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COMPARISON OF WEAR MODELS USING A LAGRANGIAN APPROACH

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Abstract. Abrasive wear of hydraulic machines is commonly simulated with grid based methods. In this work we present a mesh-free approach for modeling abrasive wear. The transport fluid is modeled with the Smoothed Particle Hydrodynamics method. The advantage of this method is the simple handling of free surfaces and complex interfaces, e.g., the interface between the fluid and the abrasive particles. The small solid particles and the boundary geometry of the machine are simulated with the Discrete Element Method. The amount of removed material on the boundary geometry is predicted with three wear models with different complexity. We simulate the impact of a free jet with particle loading on a simplified pelton bucket as working example. The resulting wear patterns of the three different wear models are analyzed and compared. In this work we take the parameters for the wear models from literature.

1 Introduction

The surface of hydraulic machines is damaged by different mechanisms. In this work we are taking damage due to abrasive wear into account. Small solid particles are transported by the working fluid and in case of a contact with the surface of the hydraulic machine they cause damage.

There are different approaches for simulating abrasive wear which were applied during the last years. We present a Lagrangian approach and three different wear models. The fluid is modeled with the Smoothed Particle Hydrodynamics (SPH) method, for the loading of the fluid the small solid particles are modeled with the Discrete Element Method (DEM) and we are using the DEM also for the boundary geometry of the machine. The removed material at the boundary is modeled with wear models.

The idea behind predicting abrasive wear with wear models is, that one estimates the amount of removed material due to an impact of a solid particle and stores and updates the value at each boundary discretization point. The wear models take different parameters

like velocity or impact angle into account for calculating the amount of removed material. There are several wear models available in literature which cover the different kind of mechanisms of abrasive wear, e.g., chip building. In this work we compare three of them and analyze the resulting shape of the wear patterns.
The impact of a free jet with loading on a simplified bucket of a pelton turbine is simulated, see Figure 1. The shape of the wear pattern of each scenario is compared. The attack angle of the free jet is changed to compare different scenarios.
This work is divided into three parts. First a short overview is given about the simulation methods which are used, i.e., SPH, DEM, and the wear models. Then, the simulation results are presented with the three wear models and finally we give a conclusion of our work.

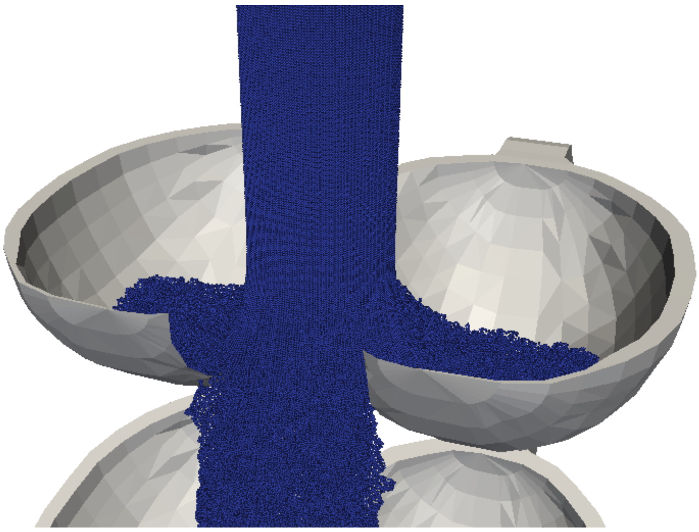

Figure 1: Impact of free jet with loading.

2 Theoretical Background

2.1 Smoothed particle hydrodynamics

The Navier-Stokes equations are used to describe the fluid in the simulations. In this work the SPH method is used for solving these equations. In doing so, the conservation of the mass of the fluid with the velocity $\mathbf{v}$ is stated

$$\nabla \cdot \mathbf{v} = 0 \tag{1}$$

and the conservation of linear momentum is

$$\rho \left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} \right) = -\nabla p + \mu \nabla^2 \mathbf{v} + \mathbf{f} \,, \tag{2}$$

with the density ρ, the viscosity μ, the pressure p of the fluid and the body forces $\mathbf{f}$. The basic ideas of the SPH method for simulating fluids, which are described with the Navier-Stokes equation, are consisting of two steps. The first one is called kernel approximation and the second particle approximation [22]. In its beginning, the SPH method was introduced in [16] and [23]. During the last years the SPH method was applied in several studies with different field of applications, e.g., free surfaces [25, 24, 28], fluid-solid interaction [10, 19], or multi-phase flow [11, 17]. Mainly there are two different approaches used in these studies, i.e., the weakly compressible SPH method and the truly incompressible SPH method. The pressure field is solved in a different way. A review of the various applications in more detail can be found in [22].

2.2 Discrete element method

For the loading of the free jet the solid particles are simulated with the Discrete Element Method [12]. In the past years the method was used not only for simulating granular but also bulk material. In [14] the method is applied to an application where particles have not a permanent contact. When some particles are coupled with breakable inner-particles bonds it is possible to model failure of different materials like in [15]. The Newton-Euler equations [27]

$$m_{\mathrm{i}}\mathbf{a}_{\mathrm{i}} = \mathbf{f}_{\mathrm{i}}\,, \tag{3}$$

$$\mathbf{I}_{\mathrm{i}} \cdot \dot{\boldsymbol{\omega}}_{\mathrm{i}} + \boldsymbol{\omega}_{\mathrm{i}} \times \mathbf{I}_{\mathrm{i}} \cdot \boldsymbol{\omega}_{\mathrm{i}} = \mathbf{l}_{\mathrm{i}} \tag{4}$$

are used to describe the motion of the particles. In (3), m_{i} is the mass, $\mathbf{I}_{\mathrm{i}}$ the inertia tensor and $\mathbf{f}_{\mathrm{i}}$ and $\mathbf{l}_{\mathrm{i}}$ are forces and torques, $\mathbf{a}_i$ the acceleration, $\boldsymbol{\omega}_i$ the angular velocity of particle i. There are different possibilities for calculating the forces between two adjacent particles [29], e.g., a classical force law corresponding to a spring damper combination. In this work a Hertian force law [20], is used where the contact force is calculated to

$$F_{\mathrm{ij}} = K_{\mathrm{ij}}\delta_{\mathrm{ij}}^{\frac{3}{2}} + d\dot{\delta}_{\mathrm{ij}} \tag{5}$$

with

$$K_{\mathrm{ij}} = \frac{4}{3\pi(h_{\mathrm{i}} + h_{\mathrm{j}})}\left(\frac{R_{\mathrm{i}}R_{\mathrm{j}}}{R_{\mathrm{i}} + R_{\mathrm{j}}}\right)^{\frac{1}{2}},$$

$$h_{\mathrm{j}} = \frac{1 - \nu_{\mathrm{j}}^2}{\pi E_{\mathrm{j}}}\,.$$

Here, R_{j} is the radius of a particle j, E_{j} Young's modulus of the granular material, ν_{j} the Poisson number, d a damping parameter and δ_{ij} the overlap of two particles.

For the coupling of the DEM and SPH particles an approach like in [26] is used. The SPH particles are smaller than the DEM particles and the contact force is calculated as a penalty force taking distance and the velocity difference into account.

2.3 Wear

Abrasive wear of the surface of hydraulic machines is caused by different mechanisms [30]. There are different approaches for modeling abrasive wear, whereby one possibility is to use a single particle wear model. In several studies corresponding models were introduced which take various mechanisms and material combinations into account. An overview and more details regarding different wear models can be found in [6, 5] and [7]. A common crucial point of the wear models is the determination of the model parameters.
In this work we have selected three different wear models. The models were originally applied to different but similar application scenarios, e.g., wear in oilfield control valves or rocket nozzles. In Figure 2 the resulting wear patterns for the simulation scenario of the impact of a free jet are shown. The resulting shape is very similar for the different wear models. The purple blade (a) is the Finnie model [3], the red blade (b) is the Bitter model [1] and the turquoise blade (c) is the Tabakoff model [8]. In this figures the blades are color coded with the amount of removed material but here at the moment only the shape of the pattern is interesting.

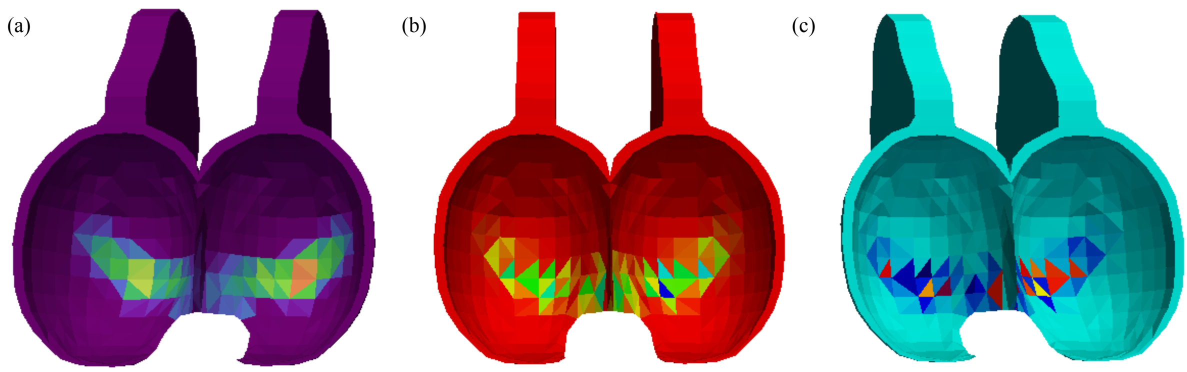

Figure 2: Comparison of the wear pattern for three different wear models.

The Finnie wear model was one of the first approaches to model the removed material of the surface of a hydraulic machine. The model was introduced in [3] and the removed material of the surface, mainly through cutting mechanism, is calculated to

$$W = \begin{cases} \dfrac{mU^2}{\psi p k}\left[\sin(2\alpha) - \dfrac{6}{k}\sin^2(\alpha)\right] & \text{for } \tan(\alpha) \le \dfrac{k}{6}, \\ \dfrac{mU^2}{\psi p k}\left[\dfrac{6\cos^2(\alpha)}{k}\right] & \text{else.} \end{cases} \tag{6}$$

Here, m is the mass of the particle, α is the attack angle, k is the ratio of the force components, ψ is the ratio of the depth of contact to the length of the swarf, U is the absolute velocity of the solid particle and p is the flow stress of the hydraulic machine. The physical assumptions for this model are that the material is removed by an idealized particle with unit width, the particle is rigid and the rotation is small during the cutting process. It attempts to predict an amount of removed material which is proportional to the kinetic energy of the solid wear particle. In Figure 3 the removed material at the surface of hydraulic machine is shown at different points.

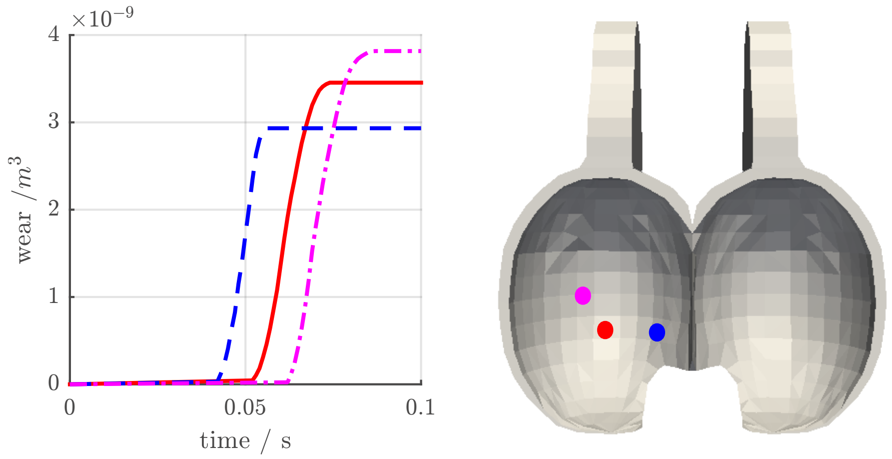

Figure 3: Abrasive wear predicted with the Finnie model at the marked points.

The second more complex model, here referred to as Bitter model, takes not only cutting mechanisms into account but also wear due to repeated deformation. The assumptions for this wear model are that these two types of wear occur simultaneously. Besides the removed material due to cutting wear, the surface of the hydraulic machine is plastically deformed because the elastic stress is exceeded due to frequent impacts of wear particles. It was introduced in [1, 2] and extended in [4]. The removed material is modeled to be

$$W = \left[\frac{100}{2\sqrt{29}} r_p^3 \left(\frac{U}{C_k}\right)^n \sin 2\alpha \sqrt{\sin\alpha}\right] + \left[\frac{M_p(U\sin\alpha - D_k)^2}{2E_f}\right] \tag{7}$$

with

$$C_k = \sqrt{\frac{3\sigma R_f^{0.6}}{\rho_p}}\,,\; D_k = \frac{\pi^2}{2\sqrt{10}}(1.59Y)^{2.5}\left(\frac{R_f}{\rho_t}\right)^{0.5}\left[\frac{1-q_p^2}{E_p} + \frac{1-q_t^2}{E_t}\right]^2. \tag{8}$$

In these equations, r_p is the radius, M_p the mass, q_p the Poisson ratio, E_p Young's modulus, ρ_p the density and R_f the roundness factor of the particle, ρ_t the density, q_t the Poisson ratio, Y the yield stress and E_f is the Young's modulus of the boundary, σ the plastic flow stress, E_f the deformation erosion factor and n the velocity exponent.

The third model was introduced in [8], here referred to as Tabakoff model. This wear model takes the same wear mechanisms into account like the Bitter wear model, but is uses a statistical approach instead of an energy based approach for estimating the wear due to deformation. The removed material is calculated to be

$$W = \frac{M_p}{\rho_t} K_1 \left[1 + C_K (K_{12} \sin 2\beta_0)\right]^2 U^2 \cos^2 \alpha (1 - R_t^2) + K_3 (U \sin \alpha)^4, \tag{9}$$

with

$$R_t = 1 - 0.0016 U \sin \alpha, \; C_K = \begin{cases} 1 & \text{for } \alpha \le 2, \\ 0 & \text{else.} \end{cases} \tag{10}$$

Here, K_1, K_{12} and K_3 are empirical constants and β_0 the impact angle where the maximum material is removed. In the next section the simulation results are discussed.

3 RESULTS

The simulation scenario is the impact of a free jet with loading to a simplified pelton turbine blade. We have simulated three cases with different attack angles of the free jet. In Figure 4 the results predicted with the Finnie model are shown, in Figure 5 the results for the Bitter model and in Figure 6 for the Tabakoff model. In these figures the removed material is color coded, but only the shapes of the patterns are compared in this work. As expected the shape of the wear patterns is similar.

The three wear models take different wear mechanisms into account and as a result the amount of removed material is different. The absolute value of the amount of removed material is depending on the model parameters which are here taken from literature [1, 2, 3, 4, 6, 8]. Depending on the application scenario the corresponding wear model has to be chosen. If the main interests are the parts of the hydraulic machine where damage takes place, a relative simple model can be applied. For different kind of loadings which influence the amount of removed material, e.g., mixtures of fine and coarse sand, a more complex wear model which takes different wear mechanisms into account should be used.

4 CONCLUSIONS

The modeling of abrasive wear with three different models is presented. The impact of a free jet with loading under three different attack angles was simulated. The simulation results are satisfying for all three models, because the wear patterns predicted with the different models are similar. It is possible to take different wear mechanisms with the applied wear models into account which lead as expected to similar wear patterns, but the amount of removed material is different. The Lagrangian approach can be used to predict wear patterns caused by different mechanisms. Depending on upcoming questions due to abrasive wear a simple wear model or a more complex one should be chosen. If only the place of damaged parts is of interest or the mixture of the loading is unknown a

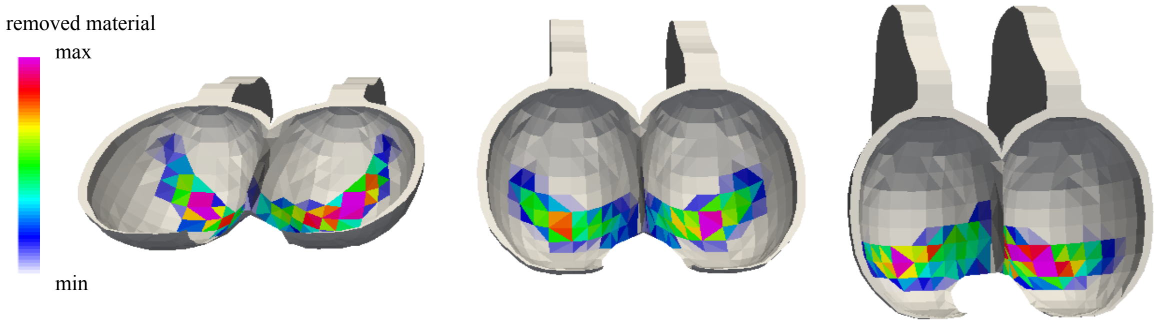

Figure 4: Wear pattern for different impact angles with the Finnie model.

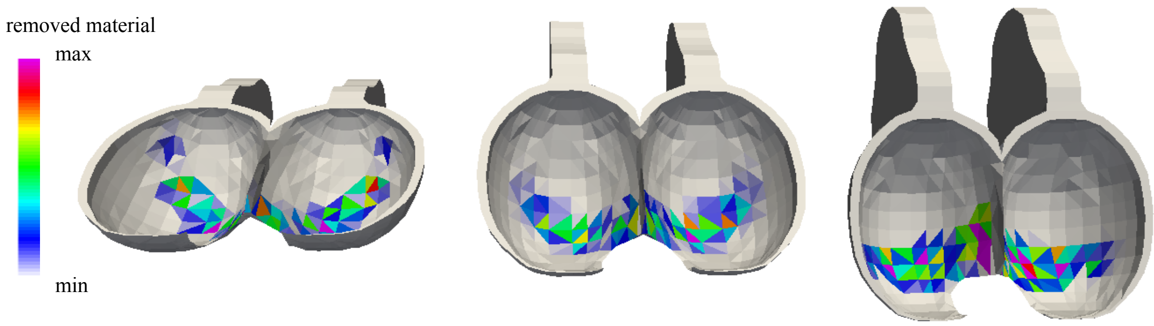

Figure 5: Wear pattern for different impact angles with the Bitter model.

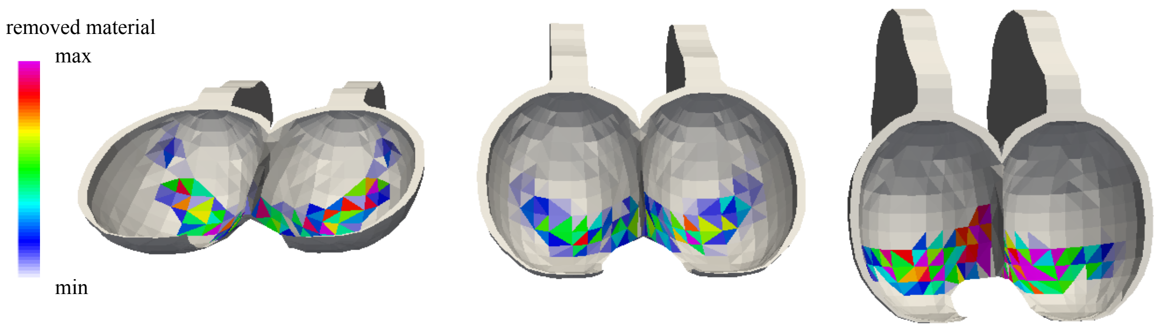

Figure 6: Wear pattern for different impact angles with the Tabakoff model.

less complex wear model can be applied. A crucial point is the exact amount of removed material as the wear models have to be calibrated with experimental studies. In this work we have taken the model parameters from literature [1, 2, 3, 4, 6, 8] but not conducted own hardware experiments.

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