

## MODELLING CENTRIFUGAL MEMBRANE DEPLOYMENT OF SOLAR SAILS WITH THE DISCRETE ELEMENT METHOD

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**Abstract:** *Spin-stabilized solar sails have been extensively studied in recent years. In this paper, a DEM-based approach is proposed for dynamic analysis of the centrifugal deployment of solar sails. In order to validate the proposed approach, the deployment of a small-scale solar sail similar to “IKAROS” is studied. The membrane is discretised into a number of particles, with no physical contact between them. Non-contact interaction is introduced to model in-plane stiffness of the membrane. In order to improve the accuracy, additional forces are applied to the mass particles to model buckling strength, crease stiffness, air drag and damping. The predicted results of the membrane deployment are compared with the experimental data and numerical results in the literature.*

### 1 INTRODUCTION

Solar sails are a form of spacecraft propulsion using the radiation pressure (also called solar pressure) from stars to push large ultra-thin membranes to high speeds [1]. Japan Aerospace Exploration Agency (JAXA) successfully launched and deployed a solar sail “IKAROS” in 2010. Later, a deployment experiment of a small-scale solar sail similar to “IKAROS” is conducted in a vacuum chamber and corresponding dynamic response was analysed by using a spring-mass model [2].

The dynamic response of solar sail is typically predicted by the finite element methods [4] and spring-mass system models [2]. This paper introduces a DEM-based approach to analyse the dynamic deployment of a solar sail membrane structure. It provides a new alternative approach to effectively analyse the dynamic deployment of solar sails or other membrane structures.

## 2 MODELLING METHOD

### 2.1 DEM basics

Mass particles are the elementary units in Discrete Element Method (DEM) , which are indeformable [5]. The interaction forces between the particles are evaluated based on the overlap between them.

The equation of motion is described as follows:

$$m_i \frac{d^2}{dt^2} \mathbf{r}_i = \mathbf{f}_i, I_i \frac{d}{dt} \boldsymbol{\omega}_i = \mathbf{t}_i \quad (1)$$

where:

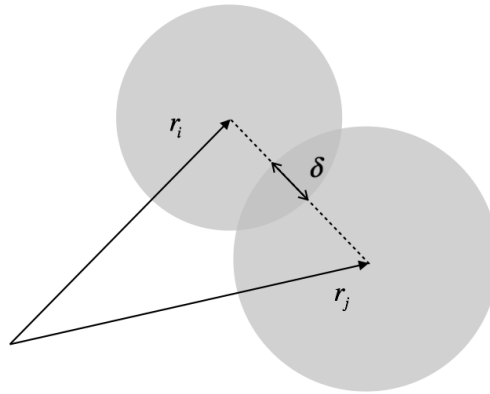
$m_i$  and  $I_i$  denote the mass and moment of inertia of particle  $i$ , respectively;

$\mathbf{r}_i$  and  $\boldsymbol{\omega}_i$  denote the position and angular velocity of it, respectively;

$\mathbf{f}_i = \sum_C \mathbf{f}_i^C + m_i \mathbf{g}$  denotes all the forces acting on it;

$\mathbf{t}_i = \sum_C (\mathbf{l}_i^C \times \mathbf{f}_i^C + \mathbf{q}_i^C)$  denotes all the torques acting on it.

As shown in Figure 1, two spherical particles  $i$  and  $j$ , with radius  $a_i$  and  $a_j$ , respectively, interact with each other when they are in contact ( $\delta > 0$ ). However, it is possible to establish a non-contact interaction when  $\delta < 0$ .



**Figure 1:** Two particles contact with overlap  $\delta$

The overlap  $\delta$  can be described as follows:

$$\delta = (a_i + a_j) - (\mathbf{r}_i - \mathbf{r}_j) \cdot \mathbf{n} \quad (2)$$

where  $\mathbf{n} = (\mathbf{r}_i - \mathbf{r}_j) / |\mathbf{r}_i - \mathbf{r}_j|$  denotes the unit vector pointing from  $j$  to  $i$ .

The force on particle  $i$ , from particle  $j$ , at contact  $C$ , can be decomposed into a normal and a tangential part as  $\mathbf{f}_i^C = f^n \mathbf{n} + f^t \mathbf{t}$ .

In linear normal contact model,  $f^n$  can be described as follows:

$$f^n = k\delta + c\dot{\delta} \quad (3)$$

where  $c$  denotes damping coefficient.

In the current work, there is no moment or torque between particles, and therefore the rotational equation of motion in Equation 1 is not taken into account. Neither the tangential force  $f^t$  is considered.

The basic method of DEM is the integration of Newton's equations of motion for all degrees of freedoms. Interaction forces are updated based on the positions and velocities of particles in each time-step.

### 2.2 Mass and stiffness

A membrane can be discretized into a number of mass particles, with non-contact interactions between them [3]. As shown in Figure 2, a triangular membrane element is discretized into 3 particles in the three corners, with interactions between them.

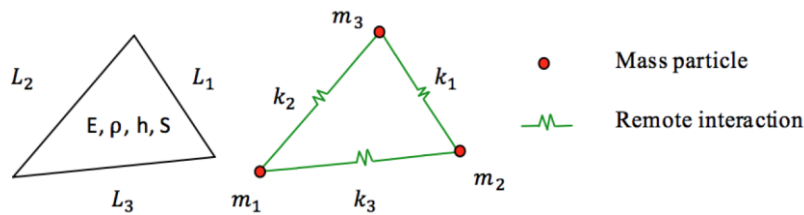


Figure 2: Triangular element

The mass of the three particles can be calculated as follows:

$$m = \frac{\rho h S}{3} \quad (i = 1, 2, 3) \quad (4)$$

In order to obtain the stiffness of the interactions, assume the strain energy of the element coincide with an actual membrane when the element is in one-axis stress states parallel to three sides [6]. Thus, we can obtain the stiffness:

$$\begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix} = B_{ij}^{-1} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, B_{ij} = \frac{p_{ij}^2 L_j^2}{E h S}, p_{ij} = 1 - \frac{4(1 + \nu) S^2}{L_i^2 L_j^2} (1 - \delta_{ij}) \quad (5)$$

where:

$E, \rho, \nu$  denote Young's modulus, density and Poisson's ratio, respectively;

$L_i$  denotes length of the sides of the triangle;

$m_i$  denotes mass of the particles;

$k_i$  denotes stiffness of the interactions;

$\delta_{ij}$  denotes Kronecker delta.

### 2.3 Buckling

In order to take buckling effects of the membrane into consideration, assume that the restoring force of the interactions keeps the same when the distance between two corresponding particles is less than a specific value  $l_{cr}$ , as shown in Figure 3.  $l_{cr}$  can be calculated as follows:

$$l_{cr} = L_i - \alpha \frac{\pi^2 h^2}{12L_i} \quad (6)$$

where  $\alpha$  is a parameter that needs to be calibrated.

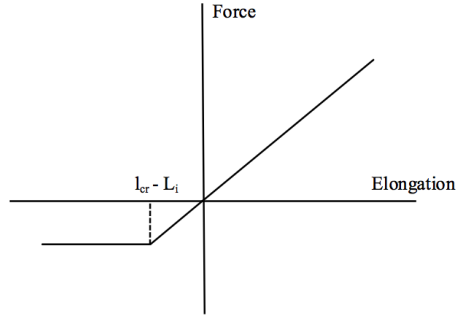


Figure 3: Contact force

### 2.4 Stiffness of crease and contact

Due to the fact that the crease of the folded membrane can generate restoring forces, the stiffness of the crease should be considered. It can be done by adding some extra restoring forces to the masses near the crease. Consider two adjacent elements ABC and ABD that contain a crease, as shown in Figure 4, restoring forces  $F_C$  and  $F_D$  induced by the crease should be added to the particles. Additionally, forces  $F_A$  and  $F_B$  need to be added as well, in order to cancel rigid-body motion of the elements.

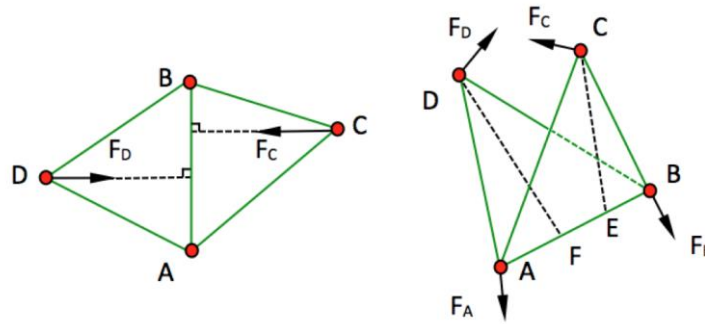


Figure 4: Restoring forces around crease

Forces  $F_C$  and  $F_D$  can be calculated as follows:

$$F_C = \beta \bar{\gamma} \frac{EJl_{AB}(\theta - \theta_0)}{l_{CE}^2}, F_D = \beta \bar{\gamma} \frac{EJl_{AB}(\theta - \theta_0)}{l_{DF}^2}, \bar{\gamma} = \begin{cases} 1 & \text{when } \theta \geq 0 \\ \gamma & \text{when } \theta < 0 \end{cases} \quad (7)$$

Forces  $F_A$  and  $F_B$  can be calculated as follows:

$$\mathbf{F}_A = -\frac{l_{BE}}{l_{AB}}\mathbf{F}_C - \frac{l_{BF}}{l_{AB}}\mathbf{F}_D, \mathbf{F}_B = -\frac{l_{AE}}{l_{AB}}\mathbf{F}_C - \frac{l_{AF}}{l_{AB}}\mathbf{F}_D \quad (8)$$

where:

$\theta$  and  $\theta_0$  denote angle and natural angle, respectively;

$\gamma$  denotes a penalty factor to take account of self contacts around the crease lines;

$J = h^3/12$  denotes geometrical moment of area per unit width.

Note that, the forces calculated in Equation 7 are actually sensitive to mesh density. Therefore, for different mesh density chosen from the reference [2], the stiffness parameter  $\beta$  needs to be calibrated.

## 2.5 Damping

In order to take account of structural damping, velocity-proportional damping forces are added to the interactions. The damping coefficient  $c$  is described as follows:

$$c = 2\zeta\sqrt{k_i m} \quad (9)$$

where:

$\zeta$  denotes damping ratio;

$m = (m_j + m_k)/2$  denotes average mass.

## 2.6 Air Drag

The deployment experiment is conducted in a vacuum chamber, but there is still some air left. Therefore, air drag is taken into consideration. Air drag forces acting on an triangular element can be described as:

$$F_a = \rho S_n V^2 / 6 \quad (10)$$

where:

$V$  denotes velocity of the mass;

$S_n$  denotes the area projected to the direction of the velocity.

## 3 DEPLOYMENT MODELLING

### 3.1 Model introduction

The small-scale solar sail in the experiment is a square membrane, with side length of 10 m and thickness of 7.5  $\mu\text{m}$ . The membrane is connected to the central spool by using tethers, and it has the same material with a real solar sail. The whole membrane consists of four trapezoidal petals, and each petal can be folded up into a strip. Kapton tapes are used to connect the four petals. Four tip masses are fixed on the four corners to generate enough centrifugal forces. Figure 5 shows the sketch of the model. In the figure, solid line represents mountain folding, and dashed line represents valley folding. The folding and deployment method can be found in the reference.

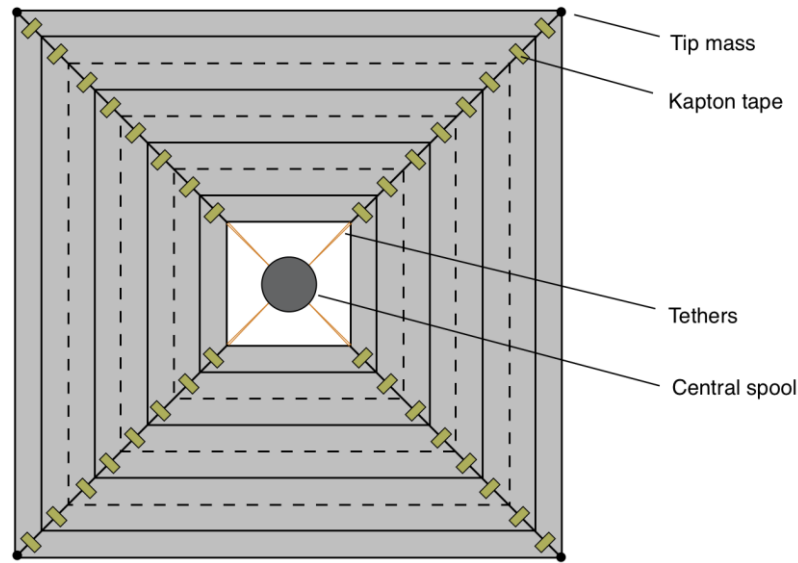


Figure 5: Model sketch

### 3.2 Model parameters

Table 1 shows the dimensions and material properties of the DEM model.

Table 1: Model parameters

Number of folding lines	7
Outer width	455 mm
Inner width	103 mm
Thickness	7.5 $\mu\text{m}$
Radius of central spool	25 mm
Crease interval	22 mm
Young's modulus (membrane)	3.2 GPa
Poisson's ratio (membrane)	0.34
Density (membrane)	1420 kg/m <sup>3</sup>
Tip mass	0.36 g
Rotation speed	6 $\pi$ rad/s

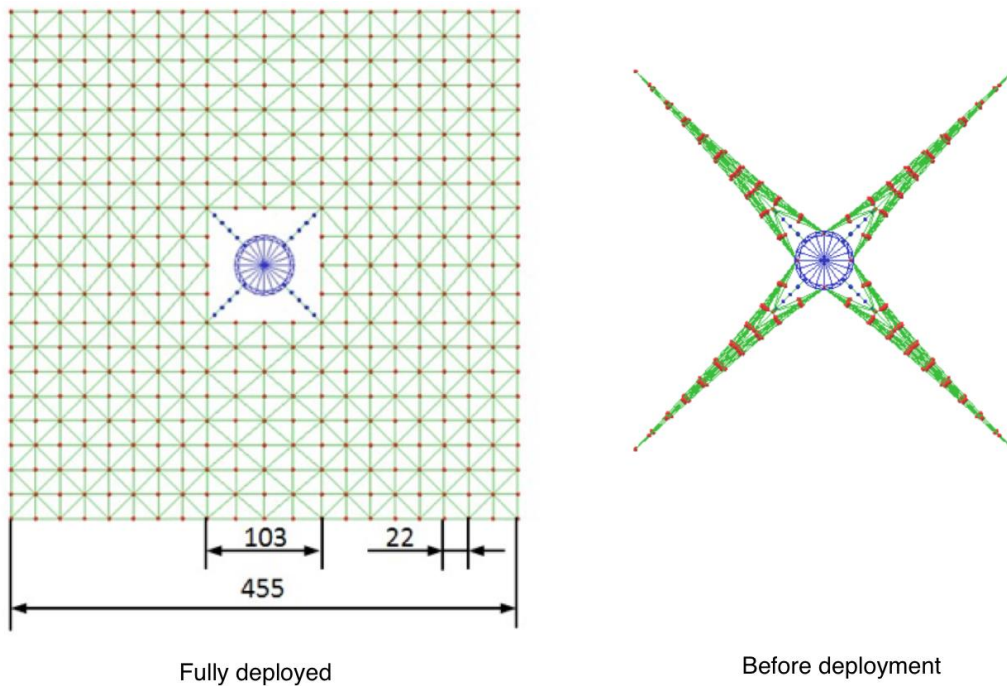
### 3.3 Numerical model

The YADE Open DEM platform [7] was selected to simulate the centrifugal deployment of solar sails. Figure 6 shows the geometrical model of the solar sail. The mass of each triangular element is equally distributed to three masses. Tip masses are added to the corresponding corner masses. The non-contact interaction force is calculated by the equations aforementioned. Considering the fact that bucking strength of the crease is relatively high, the parameter  $\alpha$  is set to be relatively large. Ignore physical contacts between the particles.

The central spool is modeled as facet cylinder. Tethers are also discretized into a number of mass particles. Two adjacent tethers are modeled as one. Tethers cannot bear compressive forces, therefore the parameter  $\alpha$  is set to zero. The contact between tether masses and the

central spool is modelled. Buckling effects, stiffness of creases, structural damping and air drag are taken into account, by adding corresponding forces to the masses.

After creating the model of a fully deployed solar sail, the folded model was obtained by moving the positions of the particles. Then, centrifugal forces were added to the masses. In order to model the constraints of the four guide bars, the corresponding particles were attached to the bars. Because the experiment is conducted on the ground, gravity is taken into account. When the system reached a state of equilibrium, the constraints of guide bars were removed and the attached particles were released. During the rotation, the membrane deployed due to centrifugal forces. The time-step  $\Delta t$  was set to be  $5 \times 10^{-6}$ . Simulation results including deployment rate and phase difference are obtained.



**Figure 6:** Geometry of the DEM model

### 3.4 Numerical results

Three cases are studied, with different simulation parameters. Several results are obtained to study the dynamic response of the membrane, including membrane shapes during deployment, deployment rate, and phase difference. The simulation parameters are shown in Table 2.

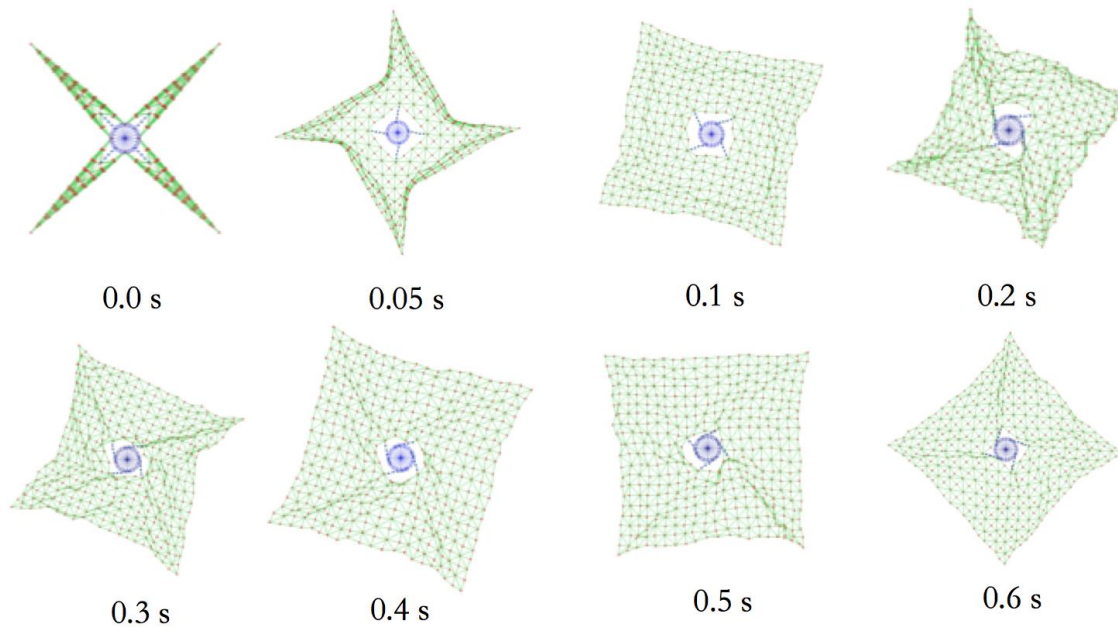
**Table 2:** Simulation parameters

Natural angle of crease $\theta_0$	0
Buckling parameter $\alpha$	100
Buckling parameter $\alpha_c$ (crease)	100, 1000
Stiffness of crease parameter $\beta$	0, 3, 30
Contact penalty factor $\gamma$	100

Damping ratio	0.02
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Figure 7 shows the membrane shapes during deployment. Figure 8 and 9 plot the deployment rate and the phase difference comparisons. The results coincide with those obtained by the reference [2]. As shown in the figures, as the increase of  $\alpha_c$  and  $\beta$ , the amplitude and period of the phase differences decrease.

However, the decrease is not as significant as the results shown in the reference, this is probably because the mesh density is different, and this may affect the interaction mechanism of the restoring forces of creases. It is possible to calibrate these parameters to tune the simulation results to the experiment data.



**Figure 7:** Membrane shapes during deployment ( $\alpha = 1000$ ,  $\beta = 30$ )



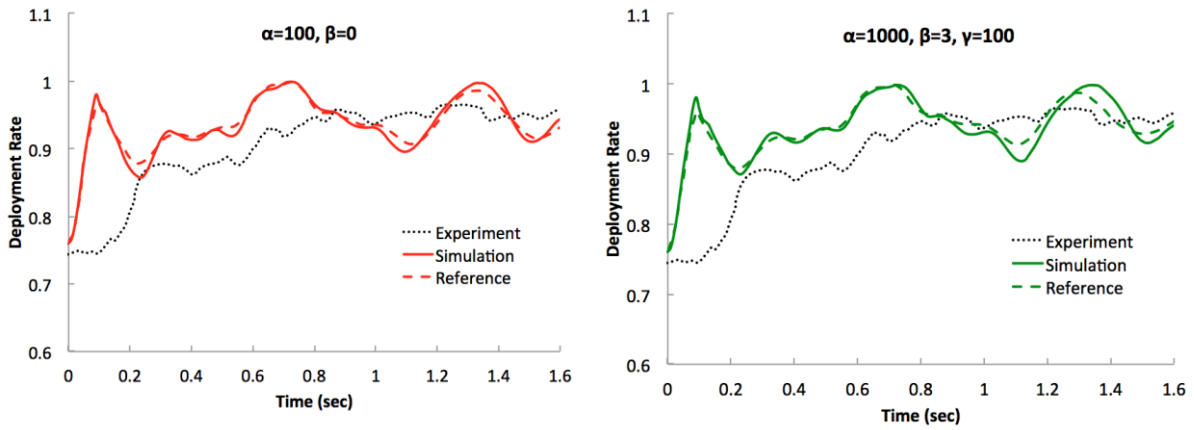


Figure 8: Results comparisons (part I)

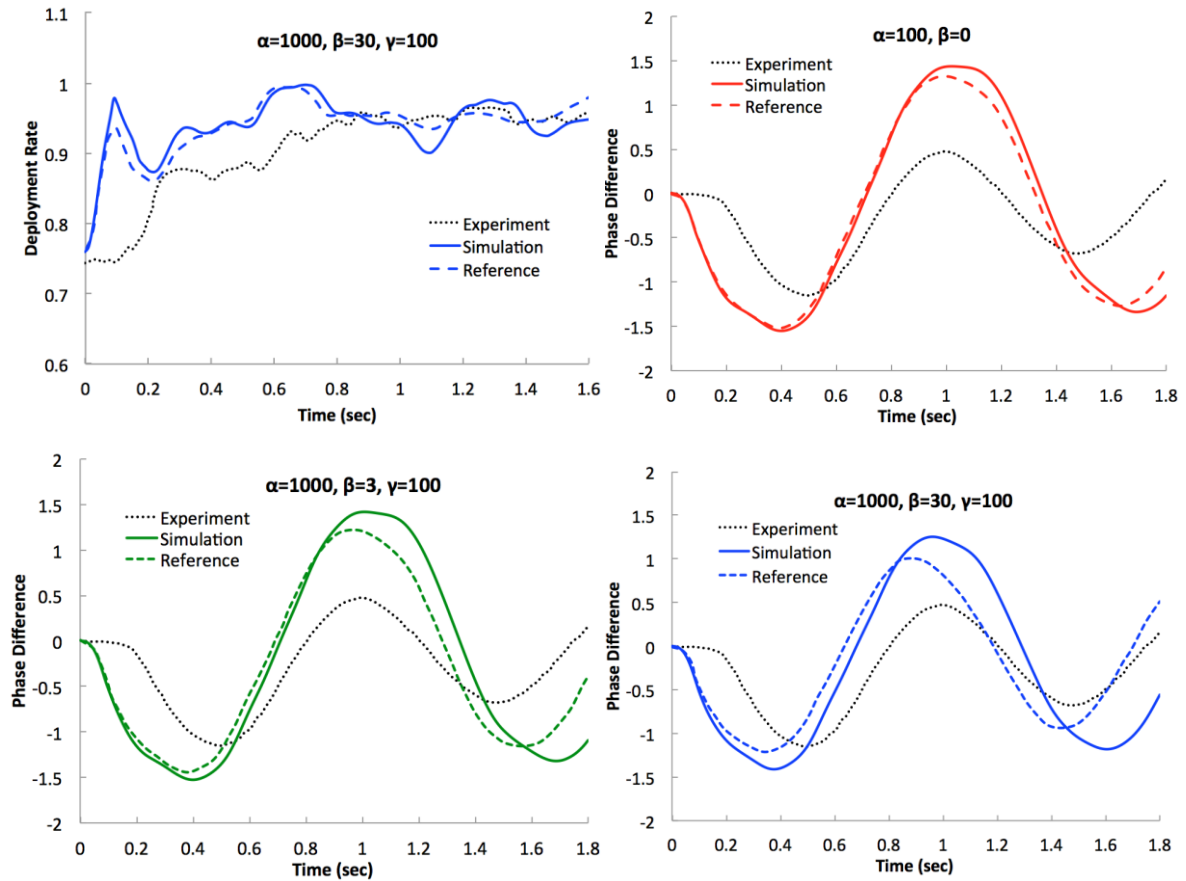


Figure 9: Results comparisons (part II)

#### 4 CONCLUSIONS

In this paper, a DEM-based approach has been used to analyse the dynamic deployment behaviour of a small scale solar sail. The current prediction was compared with test data and the results obtained by using the spring-mass approach. A good correlation has been achieved. Studies on the deployment of a large scale solar sail by using the DEM method are underway. The current study provides a new alternative to analyse the membrane deployment of a solar sail and the mechanical behaviour of other membrane structures.

#### ACKNOWLEDGEMENT

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