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WHAT IS WRONG IN LOVE-WEBER STRESS FOR UNSATURATED GRANULAR MATERIALS?

C. CHALAK[1], B. CHAREYRE[2] AND F. DARVE[3]

[1] 1. Univ. Grenoble Alpes, 3SR, F-38000 Grenoble, France
2. CNRS, 3SR, F-38000 Grenoble, France
Domaine Universitaire BP53, 38041 Grenoble cedex 9, France
caroline.chalak@3sr-grenoble.fr and URL

[2] 1. Univ. Grenoble Alpes, 3SR, F-38000 Grenoble, France
2. CNRS, 3SR, F-38000 Grenoble, France
Domaine Universitaire BP53, 38041 Grenoble cedex 9, France
bruno.chareyre@3sr-grenoble.fr and URL

[3] 1. Univ. Grenoble Alpes, 3SR, F-38000 Grenoble, France
2. CNRS, 3SR, F-38000 Grenoble, France
Domaine Universitaire BP53, 38041 Grenoble cedex 9, France
felix.darve@3sr-grenoble.fr and URL

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Abstract. This paper presents the micromechanical model for unsaturated soil in pendular regime, taking into account the roughness of the grains and the interfaces that separate the different phases present in the medium. It supplements the oral presentation with more technical content. Laplace equation is solved for two grains configuration to calculate the capillary force and all the geometric properties of the meniscus connecting the grains. Many configurations are solved and the look up table method is then used during the simulations. Results for grains moving at constant suction and constant volume are presented. It is also shown that the roughness has an important impact on the value of capillary force and it is evolution with the change of suction.

1 INTRODUCTION

Unsaturated soils has been an interest for many studies recently. Different approaches has been used to study the behavior of such materials.

One of these approaches is the thermodynamic approach([2],[5],[4],[10]). All these studies, highlighted the importance of the interfaces separating the different phases, and insisted that specific terms accounting for them must be included in the formulation of the effective stress.

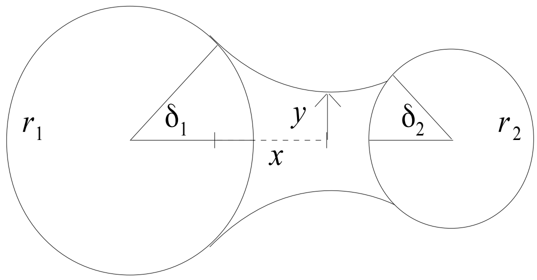


Figure 1: Illustration of a liquid bridge between 2 spherical grains.

On the other side, many recent work has been dedicated to studying unsaturated soils using the discrete element method DEM [1]. See for instance ([3], [7], [11]).

Bridging the theoretical frameworks mentioned previously, all these numerical tools need to define the interfacial areas accurately which has not been done yet.

This paper details in the first part the micromechanical model inspired by [11] and implemented in the open source code YADE and in which the interfaces that exist in the partially saturated medium are taken into account.

The model takes also into account the roughness of the grains. The influence of the roughness on the behavior of the unsaturated granular matter is discussed in the second part.

2 LAPLACE EQUATION

The geometry of the water bridges connecting the particles is determined by Laplace equation. Laplace equation relates the pressure jump p_c across the wetting-non wetting interface to its curvature through the surface tension:

$$p_c = \gamma C \tag{1}$$

Even if the form of the Laplace equation is simple, it is very difficult to solve. In the pendular regime, the capillary force is easy to compute and can be linked to the geometry of the grains and the suction in the medium. For higher saturations, the problem becomes much more complicated. This section details the resolution of Laplace in the pendular regime.

The pendular regime is over when the disconnected bridges start to overlap. The overlapping of the bridges is tested using the filling angles of the bridges on each particle.

For sake of more simplicity, the grains are assumed to be spherical in order to have an axisymmetric problem. In this case, the curvature and the Young-Laplace equation can be written as following:

$$C = \frac{1}{y(x)\sqrt{1+y'(x)^2}} + \frac{-y''(x)}{(1+y'(x)^2)^{\frac{3}{2}}} \tag{2}$$

$$\frac{p_c}{\gamma}(1+y'(x)^2)^{\frac{3}{2}} + \frac{(1+y'(x)^2)}{y(x)} - y''(x) = 0 \tag{3}$$

To generalize the study and describe the capillary phenomena for different particle sizes and different liquid types, all the variables are normalized. We introduce the factor R that is the radius of the smallest (r_2) at biggest (r_1) grains in one meniscus configuration:

$$x^* = \frac{x}{R} \tag{4}$$

$$y^* = \frac{y(x)}{R} \tag{5}$$

$$y'^* = \frac{\frac{dy(x)}{R}}{\frac{dx}{R}} = y'(x) \tag{6}$$

$$y''^* = \frac{dy'(x)}{\frac{dx}{R}} = \frac{y"(x)}{R} \tag{7}$$

$$p_c^* = \frac{p_c R}{\gamma} \tag{8}$$

$$F_{cap}^* = \frac{F_{cap}}{\gamma R} \tag{9}$$

$$V^* = \frac{V}{R^3} \tag{10}$$

$$A^* = \frac{A}{R^2} \tag{11}$$

F_{cap} refers to the capillary force, V to the volume of the meniscus and A to interfaces. The normalized Laplace equation becomes then :

$$\frac{y''^*}{(1+y'^2)^{\frac{3}{2}}} - \frac{1}{y^*\sqrt{1+y'^2}} = p_c^* \tag{12}$$

Based on the work of [8] , Laplace equation can be integrated into a first order differential equation:

Assuming $Q = 1 + y'^2$ the equation becomes:

$$\frac{dQ}{dy^*} - \frac{2Q}{y^*} = 2p_c^* Q^{\frac{3}{2}} \tag{13}$$

This is a Bernoulli type equation that gives the following solution:

$$Q = \frac{4y^{*2}}{(2K - p_c^* y^{*2})^2} \tag{14}$$

In perfect wetting condition, K is determined from the contact boundaries at the triple line connecting the bridge to the grains, or at the gorge ([6], [8]) for given spheres sizes, suction and distance separating the grains.

The calculations show that K is proportional to the capillary force.

$$K = y^* + \frac{y^{*2}p_c^*}{2} = F_{cap}^* * 2\pi = F_{cap} * 2\pi\gamma R \tag{15}$$

The final form of the integrated equation is then:

$$\frac{1}{2\pi}\left(\frac{y^*}{2\sqrt{1+y'^{*2}}} - p_c^* y^{*2}\right) = F_{cap}^* \tag{16}$$

For the same spheres sizes, suction and distance configuration, we can also calculate the volume of the liquid bridge ([13])and the values of the interfaces separating the different phases in the system.

$$V^* = \int \pi y^{*3} dx^* - V_1^* - V_2^* \tag{17}$$

V_1^* and V_2^* are the volumes of the spherical caps covered by the filling angles δ_1 and δ_2

The wetting-non wetting interface is:

$$A^*_{wn} = \int 2\pi y^{*2}\sqrt{1+y^{*'2}}dx^* \tag{18}$$

The other interfaces are calculated using the equation to calculate the interface of spherical cap using the values of the filling angles (δ_1 and δ_2).

The solid-wetting interface is:

$$A^*_{sw} = \frac{1}{R^2}(2\pi r_1^2(1-cos(\delta_1)) + 2\pi * r_2^2(1-cos(\delta_2))) \tag{19}$$

The solid-non wetting interface is:

$$A^*_{sn} = \frac{1}{R^2}(2\pi r_1^2(1+cos(\delta_1)) + 2\pi * r_2^2(1+cos(\delta_2))) \tag{20}$$

As for the rupture distance of the meniscus connecting the grains, it is determined automatically when there is no possible physical solution for Laplace equation.

3 Lookup table.

For different imposed R, p_c^* and D^* values (D refers to the distance between the particles), all these variables are calculated and saved in text data files, to be used in the simulations. The values of the imposed input values cover all the possible range of solution. R varies from 0.1 to 1 to allow simulating different particle size distribution in the packing, D^* cover the range of distance from the formation to the rupture of the bridge, and the suction varies from 0 to very high suction values. The look up table methods that consists in creating the data file that replaces the runtime computations makes the calculations much faster.

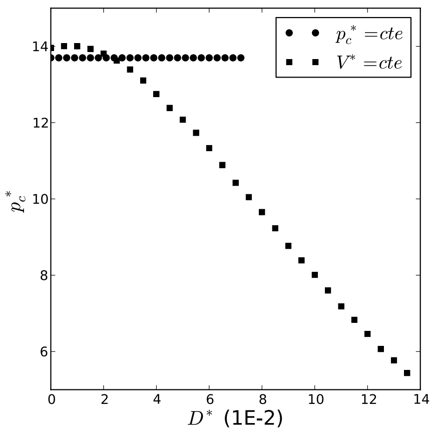
During the simulation at imposed suction, the grain size particles, their positions are known. The trio (R,p_c^*,D^*)can be deduced. The discrete set of solutions in the text files is triangulated using Delaunay triangulation, and the capillary force applied to the particles and the other variables are obtained by interpolation. This interpolation is called the P-Based interpolation and is used to simulate drained quasi-static simulations.

The values obtained from the interpolation in the text files are normalized, and need to be re-upscaled to the real particles size distribution present in the sample.

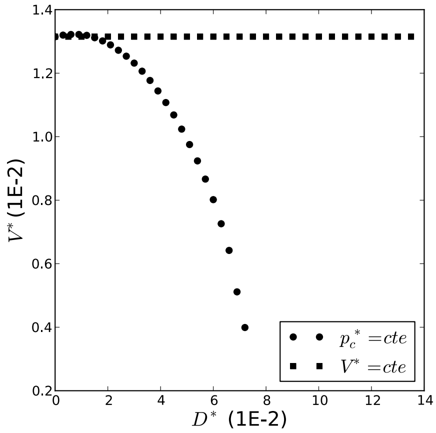
Another type of simulations is also possible in the capillary law code. The V-Based interpolation is then introduced. The input variables for the V-Based interpolation are (R,V^*,D^*) and the same procedure is repeated with different triangulation set.

Simulations are done on two grains connected by a water bridge and the variation of the force and the geometric properties of the meniscus are plotted as function of the distance separating the grains in the case of P-Based and V-Based interpolations in figure 2.

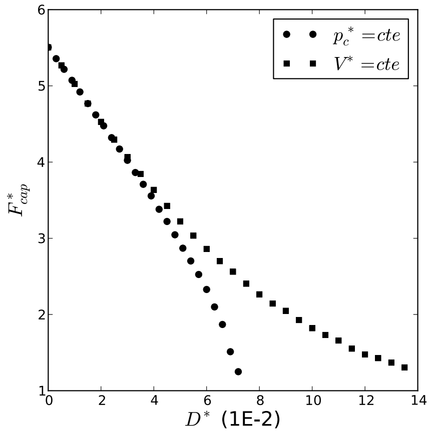
It is noticed that for the capillary force and interfaces, the results are the same for grains moving at constant suction or constant volume of water up to a critical value (D*=0.03) before they start diverging.



(a)



(b)



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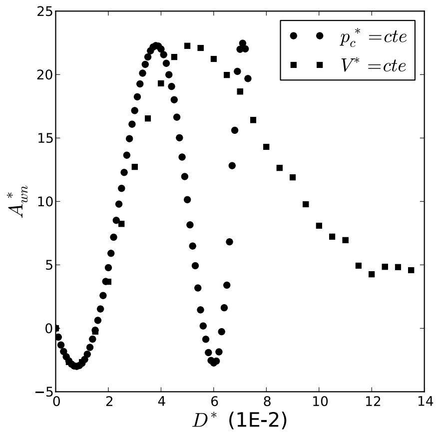

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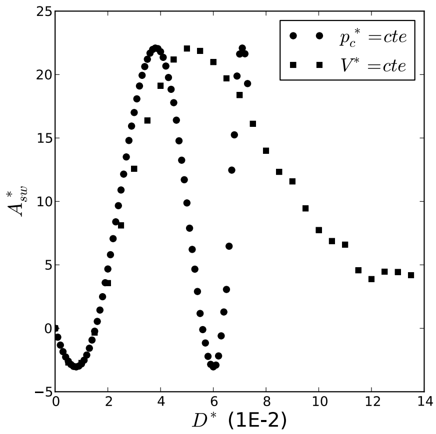

(e)

Figure 2: The variation as function of the distance of (a)-suction , (b)-volume of the bridge , (c)-capillary force, (d)-wetting-non wetting interface and (e)-solid-wetting interface for constant suction and volume.

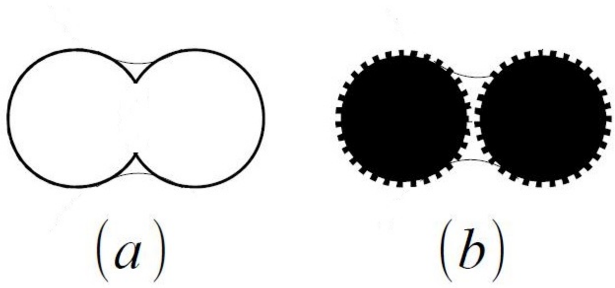

Figure 3: Illustration of 2 grains connected by a meniscus with (a) negative roughness and (b) positive roughness.

4 Roughness of the grains

Another aspect added to the capillary law in YADE is the roughness of the grains. The roughness aspect helps representing the real shape of the grains and the dependence of the capillary effect on that. The model is based on the work of [9]. The roughness is represented by considering that the meniscus is formed between spherical grains slightly smaller or bigger than the grains for the contact law. The roughness can be then either positive or negative. In the case of positive roughness, the grains are touching through asperities (figure 3(b)). When the roughness is negative, the sphere has some plane surfaces, and it's volume is smaller than the sphere used for the capillary law law(figure 3(a)). The model is not able to describe the behavior of quite angular grain shape.

The value κ of the roughness is taken into account for each grain as following:

$$r = r - r * \kappa \tag{21}$$

The way the roughness is taken into account in the DEM code is represented in figure 4.

In the case of packing, a mean value of the roughness κ_m and the value of the dispersion from the mean value in such a way to have a uniform distribution of the values of roughness are defined.

The impact of the roughness on the capillary force is shown in figure 5. It is noticed that for smooth grains in contact, the force is almost constant with the change of suction. For positive roughness, the capillary force decreases with increasing suctions. The negative roughness instead increases the value of the capillary force with increasing suction.

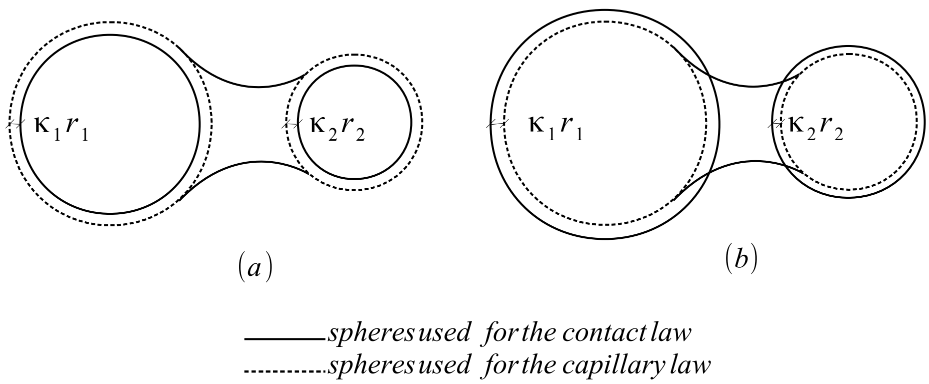

Figure 4: Illustration of a liquid bridge taking into account the roughness in the DEM capillary law for (a) negative roughness and (b) positive roughness.

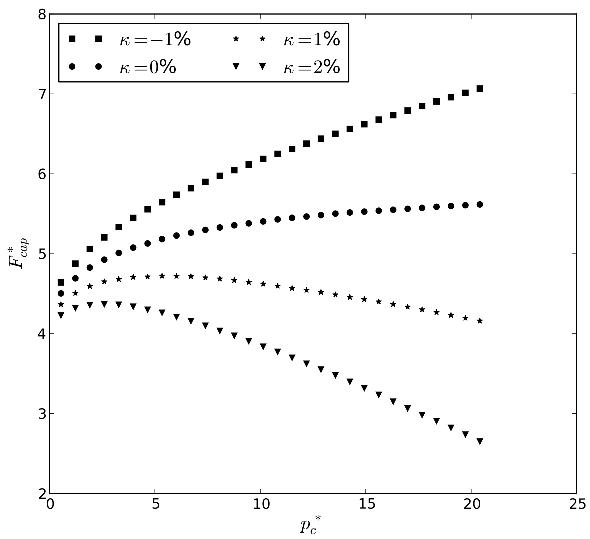

Figure 5: The influence of the roughness on the capillary force.

5 Conclusion

The numerical model presented hereabove offers a consistent framework for studying unsaturated materials. It provides accurate values of interfacial areas, a cornerstone of thermodynamics and variational methods applied to such materials. This method was used to produce results which serve as a base in the companion oral presentation. Therein, the validity of Love-Weber stress tensor $\sigma_c = \frac{1}{V} \sum f \bigotimes l$ for defining the effective stress in three phase materials ([12]) is discussed.

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