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IMPLEMENTATION AND APPLICATION OF VECTOR FORM INTRINSIC FINITE ELEMENT IN PUSHOVER ANALYSIS FOR REINFORCED CONCRETE BUILDINGS

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Abstract. This study aims to build numerical model of retrofitting analysis of Reinforced Concrete (RC) buildings with vector form intrinsic finite element (VIFIFE). Fiber element method is introduced in the paper to acquire the moment-curvature relation of the section and to define the behaviour of RC material. This paper also uses equivalent truss to build the analysis model of brick wall. In the end, lab and in-situ test specimens are utilized to assess analysis method.

The VFIFE method using in this study is one of the particle-based methods. The VFIFE method is included with the point value description, path element and convected material frame as the important characteristics. Thus, the RC structure is represented by finite particles. Each particle is subjected to the external forces and internal forces. The particle satisfies the Newton's Law. A fictitious reversed rigid body motion is used to remove the rigid body motion from the deformations of the element.

Pushover analysis is large deformation analysis. The conventional finite element method base program such as ETABS can encounter numerical difficulties in converging to a correct solution during an analysis involving large element deformation, highly non-linear plasticity or contact between surfaces. This research applied the VFIFE to solve the problems that usually occur on finite element program while performing pushover analysis. The analysis results show that for complex structures, VFIFE still could perform the pushover analysis until all the elements were collapsed but ETABS could not continue the analysis after reach the performance point. Automatically, VFIFE can predict better result in ductility rather than ETABS.

As indicated in analysis results, better performance is identified in initial stiffness, yield strength, maximum strength, or steady strength of specimens when compared with lab test results. Biases are within the tolerance range. Additionally, when failure model of specimens is compared, similar performance with lab test result is found. Hence, analysis method proposed by this paper is able to effectively simulate seismic capacity and failure behaviour of RC buildings.

1 INTRODUCTION

Currently, Seismic design criteria tend to shift from the force based procedure to performance based procedure for design and evaluation purpose. The nonlinear static analysis, which is also called pushover analysis, is a transparent and efficient method for predicting seismic behavior of the structures. The accuracy of the pushover analysis depends on the well-defined properties of nonlinear hinges in structure elements. This method appears in all publications of ATC-40 Report [1], and ASCE 41 report [2], which is the latest in a series of document developed to assist engineers with the seismic assessment and rehabilitation of existing buildings [3, 4].

Software programs such as ETABS, SAP2000, and Perform3D are used to perform pushover analysis. Finite element method is adopted by the programs. Finite element method can encounter numerical difficulties in converging to a correct solution during an analysis involving large element deformation, highly non-linear plasticity or contact between surfaces.

The nonlinear analysis of a structure is an iterative procedure. It depends on the final displacement, as the effective damping depends on the hysteretic energy loss due to inelastic deformations, which in turn depends on the final displacement. This makes the analysis procedure iterative.

The vector form intrinsic finite element (VFIFE) method can overcome the large deformation from pushover analysis. The VFIFE is a vector mechanics-based mathematical calculation method for structures with large deformations. It is based on an intrinsic finite-element modeling approach, an explicit algorithm, and a corotational formulation of kinematics [5~7]. The primary objective of this method is to handle the motion and deformation of a system of multiple continuous bodies and their interactions. The VFIFE method considers that the motion of a structure can be represented by a finite particles.

2 VECTOR FORM INTRINSIC FINITE ELEMENT (VFIFE) METHOD

2.1 Fundamentals of the Vector Form Intrinsic Finite Element (VFIFE) Method

The VFIFE is a vector mechanics-based mathematical calculation method for structures with large deformations. It is based on an intrinsic finite-element modelling approach, an explicit algorithm, and a corotational formulation of kinematics [5~7]. The primary objective of this method is to handle the motion and deformation of a system of multiple continuous bodies and their interactions.

The main objective of the VFIFE method development is to estimate the structural responses under various types of loading conditions especially from the continuous states to the discontinuous states of the structures. It is expected that VFIFE method could consider the geometrical and mechanical properties of the structure during the motion accurately. Nevertheless, the VFIFE method modelling concept of the structure is represented by finite particles, so that the conventional nonlinear structural analysis methods (CNSAM) could not be applied. The VFIFE method establish a new analysis strategy based on the intrinsic theories of mechanics[5] and avoid the difficulties such as the iterative and perturbation procedures in solving partial differential equations commonly adopted in the CNSAM. The primary concept of VFIFE is that the structure is illustrated as a system composed of particles and forces components. The forces that acting on the particles include the internal forces and external

forces. There are general assumptions that used to explain the deformation of the structure:
1. The internal forces are calculated from the deformations of structural members such as truss, frame, or solid elements.
2. Each structural members has geometry and position changes simultaneously. The changes of the geometry and the position for the deformable structure are not separated.
3. Each particles might has their own motion trajectory.

According to these assumptions, the associated analytical operations will use:

a. Point value description (PVD)

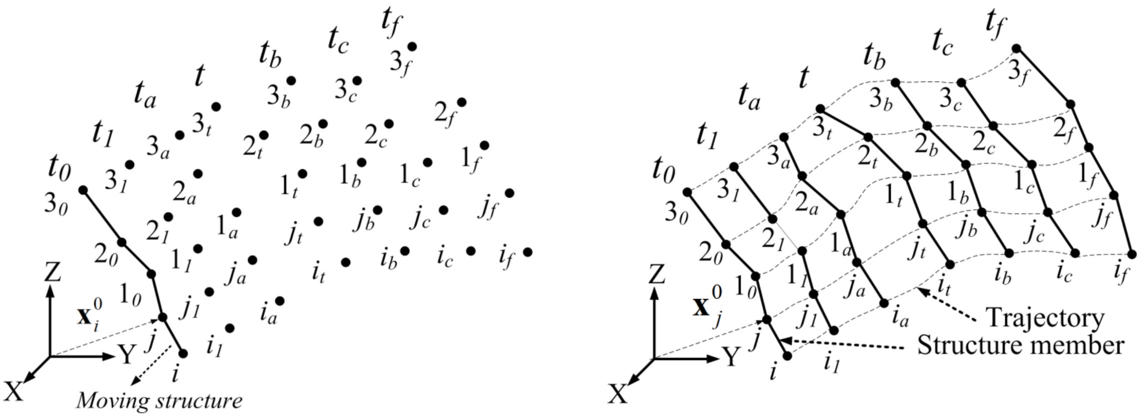
b. Path element.

c. Convected material frame (CMF).

Types of the particles and internal forces from the deformation of the frame element for each of above assumptions will be explained in the following sections.

2.2 Point value description (PVD)

The characteristic of the VFIFE analysis that use PVD concept could be seen from Figure 1. Numerical procedures that use VFIFE method can be solved by using the vector form concept and PVD. The deformations and motions of a structure are represented by the positions of the particles in spatial-temporal space and discrete times points that shown in Figure 1(a). The discrete time points is used by the PVD to describe the whole time trajectory of deformable body. The VFIFE method does not use a continuous time function such as $f(x, y, z, t)$, the response parameters such as displacement, and velocity are not continuous function of time and position in the whole time trajectory which is different from the conventional analysis methods. Therefore, it does not involve in solving the partial differential equations for structural members because the equations of motion of the particles at each different time point are built using Newton's law. The reference configuration of the structure at time t_a can be identified by connecting the representative particles ($i_a, j_a, 1_a, 2_a, and\ 3_a$) .The trajectory of any representative particles fulfil the definition of a path element at each set of time points $(e.g. i - i_1, i_1 - i_a, i_a - i_t, i_t - i_b, i_b - i_c, i_c - i_f$ in Figure 1(b)). The PVD and the function of time trajectory are shown in Fig. 1(b). The dotted line represents the particle trajectories from the positions at time t_0 to positions at time t_f.



(a) Particles in the spatial-temporal space (X, Y, Z, t)   (b) Structure configurations and the trajectories

Figure 1: Motion of a structure in VFIFE method

2.3 Path element and types of the particle

The motion of a structure is needed in order to introduce the concept of path element that shown in Figure 2(a). The whole time trajectory of the particles i and j are the end particles of the structure members. The PVD is used to describe the whole time trajectory of particles i and j. For instance, a series of time points $(t_0 < t_1 <\ldots.. t_a < t < t_b < t_c \ldots < t_f)$ is being used to represent the entire time for the particle i. If the time interval is (t_a, t_b), $\mathbf{x}_i^a$ and $\mathbf{x}_i^b$ are position vectors of the particle i at time t_a and t_b , and if the point values $(t_a, \mathbf{x}_i^a)$ and $(t_b, \mathbf{x}_i^b)$ are known. Afterwards the time trajectory of the particles in time interval (t_a, t_b) could be described by using a position function $\mathbf{x}_i(t)$.

$$x_i = x_i(t), \quad t_a \le t \le t_b \tag{1}$$

The position function $\mathbf{x}_i(t)$ is in vector form. Hence, it is defined as a position vector time function and abbreviated as position vector. The $\mathbf{x}_i(t)$ can be used to fulfil the equations of motion for the particles, or any physical conditions such as a fixed support of a structure. The motion of the frame element i-j in the time interval (t_a, t_b) as shown in Figure 2(a) could be described by using two position vectors $(\boldsymbol{x}_i(t), \boldsymbol{x}_j(t))$.

According to the physical conditions of the particles, three basic types of the particles are presented as follows:

1. Motion particle: The motion particle i has a mass, internal forces $\boldsymbol{f}_i$ and external forces $\boldsymbol{F}_i$ on the particle. This mass is a constant mass. The position vector $\mathbf{x}_i(t)$ of the motion particle must fulfil the Newton's Law as follows:

$$m_i \ddot{x}_i = F_i - f_i \tag{2}$$

where $\mathbf{m}_i$ is the mass of the particle i. The initial conditions are:

$$x(t_a) = x_i^a, x(t_b) = x_i^b, \ \dot{x}(t_a) = \dot{x}_i^a, \dot{x}(t_b) = \dot{x}_i^b \tag{3}$$

The path element can be viewed as a position vector $\mathbf{x}_i(t)$ to describe the motion of the particle only in the time interval (t_a, t_b).

2. Connected particle: The particle i in the frame element i-j is connected to the particle n in the frame element n-h. The position vectors of particles i and n fulfill the initial condition and the relationship of the two particles:

$$x_i(t_a) = x_i^a = x_n(t_a) = x_n^a, \ x_i(t) = x_n(t), \ t_a \le t \le t_b \tag{4}$$

3. Displacement particle: The position vector of the particle i in the time interval (t_a, t_b) is established by a function of time $g(t)$:

$$x_i(t) = g(t) \tag{5}$$

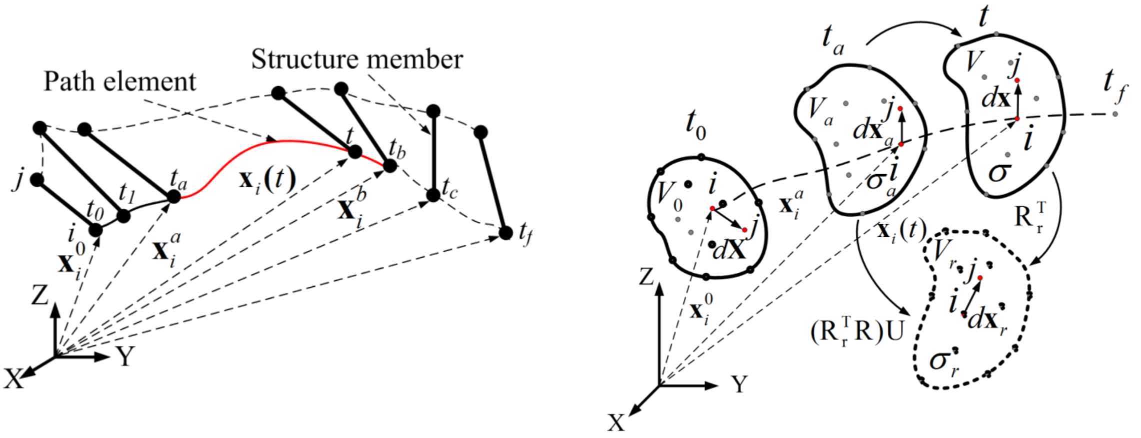
And the initial condition have to fulfil:

$$x_i(t_a) = g(t_a) = x_i^a \tag{6}$$

In general, the displacement particle is for the displacement control. For instance, setting g(t) as a constant value $\mathbf{x}_i^0$, then the position vector is:

$$x_i(t) = x_i^0 \tag{7}$$

For each type of particle, the whole time trajectory in time interval (t_0, t_f) needs the use of f path elements. Each path element determines the motion of the particle in each time interval and the initial time and final time could be redefined if the physical condition of the particle is changed suddenly.



(a) Path element in a series of time points (t0<t1<.....ta<t<tb<tc...<tf)

(b) Convected Material Frame

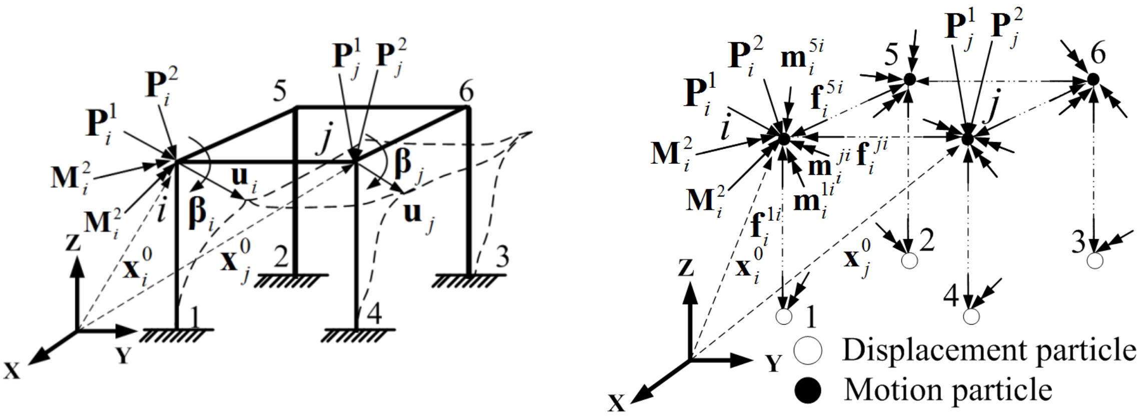
Figure 2: Path element and convected material frame adopted in VFIFE

2.4 Structural models

The structure model for a space frame in Figure 3(a) is represented in this section using many particles from Figure 3(b). Each particles is subjected to the internal forces or the external forces. The internal forces on particles could be calculated from the deformations of the frame elements.

Interaction forces between two end particles of the element are internal forces. The particles 1, 2, 3, and 4 from Figure 3(b) are specified as displacement particles. The particles i, j, 5 and 6 are specified as motion particles.

In the traditional Finite Element Method, the particle's position vector $\mathbf{x}(x, y, z, t)$ is a function of the current position (x,y,z) and at any time t from the initial states to the finial states.



(a) A space frame structure

(b) Forces on discrete particles in the space frame structure

Figure 3: Analytical model of a space frame structure

In VFIFE, the particle's position vector $\mathbf{x}(t)$ of each particle is a time function at the interval of (t_a, t_b) only. In this case, the displacement particles are act as fixed ends. The displacements and rotations of the motion particle i in the global coordinates could be calculated from the following equations:

$$\bar{m}_i \ddot{u}_i = F_i - f_i \tag{8}$$

$$\bar{I}_i \ddot{\beta}_i = M_i - m_i \tag{9}$$

where

$$F_i = \sum_{s=1}^{2} F_i^s ,\ M_i = \sum_{k=1}^{2} M_i^k \tag{10}$$

$$f_i = \sum_{n=1}^{3} f_i^{ni} ,\ m_i = \sum_{n=1}^{3} m_i^{ni} \tag{11}$$

$$\bar{m}_i = \sum_{n=1}^{3} \bar{m}_i^{ni} ,\ \bar{I}_i = \sum_{n=1}^{3} \bar{I}_i^{ni} \tag{12}$$

where s is the external force and k is the moment numbers. The n represents the particle number. The $\mathbf{F}_i$ and $\mathbf{M}_i$ are the total external forces and moments. The $\mathbf{f}_i$ and $\mathbf{m}_i$ are the total internal forces and moments. The Computation of these internal forces will be acquainted in the following sections. The $\mathbf{u}_i$ and $\boldsymbol{\beta}_i$ are the particle's displacement and rotation vectors. The $\bar{\mathbf{m}}_i$ and $\bar{\mathbf{I}}_i$ are the particles mass and the mass moment inertia. In Eq. (12), the mass $\bar{\mathbf{m}}_i^{ni}$ and the mass moment of inertia $\bar{\mathbf{I}}_i^{ni}$ of the end particles of the frame element can be computed from:

$$\bar{m}_i^{ni} = \begin{bmatrix} \bar{m}_i^{ni} & 0 & 0 \\ 0 & \bar{m}_i^{ni} & 0 \\ 0 & 0 & \bar{m}_i^{ni} \end{bmatrix},\ \bar{I}_i^{ni} = \Omega_a \hat{I}_i^{ni} \tag{13}$$

where

$$\bar{m}_i^{ni} = 1/2 \rho l^0 A^0 ,\ l^0 = \left| x_{ji}^0 \right| \tag{14}$$

$$x_{ji}^0 = x_j^0 - x_i^0 \tag{15}$$

$$\hat{I}_i^{ni} = \begin{bmatrix} \hat{I}_{i1}^{ni} & 0 & 0 \\ 0 & \hat{I}_{i2}^{ni} & 0 \\ 0 & 0 & \hat{I}_{i3}^{ni} \end{bmatrix},\ \hat{I}_{iq}^{ni} = \bar{m}_i^{ni} r_{iq}^{ni} = \bar{m}_i^{ni} \sqrt{I_{iq}^{ni} / A} ,\ q=1\sim3 \tag{16}$$

The l^0 and A^0 are the length and cross-sectional area of the frame element at the initial state. The ρ and $\bar{\mathbf{m}}_i^{ni}$ are the mass density and mass of the two ends of the frame element.

The $\bar{\mathbf{I}}_i^{ni}$ and I_{iq}^{ni} are the i^{th} particle's mass moment of inertia and the moment of inertia of the cross-sectional area with respect to the q principal direction. The transformation matrix $\boldsymbol{\Omega}_a$ and the principal directions will be explained in the following sections. The advantage of using path elements is to facilitate the computation of the material frame within the time interval (t_a, t_b). For example, geometrical changes of an element could be assumed to fulfil the following:

(a) The initial values at time t_a are required for the analysis in each path element but the deformation history before the time t_a is not considered in VFIFE model. Moreover, the rates of stress such as the Jaumann and Green-Naghdi rates of Kirchhoff stress are not even considered because the deformations of the elements in VFIFE model are total deformations in each path element. Nevertheless, in the traditional Finite element method and the explicit Finite element method, the deformation history and the current deformation of the frame must take into consideration in order to compute the incremental deformations of the structural member. It utilizes a position vector $\mathbf{x}(x, y, z, t)$ to describe the deformation of the member. Hence, it needs to consider the rates of stress.

(b) In each path elements, the element is prismatic in which the section and the material along the element length remain constant when the geometrical changes are neglected while calculating the internal forces. The stresses and strains are used. The stress and strain analysis of the frame element are a straight forward problems for the large displacement and small deformation. That is, over the entire analysis from the initial to the final states, the total geometrical change can still be very large. Moreover, the discontinuous changing of the physical properties at each time point is permitted.

3 TEASPA METHOD

This research utilizes VFIFE method that combine with TEASPA method [8] to perform pushover analysis of 3-D reinforced concrete structures compares with experimental results and also ETABS. It needs to add another parameters in order to perform pushover analysis with the most accurate result compares to the experiment result. Columns, Frame elements and a school building are taken as the source of database for this research.

The shear drift capacity of the shear damage column is estimated from the following empirical equation. This equation is based on the observation of 50 shear-critical column databases that is proposed by Elwood and Moehle [9].

$$\frac{\Delta_s}{H} = \frac{3}{100} + 4\rho" - \frac{1}{133} x \frac{\upsilon_m}{\sqrt{f_c'}} - \frac{1}{40} x \frac{P}{A_g f_c'} \geq \frac{1}{100} \tag{17}$$

where H is the length of the column; ρ " is the transverse reinforcement ratio as A_{st}/bxs; A_{st} is the transverse reinforcement area; b is the width of column section; s is the spacing of the transverse reinforcement; υ is the maximum nominal shear stress in kgf/cm^2as V/bd ; d is the depth from the extreme fiber of concrete to the center line of tension reinforcement; f_c' is the concrete compressive strength in kgf/cm^2; and A_g is the gross cross-sectional area of the column. The shear drift capacity is associated to the maximum shear stress, transverse steel ratio and also axial load. The shear drift ratio is getting lower if the axial load ratio is getting higher. As the nominal shear stress degrades, the shear drift ratio will increase, and the shear drift ratio is proportional to the amount of transverse reinforcement.

The axial drift capacity of shear damage column is estimated based on the shear friction model proposed by Elwood and Moehle which the axial drift capacity is the function of the axial load, the amount of transverse reinforcement, and the critical angle [10].

$$\frac{\Delta_a}{H} = \frac{4}{100} x \frac{1+(tan\Theta)^2}{tan\Theta + Px\frac{s}{k' A_{st} f_{yt} d_c tan\Theta}} \tag{18}$$

where f_{yt} is the yield strength of the transverse reinforcement; d_c is the depth of the column core from center line to center line of the ties.

The flexural shear strength is the required lateral load to reach the maximum flexural capacity M_n at column end. The flexural shear strength of a double curvature column can be calculated as follow:

$$V_b = \frac{2M_n}{H} \tag{19}$$

where M_n is the nominal moment strength of a reinforced concrete column.

The nonlinear moment hinge is placed at the both ends of the column to present the flexural failure mode or flexural-shear failure mode. The parameters of the nonlinear hinge of the column are calculated as follows:

$$a = \frac{\Delta_s}{H} - \frac{\Delta_y}{H} \tag{20}$$

$$b = \max\left(\frac{\Delta_a}{H}; \frac{\Delta_s}{H}\right) \tag{21}$$

$$\Delta_y = \frac{V_b}{k} = \frac{V_b H^3}{12 x \alpha x E I_g} \tag{22}$$

Moment SF = M_n ; Rotation SF = 1; α is the stiffness coefficient that consider the reduction of the element stiffness on second order analysis.

According to ACI 318-05[11], it is permitted to assume α = 0.35. The nonlinear shear hinge is placed at the middle of the column to illustrate the shear failure mode. The parameter are calculated as follows:

$$c = \min\left(\frac{\Delta_s}{H}; 0.04\right) \tag{23}$$

Force SF = V_n; Displacement SF = H.

4 CASE STUDY

Each specimens was model using ETABS program and VFIFE program. The plastic hinges of elements of the model were calculated and defined by using modified TEASPA method. Finally the pushover curves were obtained by solving nonlinear problem by using the ETABS program and VFIFE program. Based on the experimental results, the beam was remained in elastic behavior. The failures were occurred in columns and brick walls so there are no plastic hinges on beam from our models in ETABS and VFIFE.

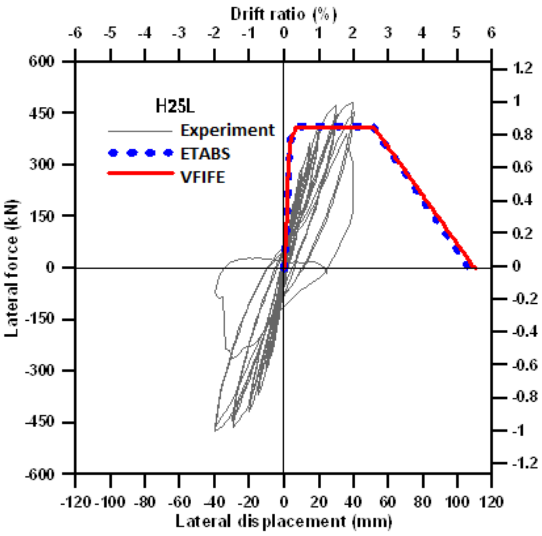


Figure 4: Base shear - Displacement comparison of H25-L

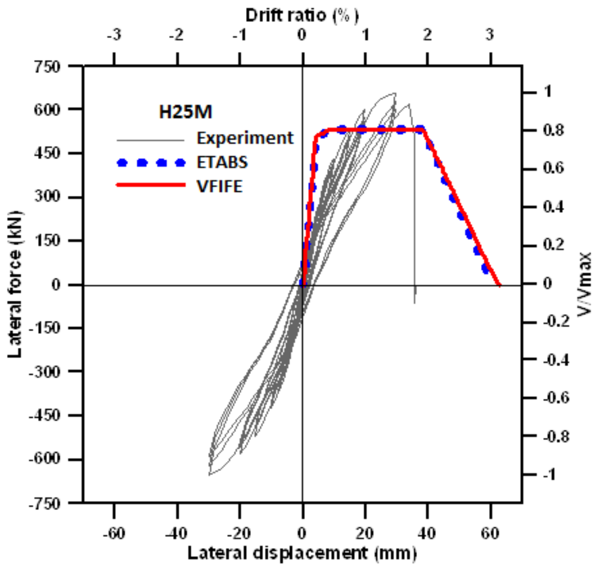


Figure 5: Base shear - Displacement comparison of H25-M

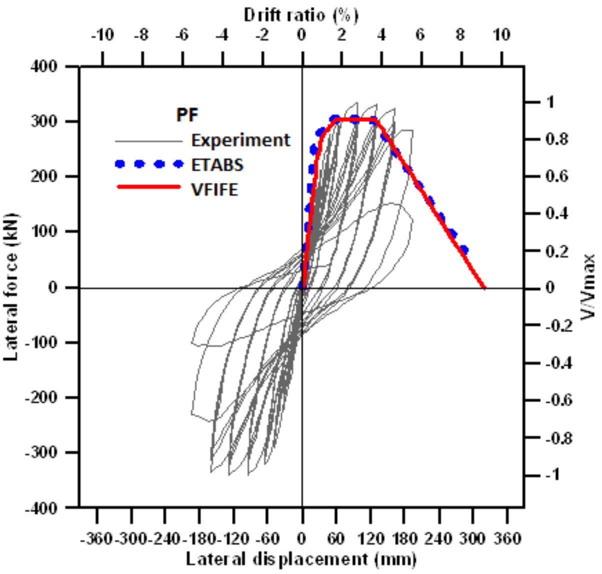


Figure 6: Base shear - Displacement comparison of PF

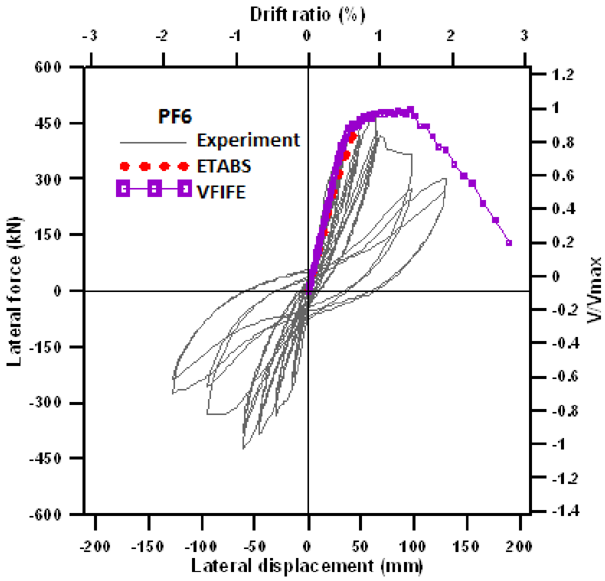


Figure 7: Base shear - Roof Displacement comparison of PF6

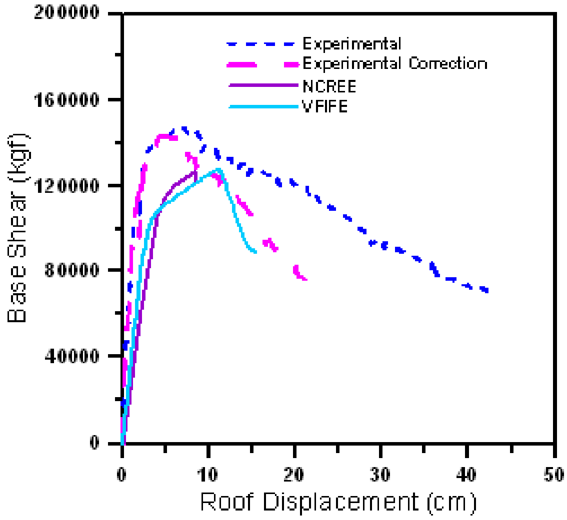


Figure 8: Base shear - Roof displacement comparison of Guanmiao

Figure 4 and Figure 5 are shown that the stiffness from ETABS and VFIFE of each column specimens has a higher stiffness compares to experimental result. It shows that the stiffness coefficient from TEASPA method is too large for the column specimen.

The ultimate strength from ETABS and VFIFE are lower than the experimental result. ETABS and VFIFE show much the same result for column specimens. For columns with high axial load have displacement almost half compares with small axial load columns but their strength are lower.

Figure 6 and Figure 7 illustrate that the initial stiffness for both specimen are approximately the same. The ultimate strength from VFIFE and ETABS are almost the same with the experimental result. The PF specimen shows the same ductility for both VFIFE and ETABS. However, The ductility from ETABS is different from VFIFE for specimen PF6. It shows that after reach the ultimate point, ETABS could not continue to obtain the displacement result. It indicates that ETABS is not suitable for a complex model for pushover analysis. VFIFE still could continue the pushover analysis until most of the elements were collapsed.

Figure 8 illustrates the comparison among the pushover curves obtained by the two analysis cases and the test. Obviously, the results of the analysis cases are conservative in comparison with the test. Figure 8 indicates that the stiffness of the pushover curve obtained from both VFIFE and ETABS are significantly smaller than the stiffness measured from the test. The imperfect condition of the test may account for the problem. The specimen was created from the real building; therefore, some non-structure elements not be taken in the analysis may contribute to increase the stiffness of the specimens.

It also indicates the same situation with the PF6 specimen for ETABS analysis. It will stop the pushover analysis after reach the performance point. It is happened because of pushover analysis is a large deformation analysis. Finite element method in this case ETABS can encounter numerical difficulties in converging to a correct solution during an analysis involving large element deformation, highly non-linear plasticity or contact between surfaces.

The nonlinear analysis of a structure is an iterative procedure. It depends on the final displacement, as the effective damping depends on the hysteretic energy loss due to inelastic deformations, which in turn depends on the final displacement. This makes the analysis procedure iterative. Difficulty in the solution is faced near the ultimate load, as the stiffness matrix at this point becomes negative definite due to instability of the structure becoming a mechanism .Therefore, the result of a specimen that starts to collapse while performing pushover analysis with finite element software will shown inaccurate result.

5 CONCLUSIONS

Several main conclusions of this research are gathered as follows:

1. Analysis results for Column specimens using ETABS and VFIFE show approximately the same results in stiffness, ultimate base shear and also the ductility. For columns with high axial load have displacement almost half compares with small axial load columns but their strength are lower.
2. For PF specimen which is one story pure frame, ETABS and VFIFE also show more or less the same results in stiffness, ultimate base shear and also the ductility. For PF6 specimen also shows approximately the same in initial stiffness and ultimate base shear. However, the ductility between ETABS and VFIFE are not the same. After reach the ultimate value, ETABS can't continue to show the deformation but VFIFE could continue to do the pushover until most of elements failed . It indicates that ETABS is not capable for Pushover Analysis for complex structures.
3. The stiffness of the pushover curve obtained from both VFIFE and ETABS are significantly smaller than the stiffness and shear strength measured from the test for Guanmiao School Building specimen. The imperfect condition of the test may account for the problem. The specimen was created from the real building; therefore, some non-structure elements not be taken in the analysis may contribute to increase the stiffness of the specimens. The ductility from ETABS has the same problem with PF6 specimen if it compares with VFIFE.
4. From all the analysis results, They show tendency that the more complex the specimen, the softer the stiffness.

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