IV International Conference on Particle-based Methods – Fundamentals and Applications
PARTICLES 2015
E. Oñate, M. Bischoff, D.R.J. Owen, P. Wriggers & T. Zohdi (Eds)

REPRESENTATION OF BULK AND SURFACE CRUSHING PHENOMENON IN DEM MODEL OF RAILWAY BALLAST

R. DUBINA AND J. ELIÁŠ

Institute of Structural Mechanics, Faculty of Civil Engineering Brno University of Technology
Veveří 331/95, Brno, 602 00, Czech Republic
dubina.r@fce.vutbr.cz, http://www.fce.vutbr.cz/STM/dubina.r
elias.j@fce.vutbr.cz, http://www.fce.vutbr.cz/STM/elias.j

Key words: Railway ballast, Discrete Element Method (DEM), bulk crushing, oedometric test

Abstract. To simulate ballast behaviour in railway track using Discrete Element Models (DEM), it is reasonable to take into account the crushing and the abrasion of grains. Nowadays, two approaches have been used in discrete modelling. The first approach represents each grain as an assembly of smaller particles with cohesive contacts between them. The crushing is represented by rupturing of contacts between particles. Such approach is universal, but demands very high computational resources. Therefore, the second approach is utilized here. At the beginning every ballast grain is modelled as one rigid body and may be replaced by several smaller particles later. Both crushing and abrasion is considered. Crushing is studied on model of oedometric test of railway ballast.

1 INTRODUCTION

Both crushing and abrasion can affect behaviour of the railway track. Therefore it is reasonable to include them in the discrete model. The grains can be crushed during the tamping of the ballast or during the cyclic loading. The Discrete Element Method (DEM) [5] has been recognized as an effective tool for studying the micromechanical behaviour of granular materials. Recently, DEM has been used to model particle fracture and crushable materials. At first the 2D models, that study the fragmentation process under compression were developed [1, 13] and compared with experiments. Nowadays, two approaches have been used in 3D discrete modelling. The first approach represents each grain as an assembly of smaller units with cohesive contacts between them [3, 7, 14]. The crushing proceeds by rupturing contacts between the particles. This approach is universal, but it demands extreme computational resources from the beginning of the simulation, because the time step is dictated by the minimum grain size. Therefore it could be better to use another approach [4, 6, 10], which is subject of our study. At the beginning every ballast grain is modelled as one rigid body, due to crushing, it might be replaced by smaller units. In work [11] it was found that the surface fracture (abrasion) rather than bulk fracture (crushing) may dominate, therefore it is desirable to consider the abrasion too. Therefore, both crushing and abrasion models are considered. Only spherical body shapes are consider to keep the model computationally efficient.

2 BULK CRUSHING

The condition that indicate crushing is based on some equivalent stress, σ_e, in the grain, which is compared to the size depending material strength, f_t. The crushing occurs when the equivalent stress exceeds material strength.

$$\sigma_\mathrm{e} > f_\mathrm{t} \tag{1}$$

The equivalent stress is computed as von Mises stress (2)

$$\sigma_\mathrm{e} = \sqrt{\frac{(\sigma_I - \sigma_{II})^2 + (\sigma_{II} - \sigma_{III})^2 + (\sigma_I - \sigma_{III})^2}{2}} \tag{2}$$

where σ_I, σ_{II}, σ_{III} are the principal stresses, which are calculated from fabric stress tensor [2]. The fabric stress tensor is expressed as

$$\sigma_{ij} = \frac{1}{V} \sum_c l_i^{(c)} F_j^{(c)} \tag{3}$$

where V is the volume of the particle, the vector $l_i^{(c)}$ connects the centroid of the grains with the c-th contact point and $F_j^{(c)}$ is the vector of contact forces. The material strength f_t is expressed as

$$f_\mathrm{t} = \frac{f_\mathrm{t0}}{r_\mathrm{eq}} \tag{4}$$

where f_t0 is the reference material strength and r_eq is the radius of a spherical particle. If the condition (1) is satisfied, then the grain is replaced by several smaller bodies, pieces. These pieces must have the spherical shape as the original grain, therefore their assembly cannot form the exact shape as the original grain. Also no overlapping of them and no protrusion from the original grain shape is allowed. Therefore, there is significant loss of volume. Three variants of the replacement are implemented and studied (Fig. 1). These three variants seem to be sufficient for our purpose. Considering even finer debris would lead to undesirable slowdown of the simulation due to decrease of the critical time step Δt_cr (5).

$$\Delta \mathrm{t} < \Delta t_\mathrm{cr} = \frac{1}{w_\mathrm{n}} \Rightarrow r\sqrt{\frac{\rho}{E}} \tag{5}$$

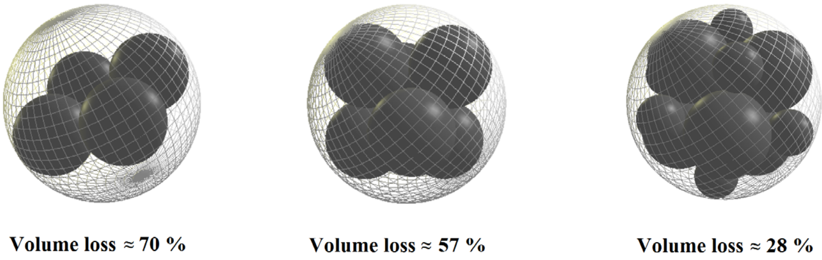

Figure 1: Three variants of replacement of the original spherical particle; the first option is replacement by four identical spheres (about 70 % of the volume is lost), the second option by six identical spheres (about 57 % of the volume is lost), the third option by fourteen spheres (more than 28 % of the volume is lost).

3 SURFACE CRUSHING

The surface crushing (abrasion) is another possibility of the railway ballast degradation. The abrasion in the model occurs when the surface pressure, p_0, here presented by maximum Hertzian pressure [8], exceeds the material strength f_{t0}.

$$p_0 > f_{\mathrm{t0}} \tag{6}$$

The maximum Hertzian pressure is given by

$$p_0 = \frac{3F_{\mathrm{N}}}{2\pi a^2} \tag{7}$$

where F_{N} is normal force, $a = (Ru_{\mathrm{N}})^{\frac{1}{2}}$, with R being effective radius of spheres in the contact and u_{N} is penetration depth of spheres. For two spheres the effective radius is $R = \frac{R_1 \cdot R_2}{R_1 + R_2}$, where R_1, R_2 are the radii of spheres. The abraded grain has to be spherical too, because the model is limited to spherical shape only. The abrasion is represented by reduction of grain radius, see Fig. 2, $r_{\mathrm{new}} = \gamma \cdot r_{\mathrm{old}}$, where parameter γ is the reduction factor. For convenience, only sufficiently large grains are subjected to bulk and surface crushing. Whenever grain radius decreases under chosen minimal value $r_{\min}$, it cannot crush anymore.

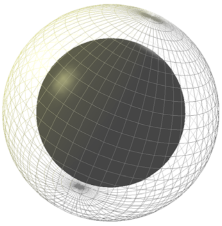

Figure 2: The surface crushing (abrasion) is managed by the reduction of radius of the spherical particle.

4 SIMULATION OF OEDOMETRIC TEST

The procedures of grain degradation described above were tested on the oedometric test which was made at the University of Nottingham by Lim [9].

The dimension of the oedometric test are 300 mm in diameter and the 150 mm in height. The boundary condition are denoted in the Fig. 3 (markers $\perp$, $\odot$ or dots in floor projection).

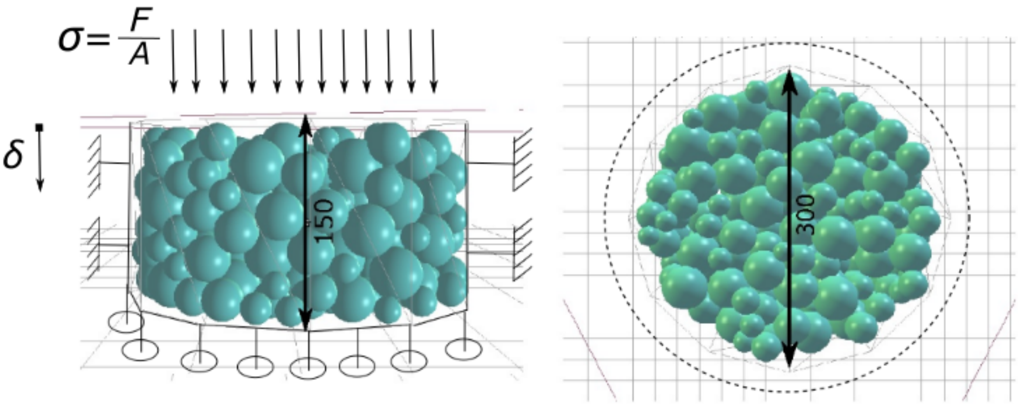

Figure 3: Scheme of the oedometric test; The diameter is 300 mm and height is 150 mm. The mean diameter of spherical grains is 32.5 mm, the standard deviation is 7 mm.

The simulation proceeds as follows. The oedometer is filled by grains, which diameter is randomly sampled from uniform distribution between 20.5 and 44.5 mm. Than the sample is vibrated by horizontal acceleration to provide sufficient number of contact between grains. The subsequent compressive cycle loaded the sample by force up to 1500 kN and then unload, the loading is defined by sinus function

$$F(t) = \frac{1500}{2}\left[\sin\left(2\pi f t - \frac{\pi}{2}\right) + 1\right] \tag{8}$$

where f is loading frequency and t is the loading time. The material properties were set according to the literature. The oedometer material was made from steel, its density was set to 7850 kg/m^3, the Poisson number to 0.25 and the friction angle to 0.0 rad [12]. The Young's modulus of the steel was considered 10 times than the Young's modulus of ballast. The ballast density was 2600 kg/m^3, the Poisson number 0.3 and the friction angle 0.8 rad [10]. The Young's modulus of ballast is set to $3 \cdot 10^{11}$ Pa based on our previous research and according to Lim [9].

5 RESULTS OF THE SIMUALTION

The results are presented using loading force F and movement of the loading plate δ. The effect of several input parameters were investigated for both modes of grain degradation. Namely type of the replacement of the origin sphere, the minimal radius r_{min}, reference value of material strength f_{t0} and the reduction factor γ in surface crushing (abrasion). All these investigated parameters have the expected influence on the F - δ diagram. The most significant influence had the volume loss (type of replacement).

Two studies are selected and presented here in detail. The first parameter is the loading frequency f. Six different values of loading frequency are used 0.25, 0.5, 1.0, 2.0, 4.0 and 8.0 Hz. The results are presented in Fig. 4. Five realizations of oedometric test for each loading frequency are calculated. These realization differ in initial configuration of spheres inside the oedometer. Particle crushing is in the graph represented by horizontal jumps. The majority of simulations has the maximum displacement about 60 mm and similar shape of the loading curve. There are however also curves which have substantially smaller displacements (dashed black and solid red), where it almost does not occur particle crushing due better force redistribution. From the study we conclude that loading frequency (in range 0.25 - 8.0 Hz) has almost no effect on results.

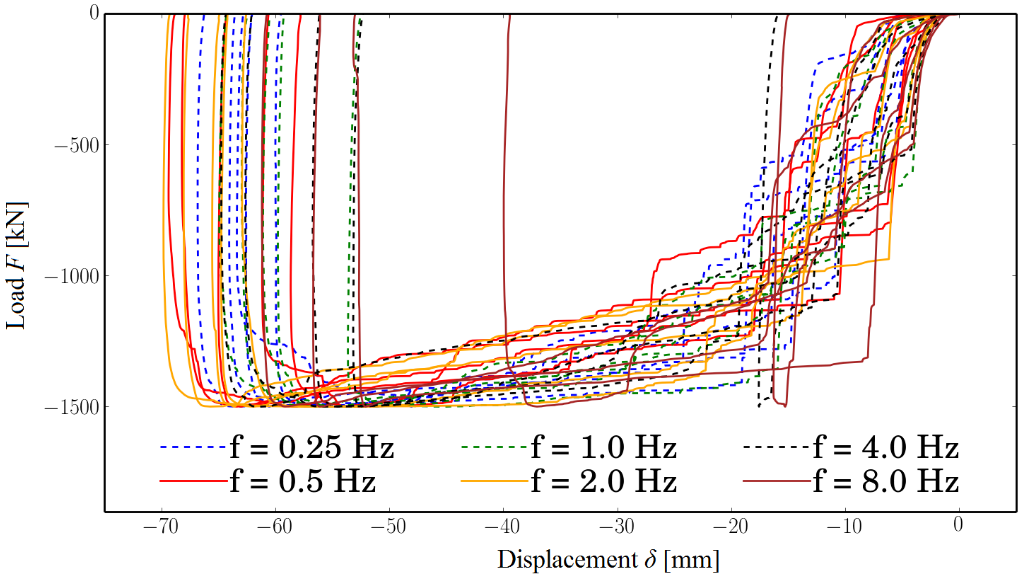

Figure 4: The Load - Displacement diagram of oedometric test simulation; Six values of loading frequency are investigated, for each frequency five realizations are computed.

The other investigated parameter is so-called crushing period, T_{c}. The crushing period represents a time period, after which the crushing condition is checked. The purpose of crushing period is to save a computational time comparing to simulation when crushing condition is checked every time step. Sixty realization of oedometric test were computed for different crushing periods. The displacements at the peak force are shown in Fig. 5. In the most left part of Fig. 5 are results, when the control of crushing is checked in every time step. In the right part of the graph the crushing condition is checked every 0.04 s.

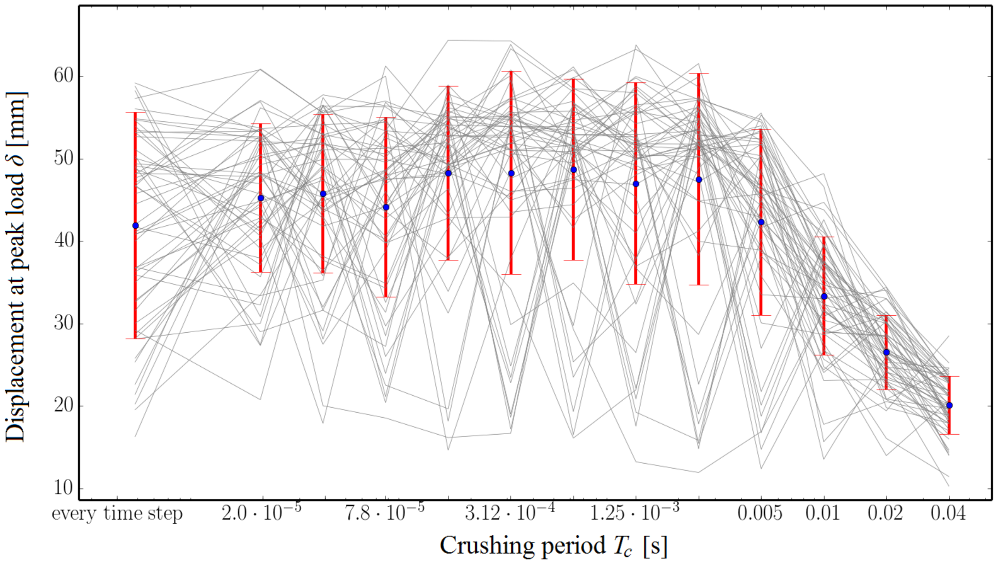

Figure 5: The dependence of displacement at peak load, δ, on crushing period, T_c ; Sixty realization for each crushing period are shown by solid grey lines; Errorbars show mean value and standard deviation.

We expect that there would exist a certain crushing period, from which the mean values of displacement (blue points in Fig. 5) will be constant. The first five largest crushing periods have an expected trend and by increasing crushing period the number of crushed particles grows and the displacement grows. As well in range $T_\mathrm{c} = 2.5 \cdot 10^{-3}$ to $3.12 \cdot 10^{-4}$s the mean displacement is almost constant. This was expected to remain for even smaller crushing periods. However, in range for smaller crushing periods decrease of mean value of displacement occurs. This decreasing is undesirable and the reason of this behaviour is the object of future research.

6 CONCLUSIONS

The developed algorithm of particle crushing suffers by large volume loss. Therefore its application is more convenient for problems where the crushing has local character. In studied range of loading frequencies, the crushing model is independent on the loading frequency. The observed dependence on crushing period will be subject of further research.

ACKOWLEDGEMENT

This outcome has been achieved with the financial support of Brno University of Technology under project No. FAST-J-15-2849. The support is gratefully acknowledged.

REFERENCES

[1] Astrm, J. A. and Herrmann, H. J. Fragmentation of grains in a two-dimensional packing. *Eur. Phys. J. B.* (1998) **5**:551–554.

[2] Bagi, K. Stress and strain in granular assemblies. *Mechanics of Materials.* (1996) **22**:165–177.

[3] Cheng, Y. P., Nakata, Y. and Bolton, M. D. Discrete element simulation of crushable soil. *Géotechnique.* (2003) **53**:633–641.

[4] Cleary, P.W. DEM prediction of industrial and geophysical particle flows. *Particuology.* (2010) **8**:106–118.

[5] Cundall, P.A. and Strack, O.D.L. A discrete numerical model for granular assemblies. *Géotechnique.* (1979) **29**:47–65

[6] Eliáš, J. Simulation of railway ballast using crushable polyhedral particles. *Powder Technology.* (2014) **264**:458–465.

[7] Ergenzinger, Ch., Seifried, R. and Eberhard. A discrete element model predicting the strength of ballast stones. *Computers and Structures.* (2012) **3-13**:108–109.

[8] Hertz, H. *Ueber die berhrung fester elastischer krper (on the contact of rigid elastic solids.* J. reine und angewandte Mathematik, **92**:156–171, (1882), (translated and reprinted in English in Hertz's Miscellaneous Papers (Macmillan & Co., London), (1896).

[9] Lim, W. L. and McDowell G. R. Discrete element modelling of railway ballast. *Granular Matter.* (2005) **7**:19–29.

[10] Lobo-Guerrero, S. and Vallejo, L. E. Discrete element method analysis of railtrack ballast degradation under cycling loading. *Granular Matter.* (2006) **8**:195–204.

[11] McDowell, G. R. and Amon, A. The application of Weibull statistics to the fracture of soil particles. *Soils and foundation.* (2000) **40**:133–141.

[12] McDowell, G. R. and De Bono, J. P. On the micro mechanics of one-dimensional normal compression. *Géotechnique.* (2013) **63**:895–908.

[13] Thakur, P. K. Vinod, J. S. and Indraratna, B. Effect of particle breakage on cyclic densification of ballast: a DEM approach. *Materials Science and Engineering.* (2010) **10**:1–7.

[14] Wang, J. F. and Yan, H. B. 3D DEM Simulation of Crushable Granular Soils under Plane Strain Compression Condition. *Procedia Engineering.* (2011) **14**:1713–1720