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APPLICATION OF KINETIC DAMPING IN DYNAMIC MATERIAL POINT METHOD FOR STATIC PROBLEMS

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Abstract. Material point method (MPM) is widely used in geotechnical engineering, owing to its powerful capability of modelling large deformation problems. But the static equilibrium problems involving very large deformation and material non-linearity can be difficult to solve using the quasi-static MPM, because of numerical difficulties with convergence in iterative procedure. An alternative method is the dynamic relaxation (DR) method, which converts the static problem to a dynamic one by considering the virtual masses and artificial damping. This paper presents a method to solve static problems using dynamic MPM with DR technique. An energy ratio and a force ratio are defined to recognize the static equilibrium state from dynamic process first. Then the kinetic damping as an DR technique is introduced into the dynamic MPM for the first time. Finally, two numerical examples are presented to illustrate the convenience and efficiency of the kinetic damping in dynamic MPM for static problems.

1 INTRODUCTION

In managing static problems involving very large deformation such as cone penetration testing (CPT), the jacking of piles and passive earth pressure problems, the classical Finite Element Method (FEM) encounters numerical difficulties due to mesh distortion and inherent problems associated with modelling slipping, separation and breakage. In contrast, the MPM developed recently， has enormous advantages in handling the large deformation problems in geomechanics[1-6].

Similar to finite element method, the quasi-static MPM[7] discrete the governing partial differential equations of a static equilibrium problem into a system of simultaneous algebraic equations, which may be written as:

$$[K]\{u\} = \{F^{\text{ext}}\} \tag{1}$$

where $[K]$ is the stiffness matrix, $\{u\}$ is the nodal displacement vector and $\{F^{\text{ext}}\}$ is the equivalent nodal external force vector.

Whenever consider the large deformation, the material properties and state variables e.g. stresses and strains are nonlinear, which makes Eqation (1) a complex system of equations and hard to solve. One alternative method is DR method, which can be much more computationally efficient than the static one.

DR method converts the static problem to a dynamic one by considering the virtual masses and artificial damping. The dynamic equations of motion, which are equivalent to Eqation (1), are as follows:

$$[M]\{a\} = \{F^{\text{ext}}\} - \{F^{\text{int}}\} - \{F^{\text{damp}}\} \tag{2}$$

where $[M]$ is the virtual mass matrix, $\{a\}$ is acceleration vector, $\{F^{\text{int}}\}= [K]\{u\}$ is nodal internal force vector and $\{F^{\text{damp}}\}$ is the artificial damping force vector. Eqation (2) can be easily solved by an explicit time integration method, which does not need to assemble the stiffness matrix. The steady state response of this dynamic system is the solution of Eqation (1), when the acceleration and velocity vectors are damped to zero ($\{F^{\text{damp}}\}$ is zero at steady static state). The selection of mass matrix $[M]$ affects the dynamic property of the system such as frequency and critic time step size[8], while the damping force vector $\{F^{\text{damp}}\}$ affects the dissipation speed of kinetic energy of the system and the iteration number to get static equilibrium state.

The common used artificial damping is viscous damping. The viscous damping force $\{F^{\text{damp}}\}=2\beta\ [M]\{v\}$ is proportional to the product of nodal velocitie vector $\{v\}$ and mass matrix $[M]$, where β is a damping factor. Zabala et al.[9] adopted an adaptive viscous damping in modeling progressive failure of Aznalcollar dam. Andersen et al.[4] proposed a similar damping scheme by decreasing the updated velocities. In order to achieve the most rapid convergence, the critical viscous damping factor of the system must be used. However, the critical damping factor might be difficult to estimate. Cundall[10] working on unstable geo-mechanical problems, suggested using kinetic damping which proved to be robust and rapidly converging when dealing with large unbalanced local forces. There is no need for prior determination of any constant. The basic idea of kinetic damping is that as an oscillating particle passes through a minimum potential energy state (equilibrium state), its total kinetic energy reaches a local maximum. For a multi-degree of freedom system, upon detection of a local energy peak, all current nodal velocities are set to zero. The process is then restarted from the current configuration and continued through resetting velocities until the energy of all modes of vibration has been dissipated then the system attains its static equilibrium state.

This paper calculate static problems using dynamic MPM with kinetic damping, which is proved as a robust and powerful DR technique. An energy ratio and a force ratio are defined to recognize the static equilibrium state from dynamic process first. Then the kinetic damping as an DR technique is first introduced into the dynamic MPM. Finally, two numerical examples, elongation of an elastic bar and progresive failure of a slope, are presented to illustrate the convenience and efficiency of the kinetic damping in dynamic MPM for static problems.

2 METHODOLOGY

2.1 Static equilibrium state

When using a dynamic code to solve static problems, A detection of static equilibrium state is required. Such state is achieved if and only if both the maximum out-of-balance force and the kinetic energy of the system vanish. The maximum out-of-balance force approaches zero, indicating that the system is reaching an equilibrium state. The kinetic energy of the system decays to zero representing a static state.

Let us define a dimensionless force ratio K_F of the maximum out-of-balance force divided by maximum external force for all the background grid nodes in the model as

$$K_F = \frac{\left\| \{F^{\text{ext}}\} - \{F^{\text{int}}\} \right\|_\infty}{\left\| \{F^{\text{ext}}\} \right\|_\infty} = \frac{\max\limits_{n_g} \left| F_i^{\text{ext}} - F_i^{\text{int}} \right|}{\max\limits_{n_g} \left| F_i^{\text{ext}} \right|} \tag{3}$$

where n_g is the number of the background grid nodes. And define a dimensionless energy ratio K_E of kinetic energy of all the material points divided by the work induced by the external forces as

$$K_E = \frac{E_k}{W^{\text{ext}}} \tag{4}$$

where n_p is the number of the material points, E_k is the kinetic energy of all the material points

$$E_k = \frac{1}{2} \sum_{p=1}^{n_p} m_p \boldsymbol{v}_p \cdot \boldsymbol{v}_p \tag{5}$$

and W^{ext} is the external work calculated by

$$W^{\text{ext}} = W_0^{\text{ext}} + \Delta W_0^{\text{ext}} = W_0^{\text{ext}} + \sum_{p=1}^{n_p} \boldsymbol{f}_p^{\text{ext}} \cdot \Delta \boldsymbol{u}_p \tag{6}$$

where $\boldsymbol{f}_p^{\text{ext}}$ and $\Delta \boldsymbol{u}$ are the external force and incremental displacement of material point p, respectively. W_0^{ext} is the external work at the beginning the current step.

The force ratio K_F and the energy ratio K_E are calculated in each computational cycle. If they are less than a pre-defined tolerance, the computation is terminated. The experience of the author shows that a tolerance of 0.01 is sufficient for both criteria. And this will be adopted in this paper.

2.2 Kinetic damping

When an oscillation particle passes through a static equilibrium position without damping, the kinetic energy curve reaches the maximum value. The kinetic damping is based on this theorem. In the computer program with kinetic damping, the kinetic energy is constantly monitored, and all velocities are set to zero when a peak in the energy is detected[11-13]. In the past few decades, the kinetic damping often used in form-finding of pre-stressed nets and membranes. A typical kinetic energy trace is shown in Figure 1 for the case of form-finding of a cable net or membrane with inaccurate initial geometry[14]. The early energy peaks (1) are associated with high frequency modes caused by large out-of-balance forces in boundary or mast-support regions. After these modes have been substantially damped out, subsequent peaks (2) are associated with the overall structural form and lowest frequency modes; the motion in these modes being generally normal to the changing surface. Near convergence, low energy peaks (3) occur rapidly, associated with slight in plane motion. The process is then ideally suited to continued modification of the form, involving local changes in boundary geometry and surface topology.

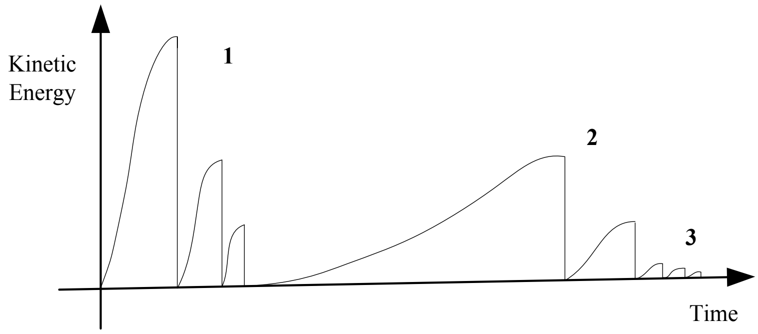

Figure 1: The dissipation of kinetic energy of system with kinetic damping [14]

The kinetic energy damping is an artificial damping, which is not a real effect, but offers a drastic reduction in the number of iterations required to find the static solution.

2.3 MPM implementation

The main steps of the material point method with kinetic damping for static problem is listed below. For more details, the reader can refer to [15-18].

(1) Map the momenta, external forces and internal forces of material points to the regular background grid nodes.
(2) Impose the essential boundary conditions on the background grid nodes.
(3) Solve global equations of background grid nodes to update the nodal velocities.
(4) Update the velocities of the material points using interpolation of the nodal accelerations.
(5) Calculate the kinetic energy of the system and if a peak value is detected, then reset the velocity of all the material points zero, otherwise update the positions of the material points using interpolation of the updated nodal velocities.
(6) Remap the velocities of the material points to the background grid nodes to calculate the incremental strain tensor and the incremental vorticity tensor of the material points. Then update the stress tensor by an objective constitutive model, as well as the density of the material points.
(7) Calculate the force ratio and the energy ratio of the system using Eqation (3) and Equation (4), respectively, to judge the static equilibrium state.
(8) Reset the deformed grid and use the initial regular background grid in the next step.

In this paper, we adopt an incremental elastoplastic constitutive model with Jaumann stress rate.

$$\sigma_{ij}^{\nabla} = C_{ijkl}^{ep} \dot{\varepsilon}_{kl} \tag{7}$$

with Jaumann stress rate:

$$\sigma_{ij}^{\nabla} = \dot{\sigma}_{ij} - W_{jk}\sigma_{ki} - W_{ik}\sigma_{kj} \tag{8}$$

where, W_{ij} is vorticity tensor

$$W_{ij} = \frac{1}{2}\left(\frac{\partial v_i}{\partial x_j} - \frac{\partial v_j}{\partial x_i}\right) \tag{9}$$

strain tensor

$$\dot{\varepsilon}_{ij} = \frac{1}{2}\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right) \tag{10}$$

3 EXAMPLES

Two numerical examples are shown to illustrate the efficiency of the kinetic damping in MPM: a simple problem of elongation of a linearly elastic bar[19] in the case of small deformation and the progressive failure of a slope.

3.1 Example 1: elongation of elastic bar with dimensionless parameters

The problem of elongation of a linearly elastic bar with the length L=10 and the side length of the cross-section a=1 is considered, see Figure 2(a). It is assumed that the left end of the bar is fixed while the point force F is applied to its right end. In the numerical model, the bar is discretized into 20 material points which lie in 10 C_1 elements. In order to apply the external surface load, a material point carrying a body force with the magnitude being F is added on the right end of the bar, see Figure 2(b). In the dynamic analysis, the force is applied immediately at simulation time t=0s. The calculations have been made with the following data: the force magnitude, F=0.001, the density ρ=1, the Young's modulus E=1 and the Poisson's ratio ν=0, see Table 1. The time step size Δt=0.01. During the calculations, the displacements and the stresses of particle A, representing the material point in the middle of the bar and labeled in Figure 2(b), are traced.

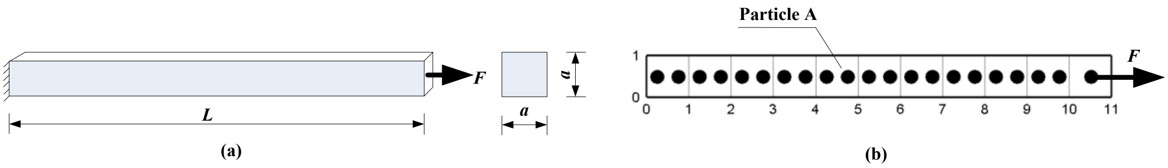

Figure 2: Model of the bar. (a) Schematic of the bar. (b) Numerical model of the bar.

Table 1: Parameters in example

Density	Young's modulus	Poisson's ratio	L	a	F
1	1	0	10	1	0.001

Figure 3 shows the kinetic energy dissipation of the bar with different damping in semilog coordinate system. For kinetic damping, the peak kinetic energy decays very fast with exponential rate. For viscous damping, the following values of the damping parameter β have been applied: 0.08, 0.1 and 0.12 respectively. We found that the best damping parameter value is 0.10 which is smaller than the "proper" damping factor $\beta = \omega_1 \approx 0.16$ with the first natural frequency of vibration:

$$\omega_1 = \frac{\pi}{2}\sqrt{\frac{E}{\rho l^2}} \tag{11}$$

For complex problem, it is difficult to estimate the best damping factor for viscous damping. But there is no need to determine any factors with kinetic damping.

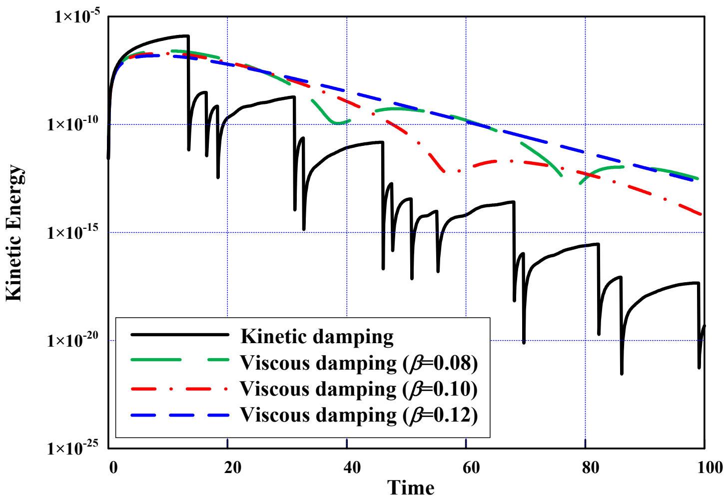

Figure 3: Kinetic energy dissipation of example 1 with different damping

The displacements and stresses of particle A with different damping are shown in Figure 4. For kinetic damping, the displacement and horizontal normal stress convergence very fast to the static solution, 4.75×10^{-3} and 10×10^{-4} respectively. But the solutions with viscous damping are slower althrough using different damping constants. The convergence time for the computation with different damping are shown in Table 2.

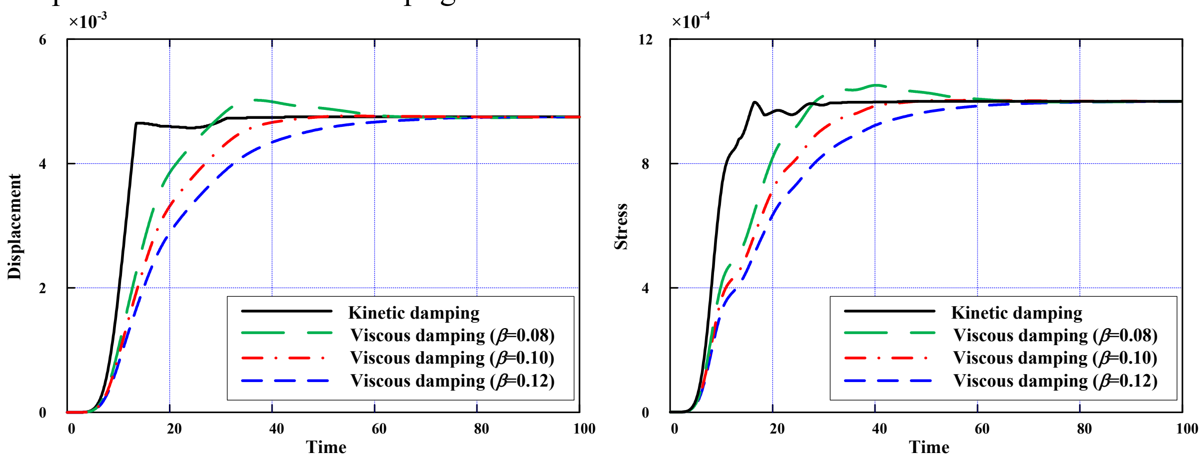

Figure 4: Displacements and stresses of particle A with different damping

Table 2: Convergence time for the computation of example 1 with different damping.

Damping type	Kinetic damping	Viscous damping		
		β=0.10	β=0.10	β=0.10
Convergence time	32.3	103.2	89.7	114.6

From Table 2, we can find that the convergence time are t=32.3 for kinetic damping, t=89.7 for viscous damping with best damping factor. The convergence time with viscous damping is nearly three time that with kinetic damping. So it is convenient and efficient to use the kinetic damping to get the static solution in dynamic material point method.

3.2 Example 2: progressive failure of cohesiveless soil slope

The second example is a homogenous high cohesiveless soil slope in plane strain condition with the height h = 40m, the slope angle θ=30°, as shown in Figure 5. The bottom is fixed on the base rock. Symmetric boundary conditions are imposed at the left and right sides of the slope. The simulations are performed in 3D MPM code with thickness being 0.5m. The element is a cube with a side length Δx =1m. Initially, there are 2×2 particles per element and the gravity is applied immediately.

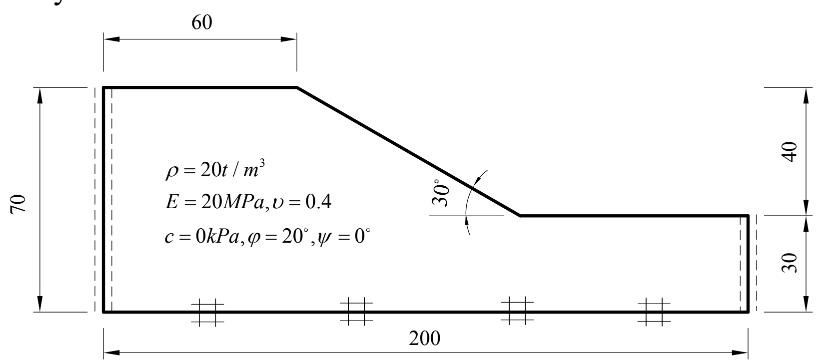

Figure 5: Schematic of the slope (unit: m)

The soil is modeled by Mohr-Coulomb linear elastic perfectly plastic model, whose Young's modulus E=20MPa, Poisson's ratio ν=0.4, cohesion c=0kPa, friction angle φ=20°, dilatancy angle ψ =0°, and density ρ=2×10³kg/m³, see Table 3. The maximum longitudinal wave speed is 146 m/s. Taking the Courant number α_c =0.2, the explicit time step size Δt = 1.366×10⁻³s.

Table 3: Material constants of soil in example 2.

Density	Young's modulus	Poisson's ratio	c	φ	ψ
2×10³kg/m³	20 MPa	0.4	0kPa	20°	0°

The kinetic energy peaks dissipation curve is shown in Figure 6 using semilog coordinate. The kinetic energy peaks is damped below 1% of the first kinetic energy peak after the first work of the kinetic damping. That is to say, the energy ratio K_E<0.01 after the first work of the kinetic damping. The simulation convergence at t=24.28s with force ratio K_F = 0.01 and the energy ratio K_E= 0.00. The horizontal displacements of the soil during the simulation are shown in Figure 7.

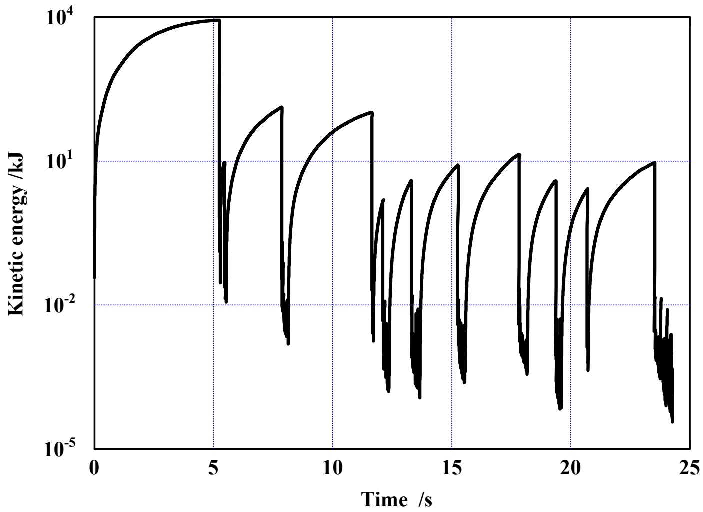

Figure 6: Kinetic energy dissipation of slope

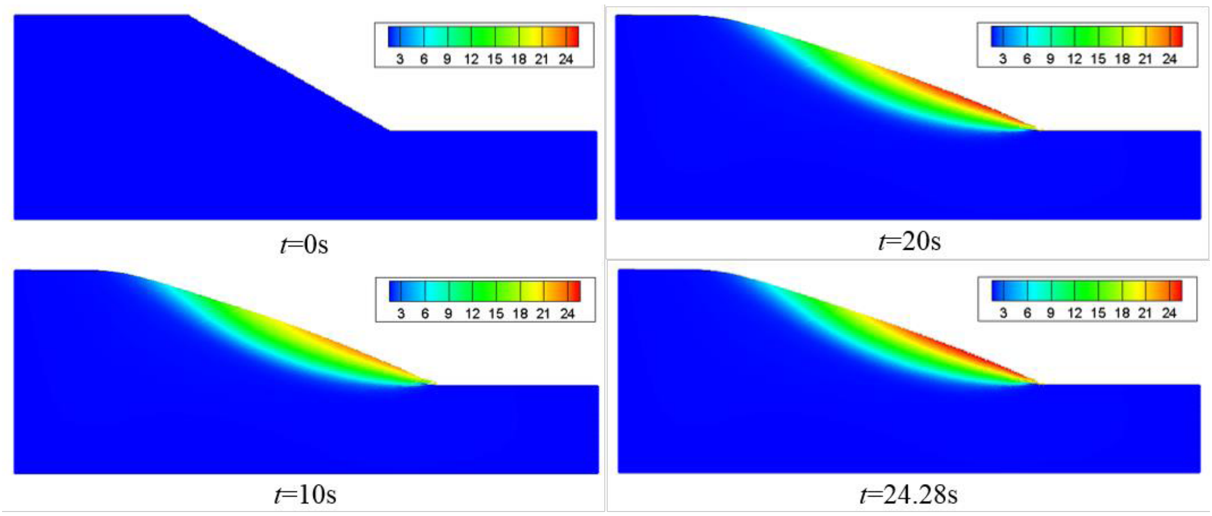

Figure 7: Horizontal displacements of the soil (unit: m)

From Figure 7, we can find that the major motion of the soil has completed before t=10s, and local movment with small magnitude after that. Finally, the angle of repose is about 20°, which is equal to the friction angle φ of the soil.

4 CONCLUSIONS

The MPM provides a convenient framework for the dynamic simulation of very large deformation problems in geomechanics. But it is not efficient for the calculation of static problems. This paper introduces the kinetic damping into MPM to calculate static problems. The energy dissipation effect of the kinetic damping is very efficient and the implementation of the algorithm is convenient, which is more impotant to some extent.

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