely when the yield stress is reached $(\tau \geq \tau_y)$a viscoplastic behaviour is obtained with $\gamma^e = 0$.

The Mohr-Coulomb criterion has been chosen to describe the cohesive and frictional behavior of flowing granular material, so that the yield shear strength of the model τ_y is defined as (see e.g. [1]):

$$\tau_y = c + p \ \tan(\varphi) \tag{7}$$

where φ is the friction angle and c is the cohesion.

To simplify the numerical solution, an exponential approximation of the Bingham-like model is then introduced. The viscosity μ is replaced by an apparent viscosity $\widetilde{\mu}$ which directly accounts for the presence of the yield limit.

$$\tau = \begin{cases} \mu\dot{\gamma} + G\gamma^e & \text{for} \quad \tau < \tau_y \\ \widetilde{\mu}\dot{\gamma} & \text{for} \quad \tau \geq \tau_y \end{cases} \tag{8}$$

where the apparent viscosity $\widetilde{\mu}$ is defined as:

$$\widetilde{\mu} = 2\mu + \frac{p \cdot \tan(\varphi)}{|\dot{\gamma}|} \left(1 - e^{-n|\dot{\gamma}|}\right) \tag{9}$$

The exponential term in (9) has only a regularization purpose [4, 10], and has not to be given a constitutive interpretation. The extension of the constitutive model to 3D is straightforward.

This model can be conveniently used to describe landslides originated from layered slopes. Furthermore, the soil transition from an initial static equilibrium state to an unstable landslide, due to an imposed ground acceleration, can be also accounted for.

4 NUMERICAL METHOD

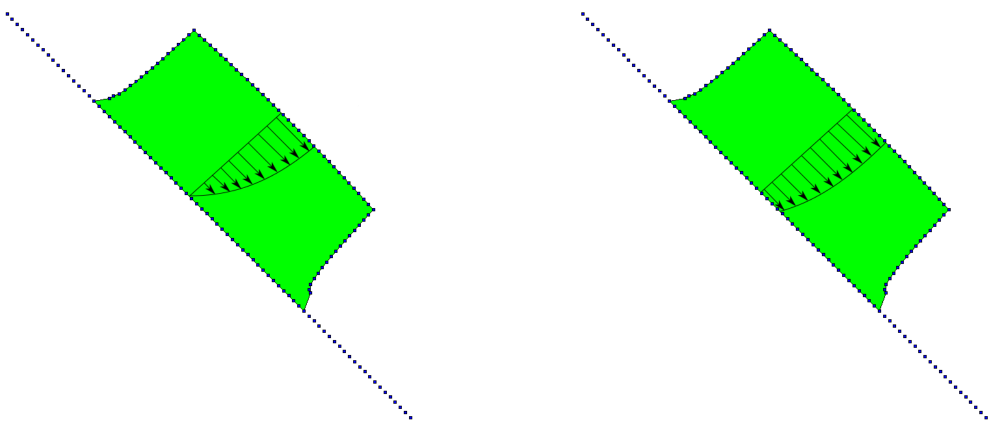
The Particle Finite Element Method (PFEM) has been chosen for the numerical solution of the differential problem (1) - (2). This method was originally developed [5, 7, 8, 9] for solving problems involving free surfaces fluid flows and fluid-structure interaction. The method is here revisited and applied to the simulation of landslides, their interaction with a basin and the generation and propagation of water waves.

A classical Finite Element procedure is used to discretize the problem in space while a backward Euler scheme is employed for the time integration. In the spirit of the Particle Finite Element Method, to avoid excessive mesh distortion, the domain is frequently remeshed. An index of the element distortion is used to check whether the mesh should be regenerated or not. When a new mesh is to be created, a Delaunay triangulation technique is used to redefine the nodal connectivity starting from the current node position. Moreover, an "alpha shape" technique is introduced to identify the free-surfaces and the interacting surfaces between water and landslide. Details on the numerical procedure can be found in [2, 3, 4, 5].

5 NUMERICAL EXAMPLES

5.1 Effect of the slip boundary conditions

In the first numerical example the effects of the slip boundary conditions are investigated. A deformable mass of granular material slides on an inclined plane subjected only to the gravity force. Two cases are considered varying the boundary conditions at the interface between the plane and the sliding mass.



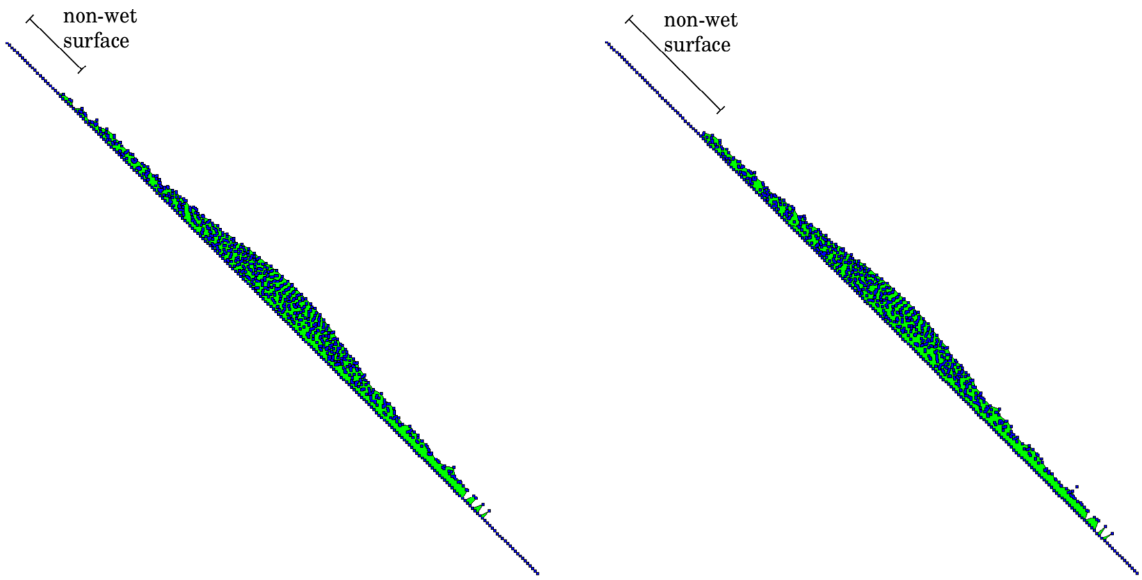
(a) no-slip boundary condition

(b) slip boundary condition

Figure 2: Velocities profiles for slip and no-slips boundary conditions.

Figure 2(a) shows the computed velocity profile obtained imposing no-slip boundary conditions, while Figure 2(b) shows the velocity profile obtained with the slip boundary condition (equation 4). As expected, the no-slip boundary conditions imposes a zero velocity at the interface. On the contrary, the slip conditions induce a discontinuity in the velocity between the inclined plane and the flowing mass, ensuring that the tangential velocity differs from zero at the interface.

Figure 3 shows a snapshot of the two cases at the same time instant, where it can be



(a) no-slip boundary condition (b) slip boundary condition

Figure 3: Velocities profiles for slip and no-slips boundary conditions.

appreciated the different behaviour of the two boundary conditions. In particular the slip boundary conditions leave fewer particles attached to the the inclined plane reducing the wet surface behind the moving mass. Different extensions of the non-wet surface can be obtained with different values of the parameter β.

5.2 Granular flow on a rigid obstruction

The estimation of the impact force of a flowing landslide against a rigid wall is critical for the safety assessment of protection structures such as earth retaining walls. In [11], small-scale tests have been conducted to measure the impact force on a rigid wall of a sand flow. In the same paper, numerical tests have also been performed in an Eulerian framework to analyze and reproduce the laboratory results. The previously described approach has been used to simulate these tests and its results have been validated against both the experimental and numerical results in [11].

Figure 4 depicts a schematic representation of the problem geometry. As suggested in [11], the following physical parameters are used:

$$\rho = 1379\text{Kg/m}^3 \quad \mu = 1\text{Pa s} \quad \varphi = 35°$$

Other details about the geometry of the problem as well as about the parameters calibration can be found in [11].

Four different tests have been performed varying the flume inclination θ. Figure 5 shows the impact force time histories for the different flume inclinations, compared with experimental and numerical outputs of [11]. In all cases, good agreement is obtained.

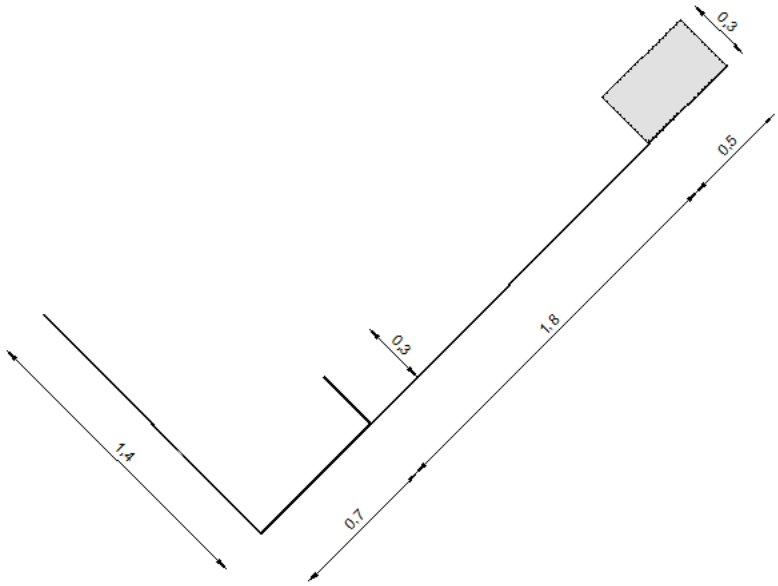


Figure 4: Granular flow on a rigid obstruction: schematic representation of the problem.

Finally, figure 6 shows snapshots of the simulation at different time steps for the case of $\theta = 55°$.

5.3 Landslide interaction with water reservoir

Water waves generated by fast landslides impinging in water basins can be very dangerous for the safety of the surrounding area. To study this phenomenon, the simplified 2D geometry of the Gilbert Inlet, at the head of the Lituya bay, Alaska, considered in [12] and reproducing the experimental setup in [13], has been used to simulate the motion of a landslide along the slope and the formation and propagation of the water waves on the opposite side.

In Figure 7 different snapshots of the simulation are shown. In [13], an experimental landslide run-up on the opposite side of 152 m has been measured, which compares well with the value of 160 m obtained with the present simulation (a run-up height of 226 m was obtained in [12]).

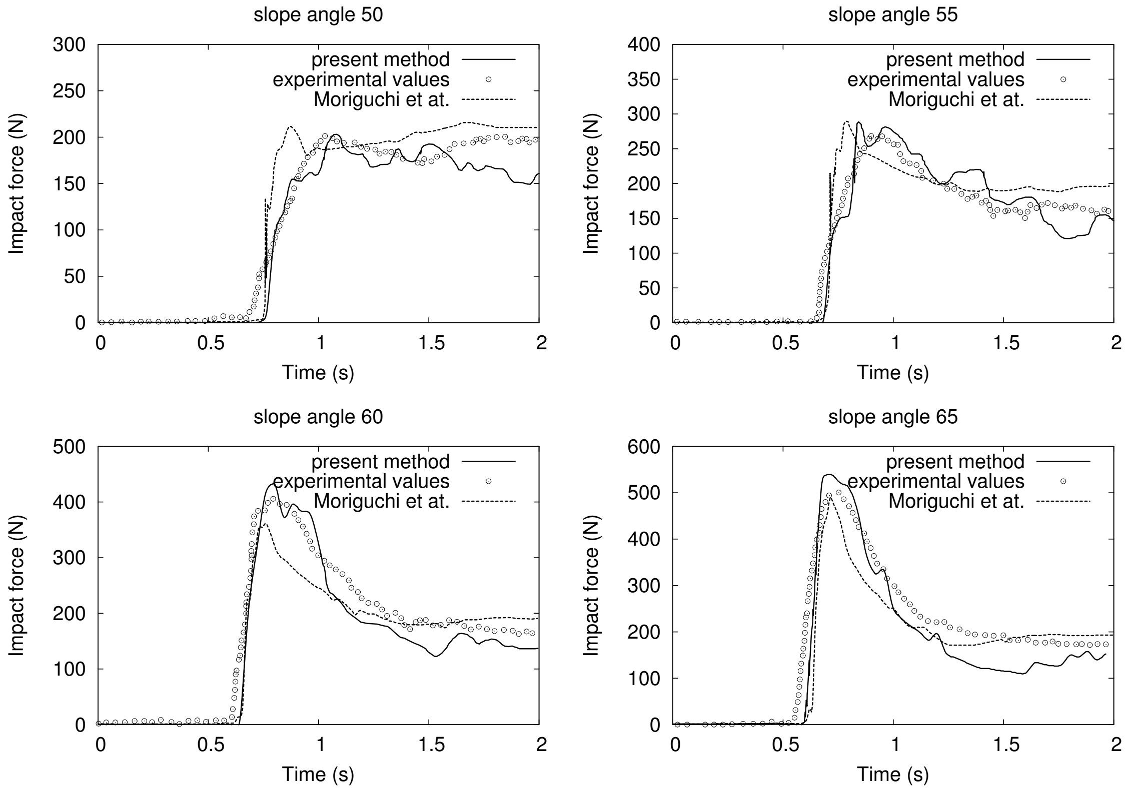


Figure 5: Granular flow on a rigid obstruction: impact force time histories for different flume inclinations.

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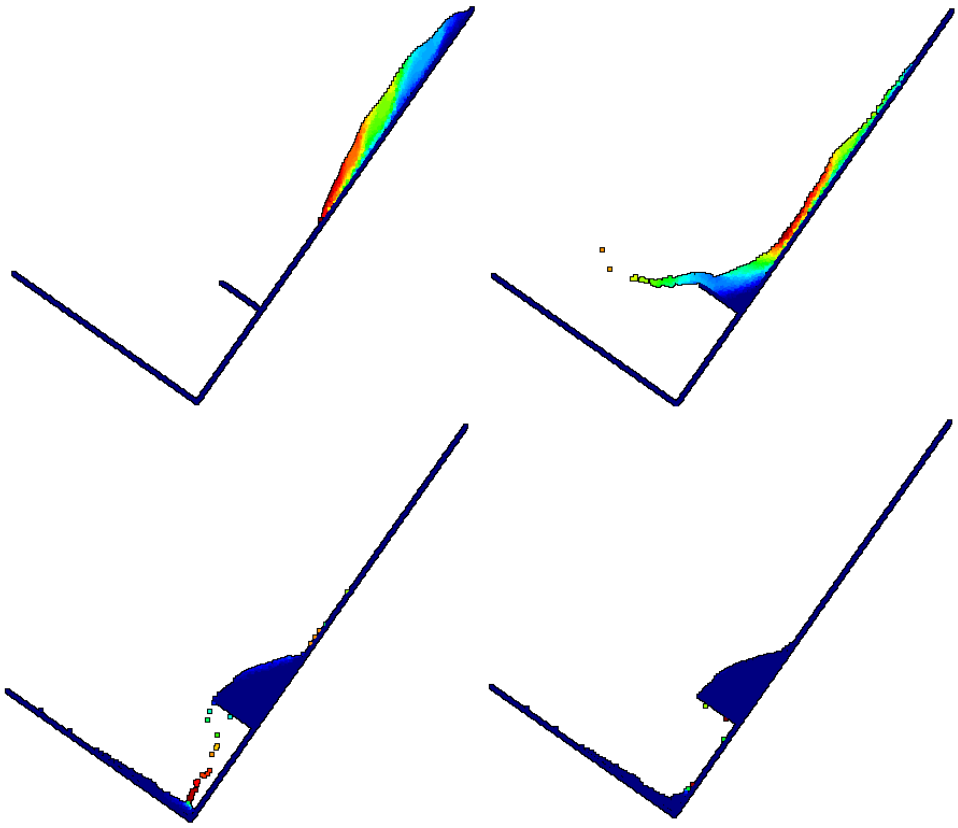

Figure 6: Granular flow on a rigid obstruction: snapshots of numerical flow.

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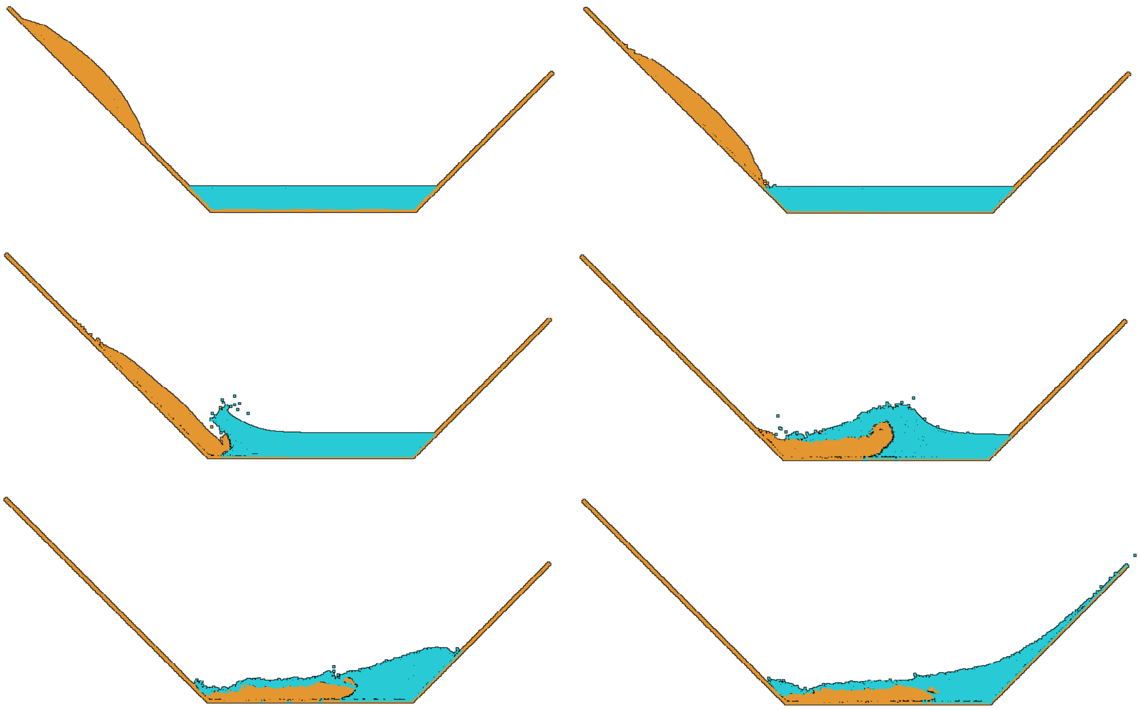

Figure 7: Landslide interaction with water reservoir. Results at different time instants.

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