IV International Conference on Particle-based Methods – Fundamentals and Applications
PARTICLES 2015
E. Oñate, M. Bischoff, D.R.J. Owen, P. Wriggers & T. Zohdi (Eds)

NUMERICAL SIMULATION ANALYSIS OF REINFORCED CONCRETE SYRUCTURE USING VFIFE FIBER-ELEMENT

REN-ZUO. WANG[1], BING-CHANG LIN[2] AND CHAO-HSUM HUANG[3]

[1]National Center for Research on Earthquake Engineering
200, Sec. 3, Xinhai Rd., Taipei City 10668, Taiwan, R.O.C.
e-mail: rzwang@ncree.narl.org.tw, web page: http://www.ncree.org/

[2]Department of Civil Engineering, Chung Yuan Christian University
200 Chung Pei Road, Chung Li District, Taoyuan City, Taiwan 32023, R.O.C.
e-mail: tbinchan@cycu.edu.tw, web page: http://www.cycu.edu.tw/

[3] Department of Civil Engineering & Disaster Prevention Engineering, National Taipei University of Technology
1, Sec. 3, Zhongxiao E. Rd., Taipei 10608 Taiwan, R.O.C.
e-mail: steve@ntut.edu.tw, web page: http://www-en.ntut.edu.tw/

Key words: Vector form intrinsic finite element, fiber-element model.

Abstract. In this paper, a vector form intrinsic finite element (VFIFE or V-5) method is used to compute the nonlinear responses of reinforced concrete (RC) structure. In addition, the fiber-element model of VFIFE frame element is formulated. Material models of both concrete and steel on the cross section of the member are considered. The VFIFE method is a particle-based method. They have three key VFIFE processes such as the point value description, path element and convected material frame [1]. The RC structure is represented by finite particles. Each particle is subjected to the external forces and internal forces. The particle satisfies the Newton's Law. A fictitious reversed rigid body motion is used to remove the rigid body motion from the deformations of the element [2]. The internal forces of the element in deformation coordinates satisfy the equilibrium equations. Comparing the results of numerical simulations and experiments of the reinforced concrete members subjected to external loads, the proposed method demonstrate accuracy and efficiency.

1 INTRODUCTION

For nonlinear material structures, most of the existing nonlinear material models can be classified into two main categories: lumped and distributed plasticity models. The lumped model is an efficient way to represent inelasticity in frames. A typical finite element method considering the geometrical and material nonlinearities requires iterations at each incremental step to achieve the equilibrium. This method encourages flexural yielding and can ensure that plastic hinge rotation will occur at the member ends rather than along the column length. The second-order plastic hinge concept based on the use of stability interpolation functions has been proposed for frame structure analysis proposed a general criterion of localization and two plastic hinges at the end of the frame member. Some researches improved the lumped plastic hinge method, such as for example the distributed plasticity model (also called plastic-zone model), which allows for the gradual spread of yielding within the member. In the distributed plasticity model, the frame element stiffness can be computed by using either the displacement or the force-based approach. This allows plastic hinges to form at any location in an element. In addition, the element cross-section can be a fiber section using different stress-strain models for different fibers within the cross-section. Without properly taking into account the internal forces due to pure deformations, most of these studies may not be able to simulate inelastic structural responses of moving structures subjected to extremely-large displacements or deformations.

Recently, the VFIFE method has been proposed by Ting, et al. [1] and Wang [2]. This method applies a unique approach to compute the effects of rigid motion, allowing the simulation of extremely large deformation of elastic motion structures. The key objective of this study is to construct a fiber-section of the frame element subjected to extremely large deformation having inelastic material properties. In this study, in order to model the reinforced concrete member, the fiber-element formulation has considered two material types such as steel and concrete is proposed. It can be used to compute nonlinear responses of reinforced concrete structure. Three numerical examples are presented to illustrate the capability and the accuracy of the proposed method.

2 FUNDAMENTALS OF VFIFE METHOD

In this study, the VFIFE method is extended in order to analyze reinforced concrete members containing multiple deformable bodies with the following characteristics: (1) interact with each other, (2) or are discontinuous, (3) undergo large deformations and arbitrary rigid body motions. The VFIFE method establish a new analysis strategy based on the intrinsic theories of mechanics and avoid the difficulties such as the iterative and perturbation procedures in solving partial differential equations commonly adopted in the conventional nonlinear structural analysis methods (CNSAM). The key VFIFE concept is that the structure is viewed as a system composed of particles and forces components. The forces on the particles include the internal forces and external forces. In order to describe the deformation of the structure, the following general assumptions are adopted: (1) the internal forces are computed from the deformations of the structural members such frame element, (2) each structural member has geometry and position changes simultaneously. In addition, the changes of the geometry and the position for the deformable structure are not separated, and (3) each particle may have a motion trajectory. Base on these assumptions, the associated analytical operations are using: (1) the point value description (PVD), (2) the path element, and (3) the convected material frame (CMF).

Figure 1 shows the characteristic of the VFIFE analysis using the PVD concept. The motions and deformations of a structure are described by the positions of the particles and discrete time points as shown in Fig. 1(a). The PVD uses many discrete time points to describe the entire time trajectory of the deformable body. VFIFE analysis is not required to solve the partial differential equations for structural members. This is because the equations of motion on particles at each time point are established using Newton's Law. The reference configuration of the structure at time t_a can be identified by connecting the representative particles (i_a, j_a, 1_a). The trajectory of any representative particle satisfies the definition of a path element at each set of time points (e.g. i-i_1, i_1-i_a, i_a-i_t, i_t-i_b, i_b-i_c, i_c-i_f in Fig. 1(b)). Details of the path element will be explained in the following sections. The PVD and the function of time trajectory are shown in Fig. 1(b). The dotted line represents the particle trajectories from the positions at time t_0 to positions at time t_f.

In order to introduce the concept of the path element, the motion of a structure member is considered as shown in Fig. 2(a). The PVD is used to describe the entire time trajectory of particles i and j. For example, a series of time points ($t_0<t_1<.....t_a<t<t_b<t_c...<t_f$) is used to describe the entire time for the particle i. Figure 2(b) shows four configurations, original configuration V_0 at time t_0, configuration V_a at time t_a, current configuration V at time t and the fictitious configuration V_r. The position vectors used to describe the motion of the deformable body are continuous functions of time. In the VFIFE method, the deformable body is represented by many particles using PVD to describe its motion as shown in Fig. 1. The entire time trajectory of the deformable body uses many path elements. For the purpose of discussion, choose the time interval ($t_{a,}$ t_b) as one of the path elements in the entire time trajectory of the deformable body. The position, material properties, stress and the geometric features of the deformable body at time t_a is known as the configuration V_a. In Fig. 2(b), the relative position $d\mathbf{x}$ between particles i and j in the current configuration V is computed from $d\mathbf{x}_a$:

$$d\mathbf{x} = \mathbf{F}d\mathbf{x}_a = \mathbf{RU}d\mathbf{x}_a \tag{1}$$

where $\mathbf{F}$ is the deformation gradient. $\mathbf{R}$ is a rigid body rotation matrix. $\mathbf{U}$ is a matrix of the deformation. The fictitious configuration V_r can be computed as follows:

$$d\mathbf{x}_r = \mathbf{F}_r d\mathbf{x} = \mathbf{R}_r^T d\mathbf{x} \tag{2}$$

where $\mathbf{F}_r$ is a virtual deformation gradient and $\mathbf{R}_r$ is a reversed rigid body rotation matrix. In this study, $\mathbf{F}_r$ is equal to a reversed rotation matrix $\mathbf{R}_r$. Substitute equations (1) into (2), the following can be obtained:

$$d\mathbf{x}_r = \mathbf{F}_r\mathbf{F}d\mathbf{x}_a = (\mathbf{R}_r^T\mathbf{R})\mathbf{U}d\mathbf{x}_a \tag{3}$$

If the reversed rotation matrix $\mathbf{R}_r$ is close to $\mathbf{R}$, the $\mathbf{R}_r$ can be used to reduce the effects of the rigid body rotation on the deformation gradient $\mathbf{F}$ of a deformable body. Then:

$$d\mathbf{x}_r = \mathbf{U}d\mathbf{x}_a \tag{4}$$

In this study, a procedure to obtain the best approximate rotation matrix $\mathbf{R}_r$ is proposed for the VFIFE method. The internal forces of the structural members are computed from using the CMF, PVD and path element.

4 REINFORCED CONCRETE FIBER FRAME ELEMENT

The internal force formulation of reinforced concrete fiber frame element (RCFFE) is introduced. The internal virtual work in the deformation coordinates of the RCFFE is:

$$\delta W = (\hat{\mathbf{d}}^*)^T\hat{\mathbf{f}}_c + (\hat{\mathbf{d}}^*)^T\hat{\mathbf{f}}_s \tag{5}$$

The deformations of the RCFFE are based on Euler beam theory. The pure deformations $\Delta\hat{u}^d$ of the RCFFE at any cross section can be computed by the pure concrete deformation $\Delta\hat{u}_c^d$ and steel deformation $\Delta\hat{u}_s^d$ along the axis of the frame element in the deformation coordinates. In Fig. 3, Total deformation of RCFFE can be represented as:

$$\Delta\hat{u}^d = \Delta\hat{u}_c^d + \Delta\hat{u}_s^d \tag{6}$$

where

$$\Delta\hat{u}_c^d = \hat{u}_m^d - \hat{y}\frac{d\hat{v}^d}{d\hat{x}} \tag{7}$$

$$\Delta\hat{u}_s^d = \hat{u}_m^d - \left(h_c - h_{sj}\right)\frac{d\hat{v}^d}{d\hat{x}} \tag{8}$$

$$\hat{u}_m^d = a_1 + a_2\hat{x} \tag{9}$$

$$\hat{v} = a_3 + a_4\hat{x} + a_5\hat{x}^2 + a_6\hat{x}^3 \tag{10}$$

where h_c is a center line of the frame section. The compatibility conditions of the frame element are:

$$\hat{x} = 0,\quad \hat{u}_m^d = 0,\quad \hat{v}^d = 0,\quad \frac{d\hat{v}^d}{dx} = \hat{\phi}_{iz} \tag{11}$$

$$\hat{x} = l_a,\quad \hat{u}_m^d = \Delta_e,\quad \hat{v}^d = 0,\quad \frac{d\hat{v}^d}{dx} = \hat{\phi}_{iz} \tag{12}$$

The deformation functions are used to compute the internal forces of the frame element. In VFIFE method, the effects of rigid body motion must be removed from the deformation functions in each path element. Then, the internal forces in the fictitious configuration V_r can be evaluated from the traditional procedures. The pure deformation $\Delta\hat{u}^d$ satisfy Eqs. (11) to (12). The concrete deformation $\Delta\hat{u}_c^d$ and steel deformation $\Delta\hat{u}_s^d$ can be written as:

$$\Delta\hat{u}_c^d = s\Delta_e - \left\{(1-4s+3s^2)\hat{\phi}_{iz} + (-2s+3s^2\hat{\phi}_{jz})\right\}\hat{y} \tag{13}$$

$$\Delta\hat{u}_s^d = s\Delta_e - \left\{(1-4s+3s^2)\hat{\phi}_{iz} + (-2s+3s^2\hat{\phi}_{jz})\right\}\left(h_c - h_{sj}\right) \tag{14}$$

where $s = \hat{x}/\hat{l}$ is a non-dimensional parameter. Because the deformation of the RCFFE is small deformation in each path element, the infinitesimal concrete strain $\hat{\varepsilon}_c$, steel strain $\hat{\varepsilon}_s$ and total axial strain $\hat{\varepsilon}_s$ are:

$$\hat{\varepsilon} = \hat{\varepsilon}_c + \hat{\varepsilon}_s \tag{15}$$

$$\hat{\varepsilon}_c = \varepsilon_{ac} + \Delta\hat{\varepsilon}_c,\ \Delta\hat{\varepsilon}_c = \frac{1}{2}\left\{\frac{\partial(\Delta\hat{u}_c^d)}{\partial\hat{x}} + \left[\frac{\partial(\Delta\hat{u}_c^d)}{\partial\hat{x}}\right]^T\right\} = \frac{d\hat{u}_c^d}{d\hat{x}} = \frac{1}{l_a}\frac{d(\Delta\hat{u}_c^d)}{ds} = \mathbf{B}_c\hat{\mathbf{d}}^* \tag{16}$$

$$\hat{\varepsilon}_s = \varepsilon_{as} + \Delta\hat{\varepsilon}_s,\ \Delta\hat{\varepsilon}_s = \frac{1}{2}\left\{\frac{\partial(\Delta\hat{u}_s^d)}{\partial\hat{x}} + \left[\frac{\partial(\Delta\hat{u}_s^d)}{\partial\hat{x}}\right]^T\right\} = \frac{d\hat{u}_s^d}{d\hat{x}} = \frac{1}{l_a}\frac{d(\Delta\hat{u}_s^d)}{ds} = \mathbf{B}_s\hat{\mathbf{d}}^* \tag{17}$$

Where

$$\hat{\mathbf{d}}^* = \begin{Bmatrix} \Delta_e \\ \hat{\phi}_{iz} \\ \hat{\phi}_{jz} \end{Bmatrix} \tag{18}$$

$$\mathbf{B}_s = \frac{1}{l_a}\begin{bmatrix} 1 & (4-6\mathrm{s})(\mathrm{h}_c - \mathrm{h}_{sj}) & (2-6\mathrm{s})(\mathrm{h}_c - \mathrm{h}_{sj}) \end{bmatrix} \tag{19}$$

$$\mathbf{B}_c = \frac{1}{l_a}\begin{bmatrix} 1 & (4-6\mathrm{s})\,\hat{\mathrm{y}} & (2-6\mathrm{s})\,\hat{\mathrm{y}} \end{bmatrix} \tag{20}$$

where ε_{ac} and ε_{as} are the concrete and steel axial strain at time t_a. The $\Delta\hat{\varepsilon}_c$ and $\Delta\hat{\varepsilon}_s$ are the concrete and steel relative axial strains from time t_a to *t*. The l_a is the length of the frame element at time t_a. In Eqs. (19) and(20), the $\Delta\hat{\varepsilon}_c$ and $\Delta\hat{\varepsilon}_s$ can be written as

$$\Delta\hat{\varepsilon}_c = \frac{\Delta_e}{l_a} + K(\mathrm{s})\,\hat{\mathrm{y}} \tag{21}$$

$$\Delta\hat{\varepsilon}_s = \frac{\Delta_e}{l_a} + K(\mathrm{s})(\mathrm{h}_c - \mathrm{h}_{sj}) \tag{22}$$

where K(s) is the incremental curvature from time t_a to *t*. The concrete stress $\hat{\sigma}_c$, steel stress $\hat{\sigma}_s$ and total axial stress $\hat{\sigma}$ of the section can be computed from:

$$\hat{\sigma} = \hat{\sigma}_c + \hat{\sigma}_s \tag{23}$$

$$\hat{\sigma}_c = \sigma_{ac} + \Delta\hat{\sigma}_c\,,\ \Delta\hat{\sigma}_c = E_{ac}\Delta\hat{\varepsilon}_c \tag{24}$$

$$\hat{\sigma}_s = \sigma_{as} + \Delta\hat{\sigma}_s\,,\ \Delta\hat{\sigma}_s = E_{as}\Delta\hat{\varepsilon}_s \tag{25}$$

where σ_{ac} and σ_{as} are the concrete and steel axial stress at time t_a. The $\Delta\hat{\sigma}_c$ and $\Delta\hat{\sigma}_s$ are the concrete and steel relative axial stress from time t_a to *t*. In Eqs. (24) and (25), the E_{ac} and E_{as} represents the instantaneous constitutive relationships of the concrete and steel at time t_a. The virtual strain energy δU :

$$\delta U = \delta U_s + \delta U_c \tag{26}$$

$$\delta U_c = \int_{\hat{V}} \delta(\hat{\varepsilon}_c)^T \hat{\sigma}_c d\hat{V} \tag{27}$$

$$\delta U_s = \int_{\hat{V}} \delta(\hat{\varepsilon}_s)^T \hat{\sigma}_s d\hat{V} \tag{28}$$

In the deformation coordinates ($\hat{x}$, $\hat{y}$), the internal virtual work δW equals to the virtual strain energy δU :

$$\delta U = \delta W \tag{29}$$

The three internal forces $\hat{\mathbf{f}}$ can be computed from:

$$\hat{\mathrm{f}} = \hat{\mathrm{f}}_a + \Delta\hat{\mathrm{f}} = \begin{Bmatrix} \hat{f}^{ij}_{jx} & \hat{m}^{ji}_{iz} & \hat{m}^{ij}_{jz} \end{Bmatrix} \tag{30}$$

$$\hat{\mathrm{f}}_a = \begin{Bmatrix} (\hat{f}^{ij}_{jx})_a & (\hat{m}^{ji}_{iz})_a & (\hat{m}^{ij}_{jz})_a \end{Bmatrix} \tag{31}$$

$$\Delta\hat{\mathrm{f}} = \left\{ \Delta\hat{f}_{jx}^{ij} \quad \Delta\hat{m}_{iz}^{ji} \quad \Delta\hat{m}_{jz}^{ij} \right\} \tag{32}$$

The remaining three internal forces can be obtained from the equilibrium equations:

$$\sum F_{\hat{x}} = 0 \quad \hat{f}_{ix}^{ji} = -\hat{f}_{jx}^{ij} \tag{33}$$

$$\sum M_{\hat{y}} = 0 \quad \hat{f}_{jy}^{ij} = -\frac{1}{l_a}(\hat{m}_{iz}^{ji} + \hat{m}_{jz}^{ij}) \tag{34}$$

$$\sum F_{\hat{y}} = 0 \quad \hat{f}_{iy}^{ji} = -\hat{f}_{jy}^{ij} \tag{35}$$

The internal forces ($\hat{\mathbf{f}}_i^{ji}, \hat{\mathbf{f}}_j^{ij}$) at the particle i and j in the deformation coordinates are (see Fig. 4):

$$\hat{\mathrm{f}}_i^{ji} = \left\{ \hat{f}_{ix}^{ji} \quad \hat{f}_{iy}^{ji} \right\}^T, \ \hat{\mathrm{f}}_j^{ij} = \left\{ \hat{f}_{jx}^{ij} \quad \hat{f}_{jy}^{ij} \right\}^T \tag{36}$$

In order to compute the actual internal forces, the internal forces in fictitious configuration V_r must be rotated back to the current configuration V. In addition, since all the force directions at each particle are defined in the global coordinates, they need to be transformed into global coordinates. Thus, the internal forces in the global coordinates can be computed:

$$\mathbf{f}_i^{ji} = \mathbf{R}_r^T \mathbf{\Omega}_a^T \hat{\mathbf{f}}_i^{ji}, \ \mathbf{f}_j^{ij} = \mathbf{R}_r^T \mathbf{\Omega}_a^T \hat{\mathbf{f}}_j^{ij} \tag{37}$$

fiber section method is used to analyse various material on crass section of member as show in Fig. 5. These materials include the reinforced steel, steel, well-confined concrete and not well-confined concrete. The fiber section method can consider the effects of frame element axial force-bending moment interaction on nonlinear dynamic structural simulation for behavior of real reaction. In this paper, we focus on numerical analysis of reinforced concrete structures. The fiber-element model can be used fiber section model in each area using different stress-strain models. The stress-strain relation of confined concrete model is adopted for each fiber as show in Fig. 6. Thus, the bending moment-curvature relation of complex section can be calculated. Wang and Restrepo [3] have suggested a formulation of Mander concrete model on rectangular section. Now, this formulation is written on VFIFE method in this paper. It can be expressed as:

$$\sigma_{cc} = \alpha_1 \alpha_2 f'_c \tag{38}$$

where

$$\alpha_1 = \left[1.4\frac{f'_{l1}}{f'_{l2}} - 0.6\left(\frac{f'_{l1}}{f'_{l2}}\right)^2 - 0.8 \right]\sqrt{\frac{f'_{l2}}{f'_c}} + 1 \tag{39}$$

$$\alpha_2 = 1.25\left(1.8\sqrt{1 + \frac{7.94 f'_{l2}}{f'_c}} - 1.6\frac{f'_{l2}}{f'_c} - 1 \right) \tag{40}$$

$$f'_{l2} = \max\left[f'_{lx}, f'_{ly} \right] \tag{41}$$

$$f'_{l1} = \min\left[f'_{lx}, f'_{ly} \right] \tag{42}$$

Thus, $f'_{lx} = k_e f_{lx}$ and $f'_{ly} = k_e f_{ly}$ was the effective lateral confining stress for rectangular confined concrete in the spiral of x and y direction and k_e was effective confined coefficient. For the stress–strain model for confined concrete, the modified Mander confined model is used to analyse numerical simulation analysis of reinforced concrete members. For steel material model, the stress-strain relation of steel by Mirza and MacGregor [4] is adopted.

5 NUMERICAL EXAMPLES

In this paper, three numerical examples are given. Comparison experimental and numerical results of reinforced concrete members are studied. Firstly, example has been studied by Yang [5]. Figures 7 and 8 are design details of the specimens for the C1 and C1W. Figures 9 and 10 show that the analytical results computed from the VFIFE method are close to experimental results. It has been demonstrated that the proposed method can be used to compute responses of the reinforced concrete members

6 CONCLUSIONS

In this paper, following the VFIFE approach, we developed a numerical procedure for the analysis of fiber-element model using different stress-strain models. The numerical procedure can be used to simulate the extremely large deformation of inelastic structures. One example has been presented. It has been demonstrated that the proposed method can be used to analyze the large static deformation responses of reinforced concrete members under loads or deformations.

7 ACKNOWLEDGMENT

This study is supported by the National Science Council of Republic of china under grant NSC102-2221-E-033 -036

REFERENCES

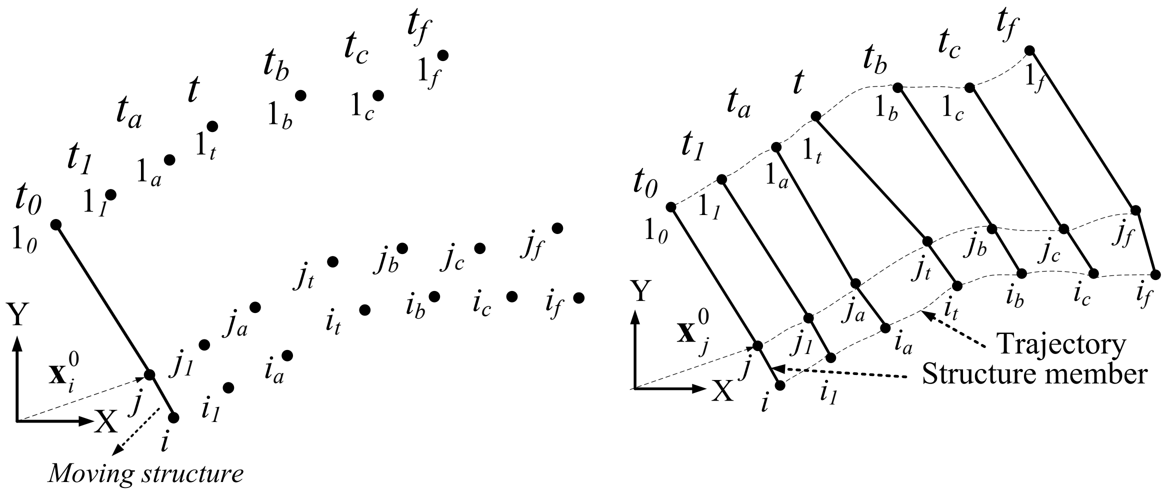
[1] Ting, E.C. and Wang, C.Y. *Vector mechanics of solids*. China Engineering Consultants Inc, Taipei, Taiwan, (2008) (in chinese).

[2] Wang, C.Y., Wang, R.Z., Chuang, C.C. Chuang and Wu, T.Y. Nonlinear analysis of reticulated space truss structures, *J. Mech,* (2006) **22**(3):199-212.

[3] Wang, Y. C., and Restrepo, J. I., Investigation of concentrically loaded reinforced concrete columns confined with glass fiber-reinforced polymer jackets, *ACI Structural Journal*, (2001) **98**(3):377-385.

[4] Mirza S. A., and MacGregor J. G., Variability of mechanical properties of reinforcing bars, *Journal of the Structural Division*, ASCE, (1979)**105**(5):921-937.

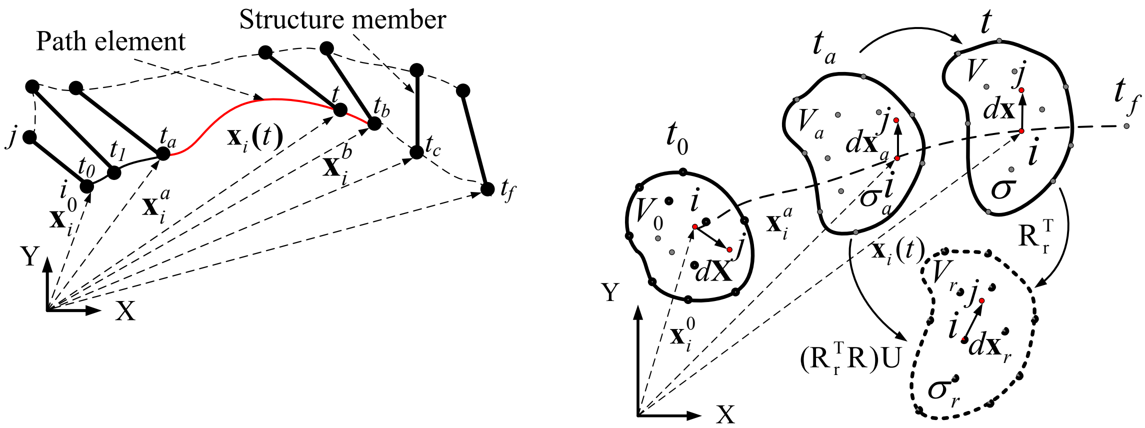
[5] Yang, C.P. Experimental study of RC columns of Hsin-Tseng Junior high school, National Taiwan Univerity of Science and Technology, Master. (in Chinese), (2005).



(a)particles in the spatial-temporal space (X, Y, t)

(b) structure configurations and the trajectories

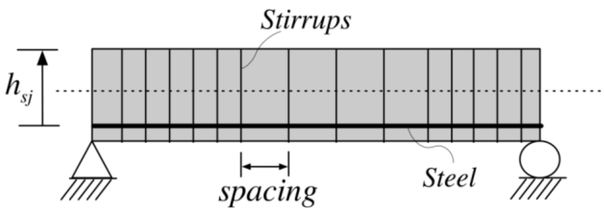
Figure 1: Motion of a structure in VFIFE method



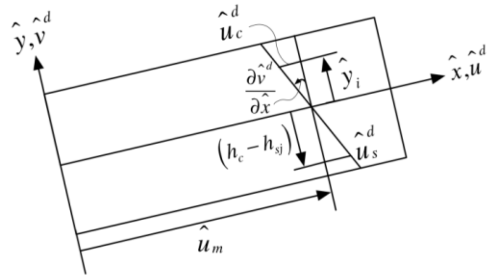
(a) Path element in a series of time points ($t_0 < t_1 < \ldots . t_a < t < t_b < t_c \ldots < t_f$)

(b) Convected material frame

Figure 2: Path element and convected material frame adopted in VFIFE



(a) Reinforced concrete simple support beam



(b) Frame element section.

Figure 3: Reinforced concrete section of the frame element.

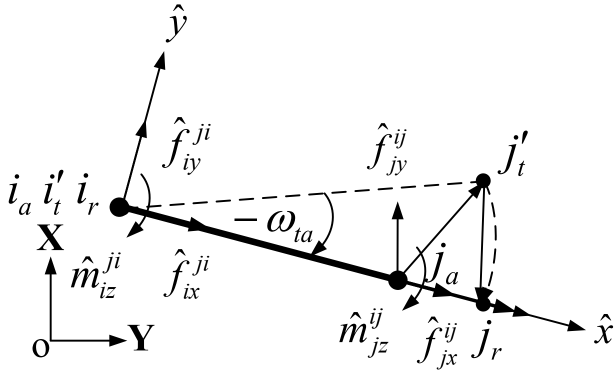


Figure 4: Internal forces in the deformation coordinates

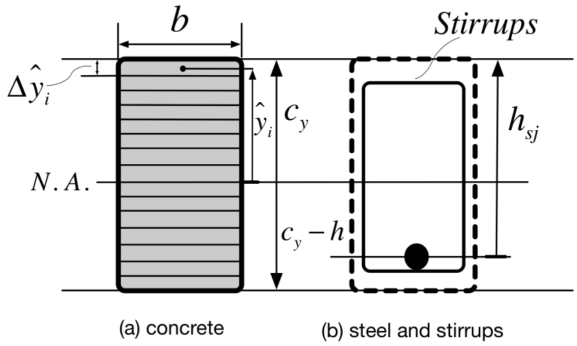


Figure 5: Fiber section of the RCCFE.

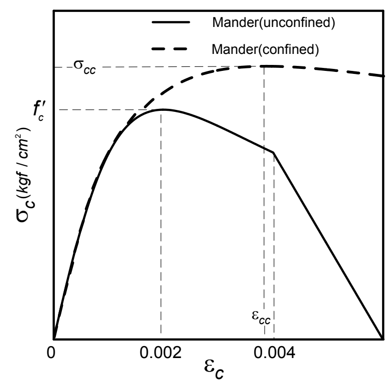


Figure 6: Mander concrete model

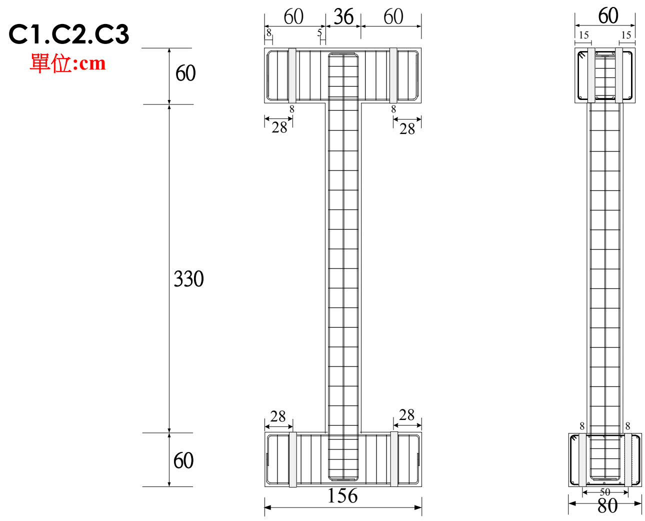


Figure 7: Design details of the specimens for the C1 by Yang [5]

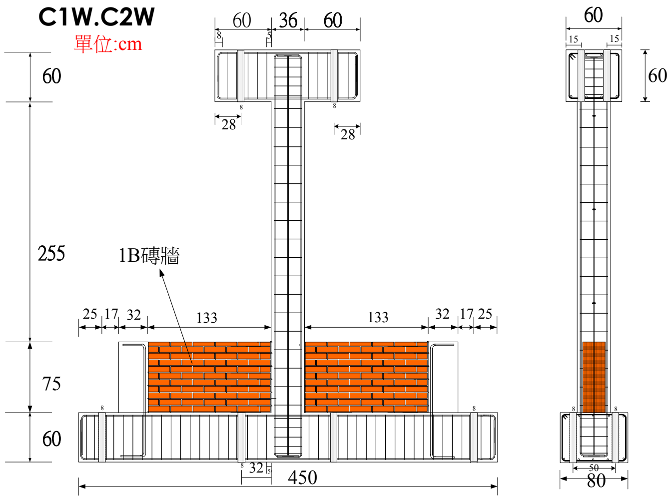


Figure 8: Design details of the specimens for the C1W by Yang [5]

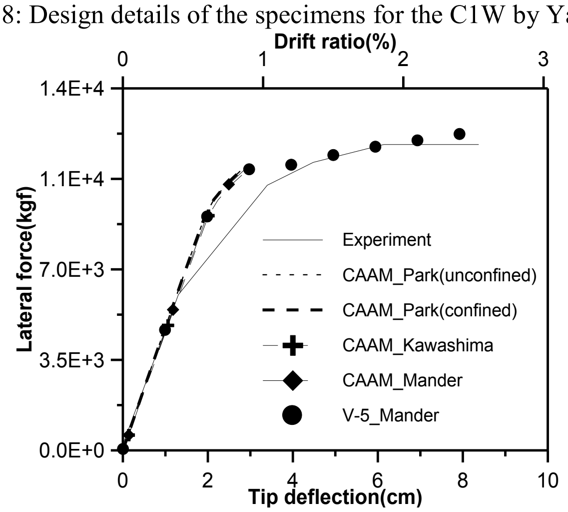


Figure 9: Comparison experimental and numerical results (C1)

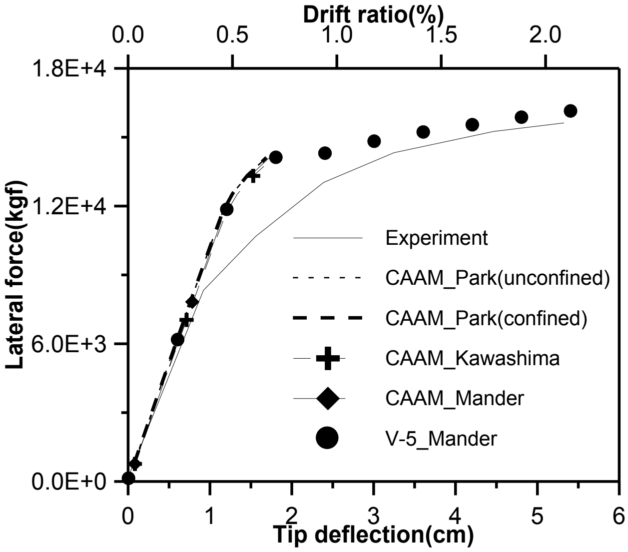


Figure 10: Comparison experimental and numerical results (C1W)