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THE AERODYNAMICS OF A JET OF PARTICLES IN A CHANNEL

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Abstract. The main cause for dust discharge is ejection, i.e. formation of directional air flows in a stream of a bulk material due to the dynamic interaction of bombarding particles with air. Discovery of induced air flow occurrence regularities enables both forecasting the level of air pollutions with aerosol emission and choosing the optimum engineering solutions of air containment and dedusting. So far we have studied solid particles flowing in a chute and a jet of loose matter. Both situations represent extreme cases of the more general problem of material flowing through a duct with different distances between flow boundaries and duct walls. Without detriment to generality of the problem we shall consider a flat flow limited by vertical walls.

1 INTRODUCTION

So far we have studied solid particles flowing in a chute and a jet of loose matter [1]. Both situations represent extreme cases of the more general problem of material flowing through a duct with different distances between flow boundaries and duct walls. Without detriment to generality of the problem we shall consider a flat flow limited by vertical walls. The flow would be symmetrical with respect to centerline axis OX with positive direction of the axis corresponding to the direction of flowing particles. Owing to the symmetry of aerodynamic field, we shall only study the airflow pattern in the first quadrant, XOY, of the coordinate system chosen by us. Basic relations for studying aerodynamic processes will be provided by dimensionless dynamics equations that could be expressed as follows provided that $N_\tau >> N$:

$$u_x \frac{\partial u_x}{\partial x} + u_y \frac{\partial u_x}{\partial y} = F_x - \frac{\partial \mathbf{P}}{\partial x} + N_\tau \frac{\partial^2 u_x}{\partial y^2}, \quad u_x \frac{\partial u_y}{\partial x} + u_y \frac{\partial u_y}{\partial y} = F_y - \frac{\partial \mathbf{P}}{\partial y} + \frac{\partial}{\partial x}\left(N_\tau \frac{\partial u_x}{\partial y} \right), \quad \frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} = 0. \qquad (1) - (3)$$

2 PLANE-PARALLEL FLOW

In case of a plane-parallel motion of solid particles the airflow initiated by them inside the duct could as well be represented by plane-parallel motion ($u_y = 0$). Combined equations (1) – (3) are thereby greatly simplified. Owing to the continuity equation it holds that $\frac{\partial u_x}{\partial x} = 0$ i.e. the velocity u_x while remaining constant over the flow line depends only on the ordinate

$$u_x = f_u(y). \tag{4}$$

The first two of the combined equations thus will assume the following form:

$$\frac{\partial \mathbf{P}}{\partial x} = F_x + N_\tau \frac{d^2 u_x}{dy^2}; \quad \frac{\partial \mathbf{P}}{\partial y} = F_y. \tag{5}$$

Considering that the projection of inter-component interaction force against on OY axis equals zero ($\upsilon_y = 0$; $u_y = 0$), it follows that $\frac{\partial \mathbf{P}}{\partial y} = 0$ and the pressure only changes along the duct

$$\mathbf{P} = f_p(x). \tag{6}$$

Then the first of combined equations (5) would transform into an ordinary second-order differential equation

$$\frac{d\mathbf{P}}{dx} = F_x + N_\tau \frac{d^2 u_x}{dy^2}, \tag{7}$$

that, as required by (4) and (6), is equivalent to the following combined equations

$$\frac{d\mathbf{P}}{dx} = \Pi; \quad F_x + N_\tau \frac{d^2 u_x}{dy^2} = \Pi, \tag{8}$$

where Π is a constant equal to $\Pi = (\mathbf{P}_к - \mathbf{P}_н)/l$, $\mathbf{P}_н, \mathbf{P}_к$ is the pressure at the beginning and at the end of a duct of length l.

It should be noted that, generally, $\upsilon = f\upsilon(x)$ and the projection of the vector of inter-component interaction force onto OX depends on x and y. This fact contradicts the initial requirement (4). Hence, there is no sense in the supposition about plane-parallel character of injected airflow inside the duct with accelerated movement of material particles.

Uniform movement of particles should be supposed in order to eliminate this inconsistency. As that would significantly restrict the application of findings, let's consider just one special case where material velocity $\upsilon = \upsilon_0$ – const greatly exceeds air velocity, and or generalized exponential distribution of particles results in

$$F_x \approx \beta_0 e^{-\left(\frac{y}{b}\right)^t} \cdot \upsilon_0^2 \tag{9}$$

and the equation (8) becomes

$$N_\tau \frac{d^2 u_x}{dy^2} = -\beta_0 \upsilon_0^2 e^{-\left(\frac{y}{b}\right)^t} + \Pi. \tag{10}$$

Under boundary conditions

$$u_x(b_0) = 0; \quad \left.\frac{du_x}{dy}\right|_{y=0} = 0 \tag{11}$$

the solution has the form

$$u_x = B_k \int_y^{b_0} \left(\int_0^y e^{-\left(\frac{y}{b}\right)^t} dy \right) dy - \Pi_k \frac{b_0^2 - y^2}{2}, \tag{12}$$

where b_0 is a dimensionless duct breadth variable; B_k, Π_k are dimensionless complexes

$$B_k = \frac{\beta_0 \upsilon_0^2}{N_\tau}; \quad \Pi_\kappa = \frac{\Pi}{N_\tau}. \tag{13}$$

In particular, with particles uniformly distributed inside duct (t → ∞)

$$u_x = \left(B_k - \Pi_k\right)\frac{b_0^2 - y^2}{2}. \tag{14}$$

In this case the airflow direction, being the same across the entire duct, is determined with a summation sign $B_k - \Pi_k$. $\Pi_k > B_k$ gives rise to a counterflow while $\Pi_k < B_k$ corresponds to a direct flow. A perfect analogy could be observed in this case with one-dimensional movement of material in a chute.

Air flow rate in this case would be determined by an obvious relation

$$q_э = 2\int_0^{b_0} u_x dy = 2\left(B_k - \Pi_k\right)\frac{b_0^3}{3}. \tag{15}$$

When concentration of material is not constant throughout its cross-section but rather varies (e.g. according to exponential law, $t = 1$), at a certain Π_k the airflow could delaminate so that some air would flow downward (along the centerline with its greater concentration of particles) and the remainder would be displaced upward. Indeed, the solution for (14) at $t = 1$ would assume the form

$$u_x = B_k b\left(b_0 - y\right) - \Pi_k \frac{b_0^2 - y^2}{2} - B_k b^2\left(e^{-\frac{y}{b}} - e^{-\frac{b_0}{b}}\right). \tag{16}$$

with

$$\frac{\Pi_k}{B_k} < 2\left(\frac{b}{b_0}\right)^2 \cdot \left(\frac{b_0}{b} - 1 + e^{-\frac{b_0}{b}}\right) \tag{17}$$

the velocity $u_x(0) > 0$ along the centerline, corresponding to a direct flow zone.

Along the straight line $y = y_0$ where y_0 is the ordinate meeting the equation

$$B_k b\left(b_0 - y_0\right) = \Pi_k \frac{b_0^2 - y_0^2}{2} + B_k b^2\left(e^{-\frac{y_0}{b}} - e^{-\frac{b_0}{b}}\right), \tag{18}$$

the velocity u_x becomes equal to zero. Finally, the $y_0 < y < b_0$ area manifests countercurrent airflow ($u_x < 0$).

In this case the straight line $y = y_0$ becomes a dividing line between direct flow and counterflow. The equality condition determining the first type of airflow is

$$q_э^{\downarrow} = \int_0^{y_0} u_x dy = B_k b\frac{2b_0 y_0 - y_0^2}{2} - \Pi_k \frac{3b_0^2 y_0 - y_0^3}{6} + B_k b^3\left(e^{-\frac{y_0}{b}} - 1 + \frac{y_0}{b} e^{-\frac{b_0}{b}}\right), \tag{19}$$

while, for the second type, it would be

$$q_э^{\uparrow} = \int_{y_0}^{b_0} u_x dy = B_k b\frac{\left(b_0 - y_0\right)^2}{2} - \Pi_k\left(b_0 - y_0\right)\frac{2b_0^2 - b_0 y_0 - y_0^2}{6} + B_k b^3\left[e^{-\frac{b_0}{b}}\left(1 + \frac{b_0 - y_0}{b}\right) - e^{-\frac{y_0}{b}}\right]. \tag{20}$$

3 ONE-DIMENSIONAL FLOW

Plane-parallel flow pattern considered before is extremely unlikely to appear in practice. Transverse overflow of air – the key necessary condition for such currents – is hardly conceivable. Solving the generalized problem analytically would pose insurmountable difficulties at $u_y \neq 0$*. Nor is it easy to solve hydromechanics equations numerically due to nonlinearity [2]. A possible alternative approach may involve equations that bind cross-sectional averages of various flow parameters. As illustrated earlier, one-dimensional problems yield satisfactory outcomes often enough. Thus we could formulate a one-dimensional problem for a jet of loose matter confined to a duct with its wall set apart by the distance b_0 from the centerline. Let's denote the half-breadth of such a jet as b_n. Consequently, there would be two flows: air moving together with material inside a band $0\leq y \leq b_n$ corresponding to an inner dual-component flow and air flowing through a gap between the wall and jet boundary surface corresponding to an outer single-component flow.

Let's suppose that falling particles are distributed uniformly across the jet

For the inner flow ($0 \leq y \leq b_n$) the equation would appear as

$$\frac{\partial}{\partial x}\int_0^{b_n} u_x^2 dy + u_y u_x\Big|_0^{b_n} = \frac{D}{\sqrt{2\upsilon}}\int_0^{b_n}\left(\upsilon - u_x\right)^2 dy - \frac{\partial}{\partial x}\int_0^{b_n}\mathbf{P}dy + N_\tau \frac{\partial u_x}{\partial y}\Bigg|_0^{b_n} \tag{21}$$

For the outer flow ($b_n \leq y \leq b_0$) the equation would appear as

$$\frac{\partial}{\partial x}\int_{b_n}^{b_0} u_x^2 dy + u_y u_x\Big|_{b_n}^{b_0} = -\frac{\partial}{\partial x}\int_{b_n}^{b_0}\mathbf{P}dy + N_\tau \frac{\partial u_x}{\partial y}\Bigg|_{b_n}^{b_0}. \tag{22}$$

To perform the averaging, suppose that pressure remains constant throughout the cross-section of the duct. Thus,

$$\int_0^{b_n}\mathbf{P}dy \approx \mathbf{P}b_n; \quad \int_{b_n}^{b_0}\mathbf{P}dy = \mathbf{P}\left(b_0 - b_n\right). \tag{23}$$

Air velocity in the inner flow, averaged by flow rate, will be designated using u while that in the outer flow will be designated using ω (positive direction matching the direction of *OX* axis):

$$\int_0^{b_n} u_x dy = b_n u; \quad \int_{b_n}^{b_0} u_x dy = \left(b_0 - b_n\right)\omega. \tag{24}$$

Positing that

$$\int_0^{b_n} u_x^2 dy \approx b_n u^2; \quad \int_{b_n}^{b_0} u_x^2 dy \approx \left(b_0 - b_n\right)\omega^2, \tag{25}$$

$$\int_0^{b_n}\left(\upsilon - u_x\right)^2 dy \approx b_n\left(\upsilon - u\right)^2 \tag{26}$$

and assuming normal admission of air on the boundary between inner and outer streams i.e.

* Due to presence of vertical boundaries along the flow this problem could not be reduced to studying self-similar airflow.

$$u_x\left(x,b_n\right)=0;\quad \left.\frac{\partial u_x}{\partial y}\right|_{y=b_n}=0, \tag{27}$$

further considering the consequence to flow symmetry,

$$u_y\left(x,0\right)=0;\quad \left.\frac{\partial u_x}{\partial y}\right|_{y=0}=0, \tag{28}$$

and accounting for a frictional shear stress at the duct wall,

$$-\overline{\tau}_{cm}=N_\tau\left.\frac{\partial u_x}{\partial y}\right|_{y=b_0};\quad \overline{\tau}_{cm}=\frac{\tau_{cm}}{\rho_2 c^2}, \tag{29}$$

integral relations (21) and (22) would lead us to the following system of ordinary differential equations:

$$\frac{du^2}{dx}=\frac{D}{\sqrt{2\upsilon}}\left(\upsilon-u\right)^2-\frac{d\mathbf{P}}{dx}\ \text{at}\ 0\le y\le b_n,, \tag{30}$$

$$\frac{d\omega^2}{dx}=-\frac{d\mathbf{P}}{dx}-\varepsilon_\tau\ \text{at}\ b_n\le y\le b_0,, \tag{31}$$

$$u+\omega\left(r-1\right)=u_0+\omega_0\left(r-1\right)=u_m-\text{const}, \tag{32}$$

where

$$r=b_0/b_n;\quad \varepsilon_\tau=\overline{\tau}_{cm}/\left(b_0-b_n\right).,\tag{33}$$

The latter equation expresses a cross-sectional flow rate conservation law in a duct with impervious walls.

The equation (30) in view of (31) and (32) could be expressed in the following form, making it easier to integrate:

$$\frac{du^2}{dx}+\frac{2\left(u_m-u\right)}{\left(r-1\right)^2}\cdot\frac{du}{dx}-\varepsilon_\tau=\frac{D}{\sqrt{2\upsilon}}\left(\upsilon-u\right)^2, \tag{34}$$

or

$$\left[\frac{2r\left(r-2\right)}{\left(r-1\right)^2}u+\frac{2u_m}{\left(r-1\right)^2}\right]\frac{du}{dx}=\frac{D}{\sqrt{2\upsilon}}\left(\upsilon-u\right)^2+\varepsilon_\tau. \tag{35}$$

This equation could be applied for analyzing the simplest case when $\upsilon=\upsilon_0$ – const and forces of friction against duct walls are negligibly small. Equation (34) would thus become

$$\frac{du}{dx}=\frac{D\left(\upsilon_0-u\right)^2}{\sqrt{2}\upsilon_0\cdot R\left(r,u\right)}; \tag{36}$$

$$R\left(r,u\right)=2\left[r\left(r-2\right)u+u_m\right]/\left(r-1\right)^2, \tag{37}$$

and resolve at initial conditions as follows: $u=u_н$ at $x=x_н$ would assume the form

$$\left[\frac{2r(r-2)}{(r-1)^2}\upsilon_0+\frac{2u_m}{(r-1)^2}\right]\cdot\frac{u-u_н}{(\upsilon_0-u)(\upsilon_0-u_н)}+\frac{2r(r-2)}{(r-1)^2}\ln\frac{\upsilon_0-u}{\upsilon_0-u_н}=\frac{D}{\sqrt{2}\upsilon_0}(x-x_н). \tag{38}$$

Let's analyze the behavior of u and ω along the duct with different b_0/b_n ratios characterizing restriction of flow by duct walls. The following values will be assumed as known initial data:

$$u_н=u_0;\quad \omega_н=\omega_0\quad \text{at } x_н=0. \tag{39}$$

The relation (38) could be transformed into

$$\frac{D}{\sqrt{2}\upsilon_0}x=\left[\frac{2r(r-2)}{(r-1)^2}\upsilon_0+\frac{2u_m}{(r-1)^2}\right]\cdot\frac{u-u_0}{(\upsilon_0-u)(\upsilon_0-u_0)}+\frac{2r(r-2)}{(r-1)^2}\ln\frac{\upsilon_0-u}{\upsilon_0-u_0} \tag{40}$$

or

$$\bar{x}=\left[\frac{2r(r-2)}{(r-1)^2}+\frac{2\bar{u}_m}{(r-1)^2}\right]\cdot\frac{\bar{u}-\bar{u}_0}{(1-\bar{u})(1-\bar{u}_0)}+\frac{2r(r-2)}{(r-1)^2}\ln\frac{1-\bar{u}}{1-\bar{u}_0}, \tag{41}$$

where

$$\bar{u}_m=u_m/\upsilon_0=\bar{u}_0+\bar{\omega}_0(r-1);\quad \bar{u}_0=u_0/\upsilon_0;\bar{\omega}_0=\omega_0/\upsilon_0;\bar{u}=u/\upsilon_0;\quad \bar{x}=xD/\left(\sqrt{2}\upsilon_0\right). \tag{42)-(43}$$

The distance between the duct origin and a cross-section where the inner air flow velocity becomes equal to

$$u=u_m, \tag{45}$$

will be expressed using x_m. This cross-section will henceforth be named critical, and x_m will be regarded as initial run of the duct. In the critical section the continuity equation (32) would make the outer flow velocity equal to zero. Due to the equality condition (41) the relative length of the initial run will be

$$\bar{x}_m=\left[\frac{2r(r-2)}{(r-1)^2}+\frac{2\bar{u}_m}{(r-1)^2}\right]\cdot\frac{\bar{u}_m-\bar{u}_0}{(1-\bar{u}_m)(1-\bar{u}_0)}+\frac{2r(r-2)}{(r-1)^2}\ln\frac{1-\bar{u}_m}{1-\bar{u}_0}. \tag{46}$$

Fig.1 plots the dependence of this length on r in various initial conditions. As it can be seen, the value $\bar{x}_m$ will rise both when the flow centerline is moved away from duct walls (with increasing r) and when initial velocities $\bar{u}_0$ and $\bar{\omega}_0$ are increased. Additional air volume is necessary to ensure increased air velocities.

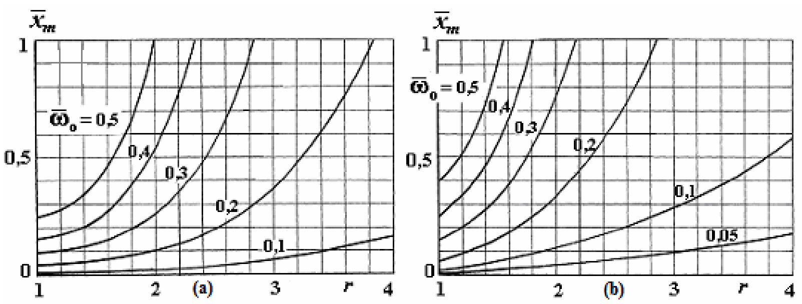


Figure 1: Relative length of the initial run as a function of flow restrictioncat $\bar{u}_0 = 0$ (a) and $\bar{u}_0 = 0,2$ (b)

Beyond the critical section lies a zone of upward outer flow ($\omega < 0$). As air moves further away from the critical section, the upward outer flow will experience increasing flow rates until a maximum is reached at a certain spot that will be named the extreme cross-section. As the equation (37) hints, the presence of an extreme cross-section is conditional on

$$R(r,u_e)=0 \tag{47}$$

or

$$u_э=-u_m/(r(r-2)), \qquad (u_e \le \upsilon_0). \tag{48}$$

As we can see, in case of a downward initial flow in the duct it would only be possible at restriction degrees

$$r<2\,. \tag{49}$$

The length of the zone $x_e - x_m$, (let's name it the initial eddy run length) is determined with the relation (41)

$$\overline{l}_н \equiv \overline{x}_e-\overline{x}_m=\left[\frac{2r(r-2)}{(r-1)^2}+\frac{2\overline{u}_m}{(r-1)^2}\right]\cdot\frac{\overline{u}_e-\overline{u}_m}{(1-\overline{u}_e)(1-\overline{u}_m)}+\frac{2r(r-2)}{(r-1)^2}\ln\frac{1-\overline{u}_e}{1-\overline{u}_m}. \tag{50}$$

The equality condition (41) determines changes in velocity on this run. Further velocity increases u become impossible because the function $R(r,u)$ turns negative and, therefore,

$$\frac{du}{dx}<0, \tag{51}$$

i.e. air begins to escape the inner flow. Air flow rate in the outer counterflow decreases to zero in the next critical section.

The differential equation (36) in this case would be rewritten as

$$\frac{du}{dx}=-\frac{D(\upsilon_0-u)^2}{\sqrt{2}\upsilon_0\left|R(r,u)\right|}, \tag{52}$$

and its in the initial condition $u = u_e$ at $x=x_e$ would become

$$\overline{x}-\overline{x}_e=\left|\left[\frac{2r(r-2)}{(r-1)^2}+\frac{2\overline{u}_m}{(r-1)^2}\right]\cdot\frac{\overline{u}-\overline{u}_e}{(1-\overline{u})(1-\overline{u}_e)}+\frac{2r(r-2)}{(r-1)^2}\ln\frac{1-\overline{u}}{1-\overline{u}_e}\right|. \tag{53}$$

The length $\overline{x}_m-\overline{x}_e$, to be named the final eddy run length, is determined with the relation

$$\overline{l}_к \equiv \overline{x}_m-\overline{x}_e=\left|\left[\frac{2r(r-2)}{(r-1)^2}+\frac{2\overline{u}_m}{(r-1)^2}\right]\cdot\frac{\overline{u}_m-\overline{u}_e}{(1-\overline{u}_m)(1-\overline{u}_e)}+\frac{2r(r-2)}{(r-1)^2}\ln\frac{1-\overline{u}_m}{1-\overline{u}_e}\right|. \tag{54}$$

As it could be seen from a comparison of the result with the equality condition (447),

$$\overline{l}_н=\overline{l}_к, \tag{55}$$

that could be explained by a constant velocity of falling particles. The total length of an eddy, resulting from an obvious relation

$$\overline{l}=2\overline{l}_н=2\overline{l}_к, \tag{56}$$

decreases with decreasing initial airflow velocity in inner and outer flows (Fig.2) with relative duct size kept constant. Lower values of r would produce more eddies in the outer flow (Fig.3). Absolute velocity in a flow of particles fluctuates around average value. At the limit $r \to 1$ it becomes equal to u_0. We have a case of a one-dimensional problem for a chute. The other extreme case could be observed with increasing r. Increasing distances between the flow and duct wall reduces the occurrence of eddies until counterflow could only be observed near the end of the duct. Finally, further increases of r result in exclusively direct flow of air along the entire duct with increasing velocities in the inner flow and decreasing velocities in the outer flow. The limit case of $r \to \infty$ corresponds to a free flow of particles whose air injection at $\upsilon = \upsilon_0 - \mathrm{const}$ could be described in view of (36) and (37) by the equation

$$\frac{du}{dx} = \frac{D}{\sqrt{2}\upsilon_0} \cdot \frac{(\upsilon_0 - u)^2}{2u}, \tag{57}$$

that resolves at $u = u_0$ at $x = 0$ as

$$\frac{\upsilon_0}{\upsilon_0 - u} - \frac{\upsilon_0}{\upsilon_0 - u_0} + \ln\frac{\upsilon_0 - u}{\upsilon_0 - u_0} = \frac{D}{2\sqrt{2}\upsilon_0} x. \tag{58}$$

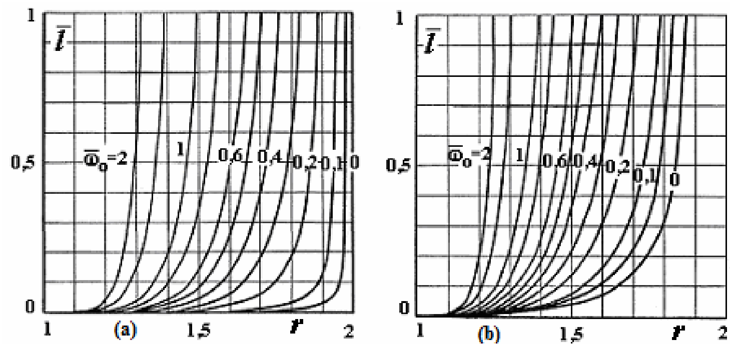


Figure2: Variation in relative eddy length as a function of restricting the flow of loose matter at $\bar{u}_0 = 0$ (a) and $\bar{u}_0 = 0{,}2$ (b)

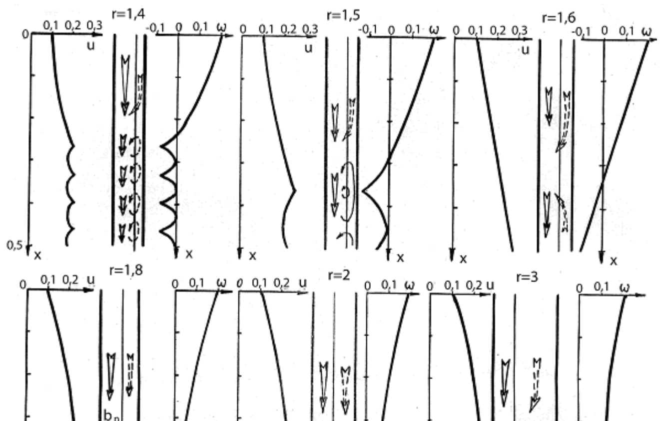
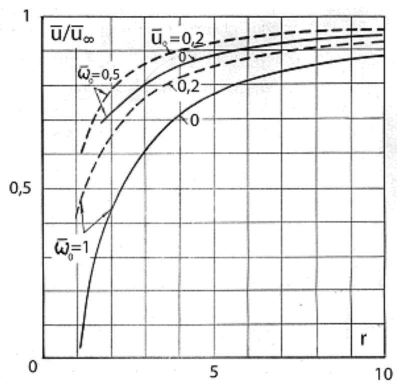


Figure 3: Variation in relative air velocity inside duct for uniformly distributed falling particles of loose matter ($D = \sqrt{2}$; $\upsilon_0 = 0.5$; $u_0 = 0{,}1$ and with $\omega_0 = 0{,}2$)

Fig.4 shows how duct breadth may change final velocity of material injected from the duct with the flow. This change is notably asymptotic in nature. Velocity almost stabilizes when duct walls become spaced by 5 ÷ 7 b_n. Walls produce no braking effect on the velocity of injected air. As material becomes closer to the flow, the quantity of injected air noticeably drops. This happens due to impaired conditions for air overflowing from the outer into the inner flow.



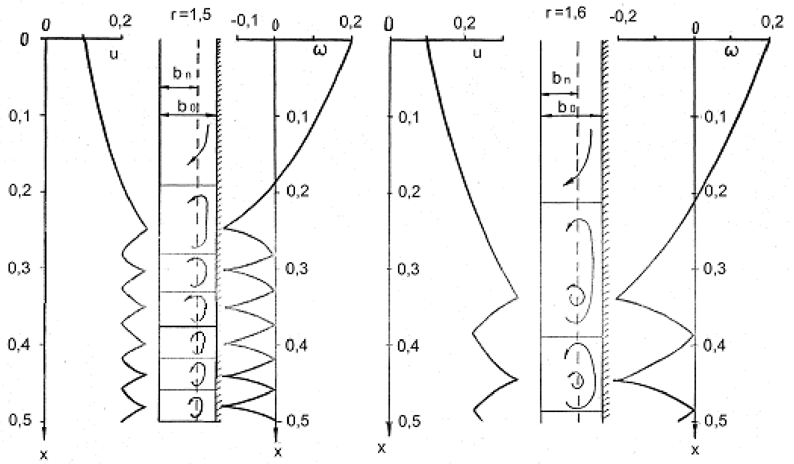


Figure 4: Variation in relative velocity of injected air at the end of the duct ($\bar{x} = 0{,}5$) as a function of flow restriction ($\bar{u}_\infty$ is the injected airflow velocity at the end of the jet at $r \to \infty$)	**Figure 5**: Variation in relative air velocity inside duct for linearly accelerated falling particles of loose matter ($D = \sqrt{2}$; $\upsilon_0 = 0.5$; $u_0 = 0{,}1$; $\omega_0 = 0{,}2$)

A similar flow pattern could be observed in case of linearly accelerated particles of loose matter. The differential equation (46) describing changes in air velocity in the inner flow at negligibly small frictional forces at duct walls could be rewritten as

$$(a_1 u + b_1)\frac{du}{d\upsilon} = \frac{D}{\sqrt{2}}(\upsilon - u)^2,\ a_1 = 2r(r-b)/(r-1)^2; b_1 = 2u_m/(r-1)^2. \qquad (59)\text{-}(60)$$

Substituting for variables

$$\hat{\upsilon} = D(\upsilon a_1 + b_1)/\left(\sqrt{2}a_1^2\right); \quad \hat{u} = D(ua_1 + b_1)/\left(\sqrt{2}a_1^2\right), \qquad (61)$$

the equation (59) could be reduced to the form

$$\hat{u}\frac{d\hat{u}}{d\hat{\upsilon}} = (\hat{\upsilon} - \hat{u})^2, \qquad (62)$$

considered by us when solving the problem of air injection with a free jet.

As an example we can plot calculated ratios using the approximation

$$(\upsilon - u)^2 \approx \upsilon^2\overline{\left(1 - \frac{u}{\upsilon}\right)^2}, \quad \overline{\left(1 - \frac{u}{\upsilon}\right)^2} \approx \left(1 - \frac{u}{\upsilon}\right)^2, \qquad (63)$$

producing satisfactory results for a free jet. The equation (59) would be easy to integrate in view of this approximation. At initial conditions $u = u_н$, $\upsilon = \upsilon_н$ at $x = x_н$ it holds that

$$a_1\frac{u^2 - u_н^2}{2} + b_1(u - u_н) = \gamma_a \frac{D}{\sqrt{2}} \cdot \frac{\upsilon^3 - \upsilon_н^3}{3\upsilon^2}(\upsilon - u)^2,\ \gamma_a = \pm 1. \qquad (64)$$

Whence we can determine

$$u=\frac{B}{2A}\left(\sqrt{1+\frac{4AC}{B^2}}-1\right), \tag{65}$$

where

$$A=a_1/2-z; B=b_1+2\upsilon z; \quad C=a_1u_н^2/2+b_1u_н+\upsilon^2 z; \quad z=\gamma_a D\cdot\left(\upsilon^3-\upsilon_н^3\right)/\left(\sqrt{2}\cdot 3\upsilon^2\right); \upsilon=\sqrt{2x+\upsilon_0^2}. \quad (66)\text{-}(68)$$

Calculation should proceed as follows. The change in injected air velocity on the initial run is determined:

$$x_н=0;\ x_m\ge x\ge x_н;\ \gamma_a=1;\ u_н=u_0;\ \upsilon_н=\upsilon_0. \tag{69}$$

The formula (65) is used to calculate velocity u. Its value grows on this section from u_0 to u_m. By further increasing x we transition into the initial run of the first eddy. Without changing initial values of $u_н$, $\upsilon_н$, $x_н$ we end up with $x_m\le x\le x_e,\ u_m\le u\le u_e=-u_m/\left[r(r-2)\right]$, if $r<2$ (the center of the eddy will not be reachable with $r\ge 2$). Further increases of x lead to a transition into the final run of the first eddy. Changes in velocity u are determined by the same relation (65) with different initial values

$$\upsilon_н=\upsilon_{нe}=\sqrt{2x_e+\upsilon_0^2};\quad u_н=u_e;\quad \gamma_a=-1. \tag{70}$$

In this area the velocity u decreases from u_e down to u_m (as x increases from x_e to x_m^I). The initial run of the second eddy occurs here. Changes in the velocity u on this spot could be determined using the formula (65), adjusted for different initial conditions

$$x_н=x_m^I;\quad \upsilon_н=\sqrt{2x_m^I+\upsilon_0^2};\quad u_н=u_m;\quad \gamma_a=1. \tag{71}$$

Velocity increases again from u_m to u_e. After that, the final run of the second eddy begins, so that initial conditions must be adjusted again in order to calculate velocities.

$$x_н=x_e^I;\qquad \upsilon_н=\sqrt{2x_e^I+\upsilon_0^2};\quad u_н=u_e;\quad \gamma_a=-1. \tag{72}$$

The calculation procedure is repeated. As it can be seen, $\gamma_a=+1$ should be posited at initial runs of eddies while $\gamma_a=-1$ should be posited at final runs. These runs differ in length owing to equal acceleration of the particle flow. Unlike the case of uniform motion considered earlier, the initial run is longer than the final run and the second eddy is longer overall than the first one. This becomes evident on Fig.5 showing calculated flow patterns for a jet in a duct using the same initial parameters that were used to produce the flow pattern for a uniformly moving flow of loose matter (Fig.3).

For airflows inside a cylindrical duct where a stream of falling particles is located coaxially, integral dynamics equations could be written based of relations (72), (33) as follows:

$$\frac{\partial}{\partial x}2\pi\int_0^{r_n}u_x^2 rdr+2\pi ru_ru_x\Big|_0^{r_n}=\frac{D}{\sqrt{2}\upsilon}2\pi\int_0^{r_n}\left(\upsilon-u_x\right)^2 rdr-\frac{\partial}{\partial x}\int_0^{r_n}2\pi\mathbf{P}rdr+N_\tau 2\pi r\frac{\partial u_x}{\partial r}\Big|_0^{r_n}\ \text{at } 0\le r\le r_n; \tag{73}$$

$$\frac{\partial}{\partial x}2\pi\int_{r_n}^{r_0}u_x^2 rdr+2\pi ru_ru_x\Big|_{r_n}^{r_0}=-\frac{\partial}{\partial x}2\pi\int_{r_n}^{r_0}\mathbf{P}rdr+N_\tau 2\pi r\frac{\partial u_x}{\partial r}\Big|_{r_n}^{r_0} \tag{74}$$

at $r_n\le r\le r_0$, where r_n, r_0 are dimensionless radii of particles and duct boundaries.

Based on the same assumptions for simplification, namely that the static pressure is constant throughout the cross-section of the duct

$$\int_0^{r_n} \mathbf{P} r dr = \mathbf{P}\frac{r_n^2}{2}; \quad \int_{r_n}^{r_0} \mathbf{P} r dr = \mathbf{P}\frac{r_0^2 - r_n^2}{2};. \tag{75}$$

air admission at the boundary of outer and inner channels occurs radially

$$u_x(x, r_n) = 0; \quad \left.\frac{\partial u_x}{\partial r}\right|_{r=r_n} = 0;. \tag{76}$$

owing to axial symmetry of currents and impermeability of duct walls

$$u_r(x, 0) = 0; \quad \left.\frac{\partial u_x}{\partial r}\right|_{r=0} = 0; \quad u_r(x, r_0) = 0;. \tag{77}$$

in presence of shearing stress at duct walls

$$-\bar{\tau}_w = N_\tau \left.\frac{\partial u_x}{\partial r}\right|_{r=r_0}, \quad \bar{\tau}_w = \tau_w / \left(\rho_2 c^2\right). \tag{78}$$

by introducing averages over cross-sections of the inner and outer flows

$$2\pi\int_0^{r_n} u_x r dr = \pi r_n^2 u; 2\pi\int_{r_n}^{r_0} u_x r dr = \pi\left(r_0^2 - r_n^2\right)\omega, \quad 2\pi\int_0^{r_n} u_x^2 r dr \approx \pi r_n^2 u^2; 2\pi\int_{r_n}^{r_0} u_x^2 r dr \approx \pi\left(r_0^2 - r_n^2\right)\omega^2, \tag{79)-(80}$$

$$2\pi\int_0^{r_n} (\upsilon - u_x)^2\, r dr \approx \pi r_n^2 (\upsilon - u)^2. \tag{81}$$

integral relations are reduced to differential equations of one-dimensional streams

$$\frac{du^2}{dx} = \frac{D}{\sqrt{2\upsilon}}(\upsilon - u)^2 - \frac{d\mathbf{P}}{dx} \text{ at } 0 \le r \le r_n, \quad \frac{d\omega^2}{dx} = -\frac{d\mathbf{P}}{dx} - \varepsilon_\tau \text{ at } r_\text{n} \le r \le r_0,. \tag{82)-(83}$$

$$u + \omega\left(n^2 - 1\right) = u_0 + \omega_0\left(n^2 - 1\right) = u_m,. \tag{84}$$

where n is the ratio between radii of boundaries surrounding the jet of material

$$n = r_0 / r_n; \quad \varepsilon_\tau = \bar{\tau}_w 2 r_0 / \left(r_0^2 - r_n^2\right).. \tag{85)-(86}$$

Therefore, combined equations for an axially symmetric flow would differ from similar equations of a plane problem only in the equation for airflow (84) that depends on relative duct size, squared. The resulting numerical relationships of the planar problem are valid for axially symmetric problem as well. In this case it is just enough to replace r with n^2 in formulations.

These findings are in a qualitative and quantitative agreement with experimental data. Indeed, the described turbulent flows were observed for the first time by A.S. Serenko who researched currents in a sand layer moving along the bottom wall in a one-meter long square pipe [3]. It was noted that air countercurrents occurred not always but only at certain position of the upper (with respect to flowing material) duct wall.

With a clearance height of 40 mm unidirectional current of injected air was observed in the duct. In this case flowing particles fill the entire cross-section of the duct ($r \to 1$). Countercurrents arise when duct clearance height is increased. Notably, air moves in line with particle layer at the beginning but reverses into a counterflow toward the end of the duct. A similar pattern was reported by O.D. Neykov and Ya.I. Zilberberg researching aerodynamics of streams of iron powder in a tilted chute [4].

A.S. Serenko's experiments have shown that the distances from duct inlet to the point where air countercurrent arises could be brought down virtually to zero by obstructing the inlet with a gate valve. In other words, the initial run becomes shorter as the original flow rate of outer flow diminishes – this agrees comfortably with our findings.

It should be noted that circulation inside a duct filled with flowing material throughout the entire cross-section (such circulation could be named “natural’) is likely only in exceptional cases. Natural circulation is hindered by a number of factors. First of all, when lumpy and grainy are handled, particles occupy virtually the entire clearance area of ducts, and the inherent transverse gradient of particle concentration deforms longitudinal velocity profile of injected air rather slightly. When aspiration develops in a descending pattern in a hollow duct area not filled with material, there is an outside positive gradient precluding the occurrence of a countercurrent. An opposite effect would be observed when handling heated material: a thermal head produced by inter-component heat exchange will promote formation of natural circulation.

4 CONCLUSIONS

It was demonstrated that when a free stream of particles is enclosed with impenetrable walls air inflow is hindered and, hence, closed circulation flows occur in $1 < r < 2$. As far as the distance between the walls and the stream surface becomes shorter ($r \to 1$) the length of these whirls and the velocity variation amplitude in the external stream is reduced to zero while flow velocity in a stream of particles tends to a constant value equal to the initial velocity. As far as the distance to the channel walls becomes longer the whirls become longer too and with $r >$there is only an external reverse flow which area is decreased inversely as r.

It was demonstrated that when the channel section is partially filled with a stream of particles averaged integral equations for a boundary layer may be used as a basis for making one-dimensional equations that describe the motion of two-component stream (internal flow) and the air flow in a cavity limited to the stream surface and the channel walls (external flow). The general solutions of these equations may be used to derive particular solutions for one-dimensional problems regarding a chute with the pseudo-uniform distribution of particles and regarding a free jet of freely-falling particles which creates the base for development of a universal methodology of computation of the induced air volumes.

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