THE MODIFIED NUMERICAL SCHEME FOR 2D FLOW-STRUCTURE INTERACTION SIMULATION USING MESHLESS VORTEX ELEMENT METHOD

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Abstract. The modification of the meshless Vortex Element Method based on the Viscous Vortex Domain Method and non-classical approach to boundary conditions satisfaction is developed for numerical simulation in 2D FSI-problems for incompressible flows. The developed approach preserves all advantages of meshless lagrangian vortex methods, however the accuracy of vortex layer computation is much higher than in 'classical' realizations of vortex methods. No-slip boundary condition is satisfied through limit value of tangential velocity vanishing on the airfoil surface while in 'classical' approach normal components of velocity are used. These approaches lead to Fredholm-type integral equation of the second kind and Hilbert-type singular integral equation of the first kind correspondingly.

The developed method allows to simulate flow around fixed, movable and deformable airfoils while the computational cost of the algorithm remains nearly the same. The discretization scheme is constructed in such a way that it provides exactly the same results in direct and inversed motion of the airfoil (i. e. when the rigid airfoil moves translationally in still media and when the airfoil is fixed in the unbounded incident flow).

In addition to number of model problems which shows advantages of the developed approach and its accuracy order, the well-known two-way coupled problem of vortex induced vibrations and wind resonance phenomenon simulation is considered. The obtained dependency of the oscillations amplitude on the eigenfrequency of linear visco-elastic mechanical system (including hysteresis phenomenon) is in good agreement with experimental data and other known numerical simulations. It is shown that meshless vortex method allows to provide numerical simulation in such problems with sufficiently small requirements to memory and computational time and small numerical viscosity.

1 INTRODUCTION

Vortex element method [1, 2, 3] is a well-known meshfree lagrangian CFD method for 2D flow simulation around airfoils. Firstly it had been developed for numerical simulation of inviscid flows, but later it was generalized to newtonian viscous flows. There are several approaches for taking viscosity influence into account: particle strength exchange method, combined mesh and meshfree methods et al. We use 'Viscous Vortex Domain' (VVD) method developed by G.Ya. Dynnikova [4, 5] since this approach is purely lagrangian and due to its high accuracy which proved in number of researches.

When using vortex element method the flow around the fixed airfoil is simulated by a thin vortex layer on the airfoil surface while for the flow simulation around moving or deformable airfoil in general case not only vortex layer but also source layer should be introduced on the airfoil surface. Their intensities depend on time, so they should be computed every time step. If airfoil motion law is known, source layer intensity and the so-called 'attached' vortex layer intensity can be found explicitly as normal and tangent projections of airfoil surface velocities. At the same time 'free' vortex layer intensity is a'priory unknown. The accuracy of the vortex layer intensity computation defines the accuracy of the boundary condition satisfaction on the airfoil surface and consequently the accuracy of vortex wake simulation around the airfoil. However, the existent well-known numerical schemes (Discrete Vortex Method), normally being used in vortex element method, sometimes lead to significant errors.

The aim of this paper is to develop modern numerical scheme for vortex element method based on alternative approaches to boundary condition satisfaction for movable and deformable airfoil and to compare its accuracy in case of flow simulation around smooth airfoils and airfoils with sharp edge.

2 GOVERNING EQUATIONS

Viscous incompressible flow is described by Navier — Stokes equations

$$\nabla \cdot \vec{V} = 0, \qquad \frac{\partial \vec{V}}{\partial t} + (\vec{V} \cdot \nabla)\vec{V} = \nu \Delta \vec{V} - \frac{\nabla p}{\rho},$$

where $\vec{V}(\vec{r},\, t)$ is flow velocity, $p(\vec{r},\, t)$ — pressure, $\rho = \mathrm{const}$ — density of the flow, ν — kinematic viscosity coefficient.

No-slip boundary condition on the movable airfoil surface and boundary conditions of perturbation decay on infinity should be satisfied:

$$\vec{V}(\vec{r},\, t) = \vec{V}_K(\vec{r},\, t), \quad \vec{r} \in K, \qquad \vec{V}(\vec{r},\, t) \to \vec{V}_\infty, \quad p(\vec{r},\, t) \to p_\infty, \quad |\vec{r}| \to \infty.$$

Here $\vec{V}_\infty$ and p_∞ are constant velocity and pressure of the incident flow.

Navier — Stokes equations can be written down in Helmholtz form [6]:

$$\frac{\partial \vec{\Omega}}{\partial t} + \nabla \times (\vec{\Omega} \times \vec{U}) = 0. \tag{1}$$

Here $\vec{\Omega}(\vec{r},\, t) = \nabla \times \vec{V}(\vec{r},\, t)$ is vorticity field; $\vec{U}(\vec{r},\, t) = \vec{V}(\vec{r},\, t) + \vec{W}(\vec{r},\, t)$, $\vec{W}(\vec{r},\, t)$ is the so-called 'diffusive velocity', which is proportional to viscosity coefficient [4]:

$$\vec{W}(\vec{r},t) = \nu \frac{(\nabla \times \vec{\Omega}) \times \vec{\Omega}}{|\vec{\Omega}|^2}.$$

Equation (1) means that vorticity which exists in the flow moves and its velocity is $\vec{U}$. 'New' vorticity is being generated only on airfoil surface, so we can consider that the vorticity distribution in the flow $\Omega(\vec{r},\, t)$ is always known.

The streamlined airfoil influence is equivalent to superposition of the attached vortex $\gamma_{att}(\vec{r},\, t)$ and source $q_{att}(\vec{r},\, t)$ layers influences and free vortex layer $\gamma(\vec{r},\, t)$ influence:

$$\gamma_{att}(\vec{r},\, t) = \vec{V}_K(\vec{r},\, t) \cdot \vec{\tau}(\vec{r},\, t), \quad q_{att}(\vec{r},\, t) = \vec{V}_K(\vec{r},\, t) \cdot \vec{n}(\vec{r},\, t), \quad \vec{r} \in K,$$

where $\vec{n}(\vec{r},\, t)$ and $\vec{\tau}(\vec{r},\, t)$ are normal and tangent unit vectors on the airfoil surface.

Using generalized Helmholtz decomposition ideas [7] flow velocity can be computed using Biot — Savart law:

$$\vec{V}(\vec{r},\, t) = \vec{V}_\infty + \frac{1}{2\pi} \int_{S(t)} \frac{\vec{\Omega}(\vec{\xi},\, t) \times (\vec{r} - \vec{\xi})}{|\vec{r} - \vec{\xi}|^2} dS + \frac{1}{2\pi} \oint_{K(t)} \frac{\vec{\gamma}(\vec{\xi},\, t) \times (\vec{r} - \vec{\xi})}{|\vec{r} - \vec{\xi}|^2} dl_K +$$
$$+ \frac{1}{2\pi} \oint_{K(t)} \frac{\vec{\gamma}_{att}(\vec{\xi},\, t) \times (\vec{r} - \vec{\xi})}{|\vec{r} - \vec{\xi}|^2} dl_K + \frac{1}{2\pi} \oint_{K(t)} \frac{q_{att}(\vec{\xi},\, t)(\vec{r} - \vec{\xi})}{|\vec{r} - \vec{\xi}|^2} dl_K. \quad (2)$$

Here $S(t)$ is current flow region, $K(t)$ is current airfoil surface; attached and free vortex layer intensities vectors are $\vec{\gamma}_{att} = \gamma_{att}\,\vec{k}$ and $\vec{\gamma} = \gamma\,\vec{k}$ and by analogy $\vec{\Omega} = \Omega\,\vec{k}$, where $\vec{k}$ is unit vector orthogonal to the flow plane; on the airfoil surface $\vec{n}(\vec{r},\, t) \times \vec{\tau}(\vec{r},\, t) = \vec{k}$.

We assume velocity of the airfoil surface $\vec{V}_K(\vec{r},\, t)$ to be known function which is continuous, so vortex layer intensity $\gamma(\vec{\xi},\, t)$ can be found from no-slip boundary condition on airfoil surface:
$$\vec{V}(\vec{r},\, t) = \vec{V}_K(\vec{r},\, t), \quad \vec{r} \in K.$$

If the velocity field $\vec{V}(\vec{r},\, t)$ is considered not only in the flow region, but in the whole two-dimensional space, it has discontinuity on the airfoil surface. It can be shown [1] that the limit value of flow velocity from the airfoil side on the airfoil surface is

$$\vec{V}_-(\vec{r},\, t) = \vec{V}(\vec{r},\, t) - \frac{\gamma(\vec{r},\, t) - \gamma_{att}(\vec{r},\, t)}{2} \vec{\tau}(\vec{r},\, t) + \frac{q_{att}(\vec{r},\, t)}{2} \vec{n}(\vec{r},\, t). \quad (3)$$

Free vortex layer intensity $\gamma(\vec{\xi},\, t)$ can be found from no-slip boundary condition on airfoil surface:
$$\vec{V}_-(\vec{r},\, t) = \vec{V}_K(\vec{r},\, t), \quad \vec{r} \in K.$$

3 NUMERICAL SCHEMES FOR VORTEX LAYER INTENSITY COMPUTATION

When solving problem using Vortex Element Method, vortex wakes normally are being simulated by discrete vortex-type singularities (vortex elements):

$$\Omega(\vec{r},\, t) = \sum_{i=1}^{n} \Gamma_i \delta\big(\vec{r} - \vec{r}_i(t)\big). \tag{4}$$

Here n is number or vortex elements, Γ_i and $\vec{r}_i$ are intensities and positions of vortex elements correspondingly, δ is Dirac delta function. Taking into account (4) we obtain

$$\vec{V}_-(\vec{r}) = \vec{V}_\infty + \sum_{w=1}^{n} \frac{\Gamma_w}{2\pi} \frac{\vec{k} \times (\vec{r} - \vec{\xi}_w)}{\left|\vec{r} - \vec{\xi}_w\right|^2} + \frac{1}{2\pi} \oint_K \frac{\vec{\gamma}(\vec{\xi}) \times (\vec{r} - \vec{\xi})}{\left|\vec{r} - \vec{\xi}\right|^2} dl_\xi + \frac{1}{2\pi} \oint_K \frac{\vec{\gamma}_{att}(\vec{\xi}) \times (\vec{r} - \vec{\xi})}{\left|\vec{r} - \vec{\xi}\right|^2} dl_\xi +$$

$$+ \frac{1}{2\pi} \oint_K \frac{q_{att}(\vec{\xi})(\vec{r} - \vec{\xi})}{\left|\vec{r} - \vec{\xi}\right|^2} dl_\xi - \frac{\vec{\gamma}(\vec{r}) - \vec{\gamma}_{att}}{2} \times \vec{n}(\vec{r}) + \frac{q_{att}(\vec{r})}{2} \vec{n}(\vec{r}), \quad \vec{r} \in K. \tag{5}$$

3.1 N-scheme, based on equality between normal components of velocities

Classical approach which is normally being used in Discrete Vortex Method [1], presupposes that the unknown function $\gamma(\vec{r})$ should be found from the equality between *normal* components of the flow velocity limit value and the airfoil surface velocity:

$$\vec{V}_-(\vec{r}) \cdot \vec{n}(\vec{r}) = \vec{V}_K(\vec{r}) \cdot \vec{n}(\vec{r}), \quad \vec{r} \in K. \tag{6}$$

This equation leads to singular integral equation and the principal value of the corresponding integral should be understood in Cauchy sense.

The equation (6) has infinitely many solutions; to select the unique solution this equation should be solved together with the following equation:

$$\oint_K \gamma(\vec{\xi}) dl_K = \Gamma. \tag{7}$$

Total circulation Γ of the vorticity layer on the airfoil can be found from problem statement; it depends on angular velocity and angular acceleration of the airfoil [8].

In order to find numerical solution of equations (6), (7) the so-called 'Discrete Vortex-type' quadrature formula is used, which allows to compute numerically Cauchy principal values but it imposes strong constraints on airfoil discretization: polygon legs (hereinafter we call them 'panels') which approximate curvilinear airfoil surface should have nearly the same lengthes, integral equation is satisfied only in 'collocation' points $\vec{k}_i$ placed precisely at centers of panels. All the vorticity from vortex layer assumed to be concentrated in vortex elements with circulations Γ_j placed in vertices $\vec{c}_j$ of the polygon (i. e., at the ends of panels).

Such an approach is hereinafter called N-scheme. It should be noted that vortex wake influence at collocation points can be computed straightforward, however, attached vortex and source layers influences computation is not a trivial procedure.

Systematic studies of the N-scheme accuracy were carried out for the case of a fixed rigid (non-deformable) airfoil. Numerical calculations show that this scheme doesn't allow to obtain high accuracy of free vortex layer intensity calculation. Moreover, significant errors were obtained not only for airfoils with sharp edges (it can be explained easily), but also for circular airfoil when there are vortex elements in the wake near the airfoil — this situation quite typical. Taking into account these disadvantages and restrictions, we can admit that N-scheme has limited applicability, and its accuracy can be not sufficient for correct flow simulation around airfoil. These disadvantages can become unacceptable in case of movable and deformable airfoils.

3.2 T-scheme, based on equality between tangent components of velocities

In order to solve problems, described in previous subsection, another approach can be implemented. It is shown [7] that 'boundary condition' (6) is equivalent from mathematical point of view to the following condition

$$\vec{V}_-(\vec{r}) \cdot \vec{\tau}(\vec{r}) = \vec{V}_K(\vec{r}) \cdot \vec{\tau}(\vec{r}), \quad \vec{r} \in K, \tag{8}$$

which corresponds to the equality between *tangent* components of the flow velocity limit value and the airfoil surface velocity.

The equation (8) in association with (5) leads to the following integral equation

$$\frac{1}{2\pi} \oint_K \left(\frac{\vec{k} \times (\vec{r} - \vec{\xi})}{\left|\vec{r} - \vec{\xi}\right|^2} \cdot \vec{\tau}(\vec{r}) \right) \gamma(\vec{\xi}) dl_\xi - \frac{\gamma(\vec{r})}{2} =$$

$$= -\left(\left(\vec{V}_\infty - \vec{V}_K(\vec{r}) \right) \cdot \vec{\tau}(\vec{r}) + \sum_{w=1}^n \frac{\Gamma_w}{2\pi} \frac{\vec{k} \times (\vec{r} - \vec{\xi}_w)}{\left|\vec{r} - \vec{\xi}_w\right|^2} \cdot \vec{\tau}(\vec{r}) + \right.$$

$$\left. + \frac{1}{2\pi} \oint_K \left(\frac{\vec{k} \times (\vec{r} - \vec{\xi})}{\left|\vec{r} - \vec{\xi}\right|^2} \cdot \vec{\tau}(\vec{r}) \right) \gamma_{att}(\vec{\xi}) dl_\xi + \frac{1}{2\pi} \oint_K \frac{(\vec{r} - \vec{\xi}) \cdot \vec{\tau}(\vec{r})}{\left|\vec{r} - \vec{\xi}\right|^2} q_{att}(\vec{\xi}) dl_\xi + \frac{\gamma_{att}(\vec{r})}{2} \right) \tag{9}$$

It should be noted that in case of smooth airfoils the equation (9) is Fredholm-type integral equation with bounded kernel. The equation (9) as well as the singular equation which follows from (6) has infinitely many solutions; in order to select the unique solution the same equation (7) should be solved together with it.

This approach doesn't have restrictions of N-scheme. Nevertheless, if we use nearly the same ideas as in Discrete Vortex Method, the accuracy will remain very low. So it seems to be rationally to use some other approach: firstly, not to concentrate vorticity into vortex elements and secondly solve integral equation (9) in weak formulation (i.e., on average along the panels) instead of its satisfying at certain collocation points.

Vortex layer intensity is supposed to be piecewise constant function, we denote its unknown value γ_i on the ith panel and intensities of attached vortex and sources layers are similarly should be approximated by piecewise constant functions $\gamma_{att,i}$ and $q_{att,i}$ correspondingly. So the integral equation (9) is approximated by the following algebraic equations:

$$\frac{1}{L_i}\left(\int_{K_i}\sum_{j=1}^{N}\frac{\gamma_j}{2\pi}\left(\int_{K_j}\frac{\vec{k}\times(\vec{r}-\vec{\xi})}{|\vec{r}-\vec{\xi}|^2}dl_\xi\right)dl_r\right)\cdot\vec{\tau}_i-\frac{\gamma_i}{2}=$$

$$=-\frac{1}{L_i}\left(\int_{K_i}\sum_{w=1}^{n}\frac{\Gamma_w}{2\pi}\left(\frac{\vec{k}\times(\vec{r}-\vec{r}_w)}{|\vec{r}-\vec{r}_w|^2}\right)\cdot\vec{\tau}_i\,dl_r+\int_{K_i}\sum_{j=1}^{N}\frac{\gamma_j^{att}}{2\pi}\left(\int_{K_j}\frac{\vec{k}\times(\vec{r}-\vec{\xi})}{|\vec{r}-\vec{\xi}|^2}\cdot\vec{\tau}_i\,dl_\xi\right)dl_r+$$

$$+\int_{K_i}\sum_{j=1}^{N}\frac{q_j^{att}}{2\pi}\left(\int_{K_j}\frac{(\vec{r}-\vec{\xi})\cdot\vec{\tau}_i}{|\vec{r}-\vec{\xi}|^2}dl_\xi\right)dl_r\right)-(\vec{V}_\infty-\vec{V}_{K,i})\cdot\vec{\tau}_i-\frac{\gamma_i^{att}}{2},\quad i=1,\ldots,N.\quad(10)$$

Here L_i is the ith panel length which denoted as K_i; Γ_w and $\vec{r}_w$ are circulations and positions of vortex elements which simulate vortex wake, $w=1,\ldots,n$; $\gamma_{att,j}$ and $q_{att,j}$ are average values of attached vortex and sources layers along the jth panel correspondingly; $\vec{V}_{K,i}$ is average velocity of the airfoil surface along the ith panel.

Average values in (10) should be calculated by using the following formulae:

$$\gamma_i^{att}=\frac{1}{L_i}\int_{K_i}\gamma_{att}(\vec{\xi})dl_\xi,\quad q_i^{att}=\frac{1}{L_i}\int_{K_i}q_{att}(\vec{\xi})dl_\xi,\quad \vec{V}_{K,i}=\frac{1}{L_i}\int_{K_i}\vec{V}_K(\vec{\xi})dl_\xi,$$

but in practical calculations approximate equalities are used:

$$\gamma_i^{att}\approx\gamma_{att}(\vec{k}_i),\quad q_i^{att}\approx q_{att}(\vec{k}_i),\quad \vec{V}_{K,i}\approx\vec{V}_K(\vec{k}_i).$$

We will call the proposed approach T-scheme. For fixed rigid airfoil this approach is developed in [9]. Coefficients of system (10) can be computed analytically; formulae for all integrals in this system are also derived in [9].

The equation (7), which allows to obtain the unique solution, in case of using T-scheme is approximated by equation

$$\sum_{i=1}^{N}\gamma_i L_i=\Gamma^*.$$

The T-scheme allows to obtain much more accurate results in comparison with N-scheme when simulating flows around fixed rigid airfoils, so it is preferable to use T-scheme for numerical flow simulation around movable and deformable airfoils.

4 NUMERICAL EXPERIMENTS

Now we compare numerical results which can be obtained by using N- and T-schemes. We consider airfoils to be installed at a certain angle of incidence and we assume that they move in the flow with constant horizontal velocity $V = 1$ (direct motion) as well as the same airfoils being immovable in the horizontal constant incoming flow $V_\infty = 1$ (inverse motion); stationary solutions of these problems are the same. Two possible ways of airfoils splitting into panels are considered.

4.1 The numerical scheme with splitting using conform mapping

The simplest implementation is to split the airfoil into panels using the conformal mapping.

Simple shape of some airfoils (circular, elliptical and Zhukovsky airfoils) makes it possible to construct conformal mapping $\xi = \hat{\zeta}(z)$ of the airfoil exteriority on the circle exteriority, that satisfies the necessary conditions at infinity, and the inverse mapping $z = \zeta(\xi)$, transforming the circle exteriority to the airfoil exteriority.

For elliptical and Zhukovsky airfoils these mappings are the following:

$$\hat{\zeta}(z) = \frac{1}{R}\left(z + \sqrt{z^2 - a^2} - H\right), \qquad \zeta(\xi) = \frac{1}{2}\left(R\xi + H + \frac{a^2}{R\xi + H}\right). \tag{11}$$

For elliptical airfoils $a = \sqrt{a_1^2 - b_1^2}$, $R = a_1 + b_1$, $H = 0$, a_1 and b_1 are major and minor semiaxes of the ellipse. For Zhukovsky airfoil $R = \sqrt{(a + d\cos\phi)^2 + (h + d\sin\phi)^2}$, $\phi = \arctan\frac{h}{a}$, $H = ih - de^{-i\phi}$, a, d and h are arbitrary parameters, which correspond to length, width and curvature of the airfoil.

Therefore the circle with radius R is approximated with regular n-gon with vertices $\xi_i = Re^{\frac{2\pi i}{N}}$, $i = 0, \dots, (N-1)$, so, vertices of the n-gon which approximates the airfoil are $z_i = \zeta(\xi_i)$. Ends of panels are considered as points of vortices birth $\vec{c}_i$, collocation points $\vec{k}_i$ are placed at centers of the panels.

Using these conformal mapping we obtain panels with different lengthes, and lengthes of panels are smaller near ends of the major axis of the elliptical airfoil and near forward part and sharp edge of Zhukovsky airfoil. Significant difference between the lengthes of the largest and the smallest panels (almost exactly equal to the ratio of the semiaxes of the ellipse) is not essential, because lengthes of neighbor panels differ slightly. Therefore, according to the formal criteria for N-scheme applicability, both N-scheme and T-scheme can be used for vortex layer intensity calculation.

When flow around Zhukovsky airfoil simulating the lengthes ratio of the largest panel to smallest increases almost proportionally to the number of panels, and there are always panels (near sharp edge) with significantly different lengthes, which are neighbors (for Zhukovsky airfoil with parameters $a = 3.5$, $d = 0.4$, $h = 0.3$ this ratio is about 3. Therefore, N-scheme is unusable for the flow simulation around Zhukovsky airfoil: near the sharp edge there will be significant error. T-scheme for this problem is preferred.

Model problems of vortex layer intensity calculation for elliptical airfoil ($a_1/b_1 = 10$) and Zhukovsky airfoil ($a = 3.5$, $d = 0.4$, $h = 0.3$) have been solved. Both airfoils were installed at the angle of incidence $\beta = \pi/6$. Computations were performed for different number of panels N in cases of direct motion (the airfoil moves in a stationary media) and reversed motion (stationary airfoil in the incoming flow).

If there is no vortex wake near the airfoil these problems can be solved analytically using conformal mappings technique. Using the formulae [9], we can obtain the exact solution $\gamma_{exact}(\vec{r})$ for the vortex layer intensity.

For error estimation in case of using N-scheme we calculate the value

$$\|\Delta\Gamma\|_N = \max_i |\Gamma_i^{ex,c} - \Gamma_i^*|, \quad \Gamma_i^{ex,c} = \int_{K_{i-1}}^{K_i} \gamma_{exact}(\vec{\xi}) dl_\xi. \tag{12}$$

Here Γ_i^* is calculated circulation of vortex elements, $\Gamma_i^{ex,c}$ is circulation obtained from the exact value of the vortex layer intensity on the panels around i^{th} point of birth $\vec{c}_i$.

When using T-scheme for error estimation we use the following formula:

$$\|\Delta\Gamma\|_T = \max_i \left[\left(|\gamma_i^{exact} - \gamma_i|\right) L_i\right], \quad \text{where} \quad \gamma_i^{exact} = \frac{1}{L_i} \int_{K_i} \gamma_{exact}(\vec{\xi}) dl_\xi. \tag{13}$$

The results of flow simulation around elliptical airfoil in cases of direct and reversed motion using N-scheme are shown on fig. 1 on double logarithmic scale.

It is seen that in case of elliptical airfoil N-scheme provides second-order accuracy by increasing the number of panels in both forward and inverse motion; at the same time for inverse motion (fixed airfoil in the flow) the error is larger. If we use conformal mapping for splitting Zhukovsky airfoil into panels calculated error $\|\Delta\Gamma\|_N$ of N-scheme is unacceptably high.

When using T-scheme it is possible to obtain results quite close to the exact solution for both elliptical and Zhukovsky airfoils. The errors $\|\Delta\Gamma\|_T$ of vortex layer intensity calculations for elliptical and Zhukovsky airfoils using T-scheme are shown on fig. 2 on double logarithmic scale.
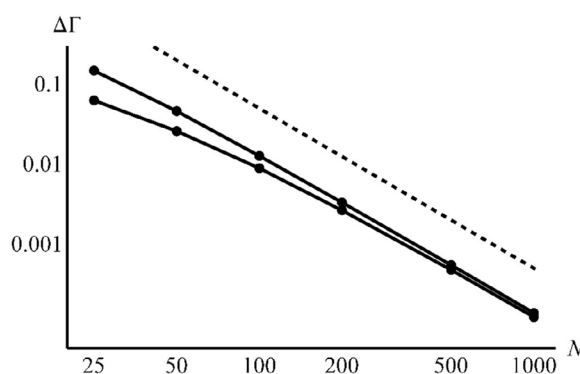

Figure 1: Error $\|\Delta\Gamma\|_N$ of flow simulation around elliptical airfoil (on double logarithmical scale) using N-scheme with different number of panels; dashed line corresponds to $O(N^{-2})$ error

4.2 The numerical scheme with equal lengthes of panels

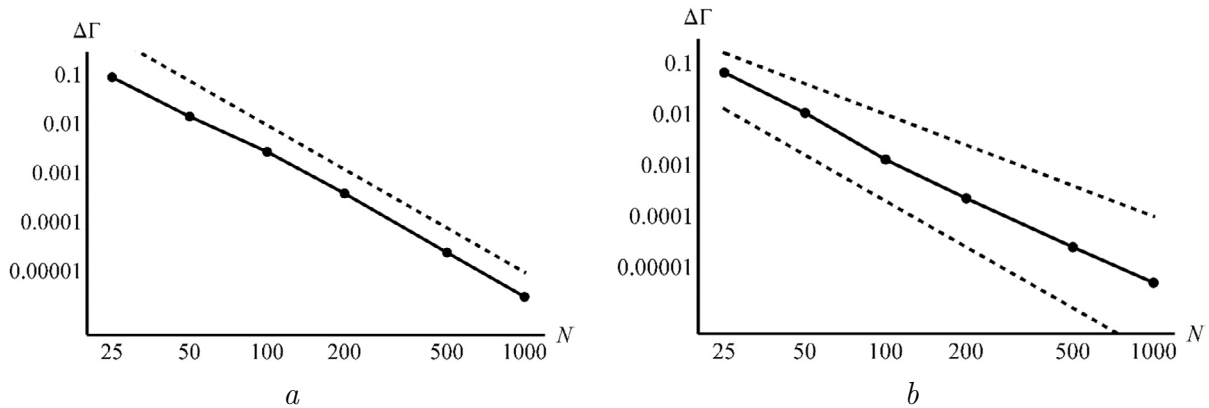An alternative approach to splitting airfoil into panels assumes that all panels have equal (or nearly equal) lengthes.

Figure 2: Error $\|\Delta\Gamma\|_T$ of flow simulation around airfoil (on double logarithmical scale) using T-scheme with conformal mapping usage for airfoil splitting: a — elliptical airfoil, dashed line corresponds to $O(N^{-3})$ error; b — Zhukovsky airfoil, dashed lines correspond to $O(N^{-2})$ and $O(N^{-3})$ errors

As before, in case of using N-scheme the results of the calculation are different for direct and inverse motion, in case of using T-scheme solutions coincide.

The dependence of error of flow simulation around an elliptical airfoil using schemes with equal lengthes of panels is shown in fig. 3. It is seen that simulation for direct motion using N-scheme allows to obtain the second accuracy order, and for reversed motion — the third order (fig. 3, a). For large number of panels the simulation of the flow around an elliptical airfoil using T-scheme allows to get the third accuracy order (fig. 3, b).
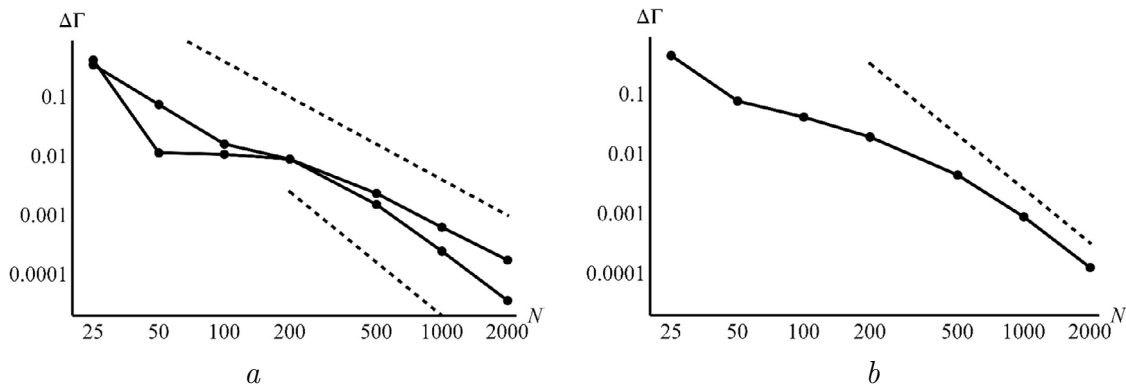

Figure 3: Errors of flow simulation around elliptical airfoil (on double logarithmical scale) using schemes with equal lengthes of panels: a — N-scheme, dashed lines correspond to $O(N^{-2})$ and $O(N^{-3})$ errors; b — T-scheme, dashed line corresponds to $O(N^{-3})$ error

When simulating of the flow around Zhukovsky airfoil using N-scheme with uniform splitting into panels it is possible to obtain correct approximate solution, but only for inverse motion (fig. 4, a); and the error decreases very slowly ($O(N^{-0.5})$). In case of direct motion, the error is unacceptably high and it isn't decrease with increasing of N.
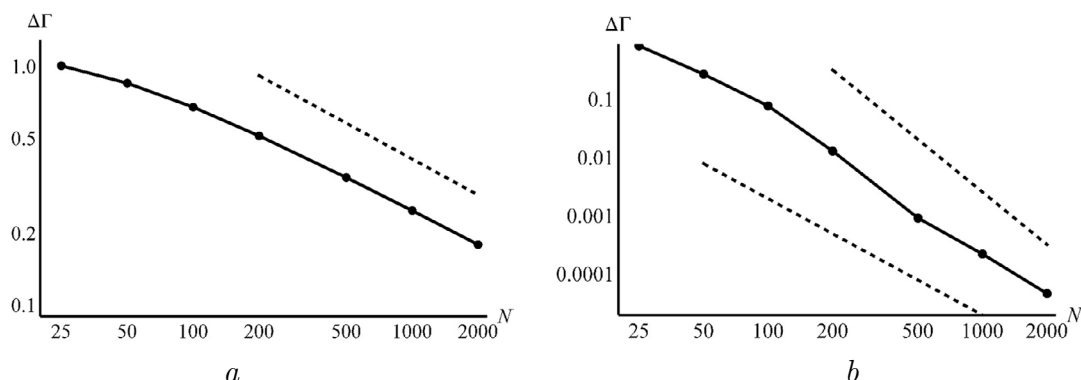
Figure 4: Errors of flow simulation around Zhukovsky airfoil (on double logarithmical scale) using schemes with equal lengthes of panels: a — N-scheme, dashed line corresponds to $O(N^{-0.5})$ error; b — T-scheme, dashed lines correspond to $O(N^{-2})$ and $O(N^{-3})$ errors

It is seen that the T-scheme with uniform splitting of airfoil into panels in case of flow calculation around Zhukovsky airfoil ensures a close to second order accuracy for sufficiently large number of panels.

4.3 Wind resonance phenomenon simulation

For verification of the developed scheme for movable airfoils we consider the motion of the circular airfoil with diameter D across the stream (with one degree of freedom) [10]. Constrain of airfoil assumed to be linear viscoelastic Kelvin — Voigt-type (fig. 5) and its motion is described by the following equation:

$$m\ddot{y}_* + b\dot{y}_* + ky_* = F_y. \tag{14}$$

Figure 5: The circular airfoil with Kelvin — Voigt viscoelastic constraint and vortex wake behind it

Here m is the airfoil mass, b is small damping factor, k is the elasticity coefficient of the constraint, F_y is lift force, y_* is the deviation from the equilibrium. The natural frequency of the system $\omega \approx \sqrt{k/m}$ can be changed by varying of the coefficient k.

In all numerical simulations with the following values of dimensionless parameters have been chosen: Re = 1000, $V_\infty = 3.0$, $m = 39.15$, $b = 0.731$. The dimensionless natural frequency of the system is in the following range:

$$\mathrm{Sh}_\omega = \frac{\omega}{2\pi} \cdot \frac{D}{V_\infty} = 0.150 \ldots 0.280. \tag{15}$$

About 80 computations have been produced by using the developed T-scheme for different values of the dimensionless frequency and the unsteady process have been simulated.
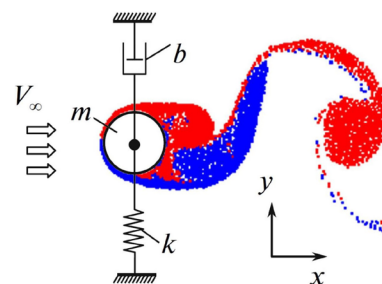
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At the initial time there were still flow and the airfoil in equilibrium position. Time step Δt was equal to 0.01, number of panels which approximates the airfoil $N = 200$. Then the velocity of the incident flow became greater; at time moment $t = 1.0$ (after 100 time steps) it was equal to $V_\infty = 3.0$ and then remained constant. After the transient mode oscillation of airfoil in all cases became close to periodical, their amplitudes dependency on the natural frequency Sh_ω is shown on fig. 6, a (dots connected by line).
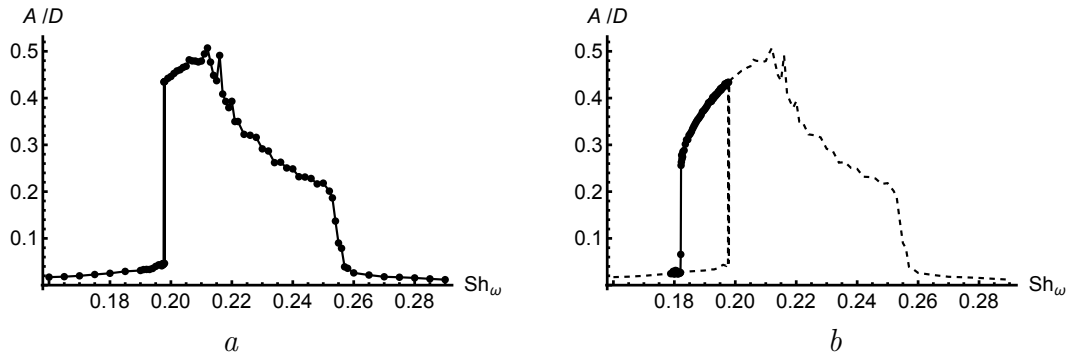

Figure 6: Maximum amplitude of the circular airfoil oscillations simulated using vortex element method: a – initial state of airfoil in equilibrium position, b — initial state of airfoil is close to resonance oscillations

The fig. 6, a shows that there is a sharp increase in the amplitude of oscillations at $\mathrm{Sh}_\omega \approx 0.198$. It's well known that there is hysteresis-type phenomenon [11] and in order to simulate it the following computations were performed: from $t = 0$ to $t = 100$ (10 000 time steps) Sh_ω was equal to 0.21; at this time the oscillations become steady with amplitude $A/D \approx 0.47$, then the elasticity coefficient of the constraint was changed abruptly to the values which correspond to Sh_ω from 0.178 to 0.198 with step 0.00025. In each case after the transient mode new steady oscillations were generated, and their amplitudes are shown on fig. 6, b (dots connected by solid line).

The obtained results for maximum amplitude of oscillation, the resonance frequency and hysteresis properties are in good agreement with the results given in [11, 12].

5 CONCLUSIONS

The problem of numerical simulation of flow around moving and deformable airfoils is considered. Two different numerical schemes can be used: classic N-scheme and developed T-scheme. They differ from each other in approaches to boundary condition satisfaction on the airfoil surface: equality of normal and tangential velocity components of the flow and airfoil respectively.

The algorithms suitable for the solution of practical problems are developed based on T-scheme.

It is shown that T-scheme allows to obtain the solution for wider class of problems in comparison with N-scheme, it it also provides higher accuracy. An important features of

the developed T-scheme is its 'symmetry' — in case of translational motion of the airfoil in the flow, it allows to obtain the same solution in direct and inverse motions.

The model problem of wind resonance of the circular airfoil is considered. The developed approach allows to simulate this phenomenon (including hysteresis) with high accuracy and small computation cost.

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