AN IMPORTANT ROLE OF ELASTIC VORTICES IN UNSTEADY PROPAGATION OF LONGITUDINAL SHEAR CRACKS IN BRITTLE MATERIALS

EVGENY V. SHILKO[1,2], SERGEY G. PSAKHIE[1,2] AND VALENTIN L. POPOV[1,3]

[1]Institute of Strength Physics and Materials Science SB RAS (ISPMS SB RAS)
2/4, pr. Akademicheskii, 634021 Tomsk, Russia
shilko@ispms.tsc.ru, http://www.ispms.ru

[2]Tomsk State University
36 Lenin prospect, 634050 Tomsk, Russia
shilko@ispms.tsc.ru, http://ff.tsu.ru

[3]Berlin University of Technology (TU Berlin)
Sekr. C 8-4, Str. Des 17. Juni 135, D-10623 Berlin, Germany
v.popov@tu-berlin.de, http://www.tu-berlin.de/reibungsphysik

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Abstract. The paper is devoted to the numerical study of some fundamental aspects of longitudinal shear crack propagation in sub-Rayleigh and supershear regimes. The simulation was carried out with use of particle-based movable cellular automaton method. It is shown that a well-known phenomenon of shear crack acceleration towards longitudinal wave speed is explained by the formation and development of elastic vortex traveling ahead of the crack tip at a shear wave velocity. The stress concentration area ahead of the crack tip revealed by different authors is connected with the elastic vortex. Shear crack accelerates towards the longitudinal wave speed through the formation of a daughter crack by the mechanism of shearing (the daughter crack is formed in the center of elastic vortex). Analysis of sub-Raleigh-to-intersonic transition showed that the condition of this transition can be expressed in terms of dimensionless geometrical crack parameter.

1 INTRODUCTION

Dynamic crack propagation has been studied both experimentally and numerically for many years. Nevertheless some fundamental questions concerning the material deformation in the vicinity of the crack tip are still not fully understood. In particular, why the near-circular or elliptical area with high stress concentration is formed near the tip of propagating longitudinal shear crack? This question is of fundamental importance because under mode II loading (longitudinal shear) the effect of stress concentration ahead of the crack tip is most pronounced and determines the main features of dynamic crack development [1-3]. Continuum mechanics based analytical and numerical solutions explain such stress concentration by elastic strain energy transport/influx from unloaded material behind the crack tip. However a satisfactory physical explanation of processes and the mechanism leading to the formation of relatively small (localized) elliptical region with high stress concentration near the tip of dynamically growing mode II crack has not yet been given.

Formation of a maximum of shear stresses ahead of mode II crack growing in unstable regime was first shown analytically by Burridge [1]. In the following decades the evolution of such stress peak was studied in detail by numerical modeling and laboratory experiments [2,4-7]. In particular, it was shown that this stress peak forms at the initial stage of unstable crack growth and then propagates ahead of the rupture front at a shear wave velocity [2]. Moreover, during the course of dynamic crack propagation an area of concentration of shear stresses ahead of the crack tip is enlarged, and the magnitude of the stress peak increases. Under certain conditions stress peak magnitude can reach the shear strength of the material. In this case small fracture ahead the shear crack tip is nucleated (it was called by Abraham and Gao [8] a daughter crack). This daughter crack is capable to propagate faster than the shear wave speed (in intersonic regime). An effect of stress peak induced acceleration of mode II cracks towards the longitudinal wave speed is quite general as it was observed numerically or instrumentally at various spatial scales from the atomic one [8] to the scale of tectonic faults [9,10]. It was found that important features of shear crack growth in supershear regime are many times higher values of velocity and displacement amplitudes, as well as duration of the oscillations, as compared to the conventional sub-Rayleigh regime.

Therefore, understanding of physical causes of formation of elliptical region of stress concentration ahead of longitudinal shear crack and the conditions providing the ability of the shear crack to develop faster than the speed of transverse elastic wave are the keys to identify general features of dynamic fracture of solids at different spatial scales under shear dominated loading. Presented study is devoted to the numerical analysis of this problem.

2 PROBLEM STATEMENT

Analysis of the results of theoretical and experimental studies of unstable development of mode II cracks gave the reason to believe that the formation and development of a localized region of maximum shear stress ahead of the crack tip has to be a consequence of the collective vortex-like elastic displacement of material particles near the tip. Note that circular (vortex-like) motion profile of material particles in a certain plane is conventionally associated with surface waves (Raleigh, Love or Lamb waves) and waves propagating along the interfaces between different phases (Stoneley wave). The trajectories of material particle motion in such waves have pronounced vortex nature. We can assume that in the case of propagating longitudinal shear crack the opposite direction of motion of the material layers separated by a crack will provide conditions for the formation of elastic circular motion of material points near the crack tip in much the same way as dynamic vortex structures form in liquids or gases at flow past an obstacle.

One of the most important characteristics of elastic waves is the concentration of elastic energy concerned with volume strain (in longitudinal waves) or shear strain (in shear waves). In the case of collective vortex-like elastic motions (in particular, in surface waves), which are often considered as the result of interference of a large number of differently oriented shear waves, the concentration of the elastic strain energy must be related, primarily, with elastic strain energy of distortion (in other words, with shear stresses). Therefore it is logically to assume that the area of concentration of shear stresses in elastic vortex should have a circular or elliptical shape.

To study the question of whether collective vortex-like elastic motions of material points could be the real physical mechanism of formation of elliptical stress concentration region ahead of the mode II crack tip (formation of such a region was shown by different authors [10,11]), we have performed computer-aided simulations.

The simulation was carried out with use of particle-based movable cellular automaton (MCA) method. The MCA method belongs to the group of *distinct element methods* within a wide class of discrete element methods (DEM). The DEM treats the solid as an ensemble of linked (chemically bonded) or contacting interacting particles and hence is an efficient tool to simulate fracture phenomena (including multiple fracture) accompanied by contact interaction of surfaces [12,13]. These advantages have determined the choice of this numerical technique for analysis of shear crack propagation. The main advantage of the MCA method as being compared to other representatives of the group of distinct element methods is multi-body formulation of element-element interaction forces [14,15]. Such a formulation makes possible to interconnect average strains and stresses in the volume of distinct element with forces of element interaction with neighbours (such approximation to description of element deformation is called approximation of *simply deformable element*). This allows implementation of various constitutive laws of solids (each automaton behaves following applied constitutive law and hence the mechanical response of the whole ensemble of elements corresponds to applied constitutive equations) and overcoming some critical limitations of the distinct element method including packing related artificial anisotropy and fundamental problems in correct simulation of irreversible strain accumulation in ductile materials.

To treat the considered problem the model of many-body interaction, which provide macroscopically isotropic linear-elastic response of particle ensemble, was applied [14,15]. Equivalent stress criterion was used as a criterion of breaking the bond between linked particles (i.e. as a fracture criterion) [14].

The numerical study was carried out with use of the two-dimensional model slab (Fig. 1). It consists of two bonded parts. Parts have the same properties and are isotropic, linear-elastic and high-strength. The following mechanical properties of material of the plates were used in the study: Young modulus E=200 GPa, Poisson's ratio ν=0.3, density ρ=5700 kg/m^3. Ideal bonding between the parts was assumed. This assumption means that thickness of the interface zone is much smaller than automaton (element) size. In the framework of such assumption the only interface property taken into account in the model is interface strength. In the study we assumed much smaller interface strength (critical value of equivalent stress σ_{eq}^{is} was assigned to be 250 MPa) in comparison to strength of plates themselves (σ_{eq}^{ps}=2000 MPa). A short initial section (preliminary crack) at the interface was cut by means of breaking bonds between corresponding linked elements belonging to different parts of the slab. Note that such kind of models with "weak" (low-strength) interface is typically used in studies of shear crack development because shear cracks always tend to kink into the plate or branch out [3].

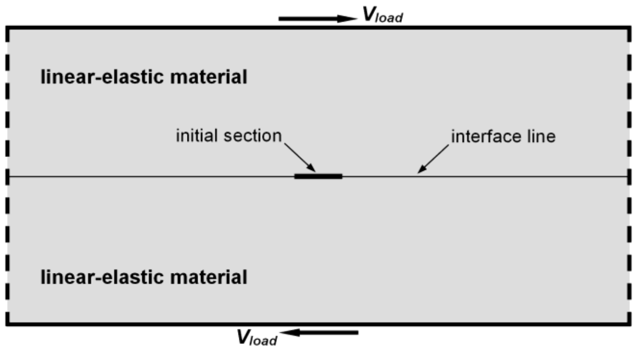

Figure 1: Schematic representation of the geometry of two-dimensional plane-strain model and the loading conditions. Horizontal solid bold lines delineate the upper and the lower external boundaries. Vertical dashed bold lines mark the side boundaries subjected to periodic boundary conditions in horizontal direction. Longitudinal shear loading is modelled by displacement of the upper and the lower external boundaries in horizontal direction at a constant velocity V_{load}. Vertical positions of these boundaries are fixed.

To simulate shear loading, the upper and the lower layers of elements defining the opposite horizontal faces of the slab were displaced in opposite directions along the interface line with very low constant velocity V_{load} (Fig. 1). This loading scheme models the condition of simple shear of the system in quasi-static regime. Periodic boundary conditions in horizontal direction (along the interface line) were applied to opposite vertical (side) faces to avoid the influence of contortions at these faces of the slab. Plane strain state approximation was used as a boundary condition in the normal direction to the considered plane. The applied loading and boundary conditions remained unchanged during the simulation.

Under such a loading the course of deformation of the slab consists of two stages. In the first stage the slab is elastically deformed and accumulates elastic energy. Maximum shear stresses in the specimen are concentrated at the crack tip. Upon reaching the threshold value of shear stress τ_0 (shear strength τ_0, which depends on the length of the initial crack and amounts a portion of the shear strength of intact interface) the second stage related to the dynamic crack propagation at the interface begins.

3 SIMULATION RESULTS AND DISCUSSION

3.1 Formation and development of elastic vortices

Fig. 2 shows several consecutive snapshots of distribution of element velocities in the area surrounding the right tip of dynamically propagating shear crack. It is clearly seen that at the beginning of unstable crack growth the collective vortex-like elastic motion of material particles (hereinafter referred to as elastic vortex) is formed ahead of the tip of growing crack. As the crack develops the elastic vortex occupies a larger area in front of the crack. The development of an elastic vortex is inseparably linked with development of a crack. During the short initial period of crack propagation the crack velocity rapidly increases to the value comparable with Raleigh wave speed (this period depends on material and geometrical parameters of the sample and crack and takes several microseconds in the example shown in Fig. 2) and then changes slightly [3,4,8]. During this period the elastic vortex is formed as a self-dependent dynamic structure.

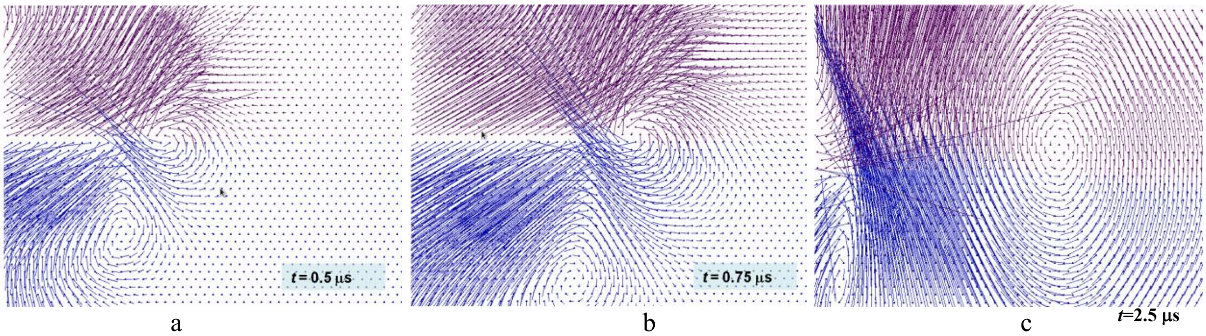

Figure 2: Fields of velocities of distinct elements near the right tip of growing shear crack 0.5 μs (a), 0.75 μs (b) and 2.5 μs (c) after propagation beginning. Crack is propagating from the left to the right. In the shown example initial crack length was 0.6 mm, the height of the slab was 14 mm, the size of the distinct element was 0.1 mm. Transition to steady-state regime of vortex and crack propagation takes just about several microseconds.

Formation of such a vortex at the beginning of dynamic crack propagation is not surprising. Previous study by the authors have shown that dynamic loading or dynamic change of specimen stress state can lead to formation of elastic vortices near grain or interphase boundaries as well as near free surfaces [16]. In the considered case the formation of a vortex is also concerned with a presence of free surfaces. These are surfaces of the initial crack which provide conditions for bending around the crack tip.

Formation and propagation of elastic vortices is an important mechanism of dynamic redistribution of elastic strain energy in the material. In the considered case of unstable propagation of a shear crack the elastic strain energy comes ahead of the crack tip from unloading material behind the tip. Crack tip advancement is accompanied by divergence of surfaces behind the tip (crack opening). Such opening leads to increase in crack normal (tensile) component of crack tip strain. Fracture is accompanied by dynamic liberation of elastic strain energy accumulated in material of slab parts behind the crack tip (elastic energy transforms to kinetic energy). Due to the opening of growing mode II crack the dominating part of this kinetic energy in a short segment behind the crack tip is concerned with transversal or crack normal component of particle velocities. This can be illustrated by Fig. 3 showing typical distributions of crack normal and crack parallel components of kinetic energy density along the interface at different time moments after crack propagation beginning. Distributions are built for the thin horizontal layer of the upper part of the slab adjoining the interface. It is seen that around the crack tip the crack normal component of kinetic energy is order of magnitude larger than crack parallel. Maximum values of crack normal velocities are strongly localized within the thin vertical layer containing the crack tip. Horizontal gradient of crack normal (vertical) component of velocity vectors reaches here the maximum values as well. This indicates high shear strain rate in the thin vertical layer containing the crack tip and leads to intensive transfer of elastic energy ahead of the crack tip by the transverse (shear) elastic wave. Note that the magnitude of crack normal component of velocity vector gradually decreases with distance from the crack tip in vertical (crack normal) direction. This causes the vertical gradient of the strain rates. Strong horizontal and vertical gradients of velocities (and elastic displacements) in the slab parts at and behind the crack tip leads to the formation of collective vortex-like elastic motion ahead of the crack tip.

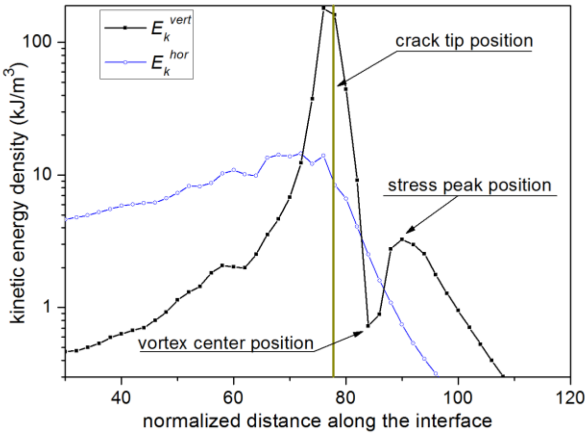

Figure 3: Typical distributions of crack normal (E_k^{vert}) and crack parallel (E_k^{hor}) components of kinetic energy density along the interface. An example shown in Fig. 2 is presented. The plot corresponds to the time moment shown in Fig. 2c. The abscissa is the distance to the initial position of the crack tip, divided by the distinct element size. Crack is propagating from the left to the right. Bold vertical line in the center of plot indicates instantaneous position of the crack tip.

Localized collective circular displacements of material points in elastic vortex indicate the concentration of shear stresses. Fig.4 shows typical equivalent stress distributions near the right tip of growing mode II crack at different stages of vortex development. Strongly pronounced elliptical region of high shear stresses (compared to background value far from the crack) accompanies the vortex-like collective behavior of material ahead of the crack tip. This elliptical region is situated in the frontal part of the vortex. Horizontal coordinate of the center of this region (maximum of equivalent stress distribution) corresponds to the coordinate of the maximum value of crack normal component of element velocities in the frontal part of the vortex (see Fig. 3). Note that this coordinate corresponds to position of stress peak ahead of the crack tip described by Burridge [1] and Andrews [2]).

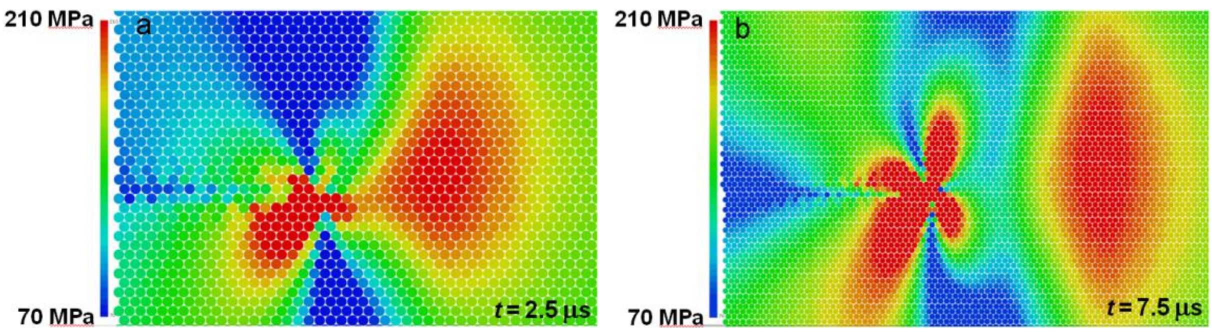

Figure 4: Snapshots of the distributions of equivalent stress near the right tip of dynamically growing shear crack 2.5 μs (a) and 7.5 μs (b) after growth start. An example shown in Fig. 2 is presented. Stress distribution in Fig. 4a corresponds to the time moment shown in Fig. 2c. Pictures demonstrate stress patterns before (a) and after (b) detaching of elastic vortex from the crack.

Since the formation of the elastic vortex is due to the influx of elastic energy, during the course of dynamic crack propagation the vortex increases in size (it occupies larger area ahead of the crack tip), and the concentration of shear stress in the vortex increases as well.

The velocity of propagation of the vortex rapidly approaches the shear wave speed V_S. At the same time the crack advances at a velocity lower than Raleigh wave speed V_R. So, during the course of propagation the vortex gradually moves away from the crack tip and finally detaches from it. Two snapshots in Fig. 4 show equivalent stress distributions before (Fig. 4a) and after (Fig. 4b) vortex detaching from the crack. The elastic strain energy in the vortex increases until the moment of its separation from the crack (i.e. while the vortex has energy supply). After separation the elastic vortex becomes a self-dependent dynamic object, which propagates independently on the source of its origin. During the course of subsequent (independent) vortex propagation the concentration of shear stresses gradually decreases.

Intensive transfer of elastic strain energy of distortion ahead of the tip of advancing mode II crack by the shear elastic waves (i.e. at a shear wave speed) provides the conditions for the formation of a new vortex at the crack tip after separation of the first one. The history of new vortex development and direction of rotation are similar to those for the first vortex. Fig. 5 shows an example of equivalent stress distribution near the right tip of the mode II crack after the second vortex formation. Note that the second vortex repeats the “fate” of the first one. The maximum concentration of equivalent stress in the second vortex is achieved to the moment of separation from the crack tip (this moment is shown in Fig. 5). After that the second vortex moves independently (it follows the first one) and gradually attenuates.

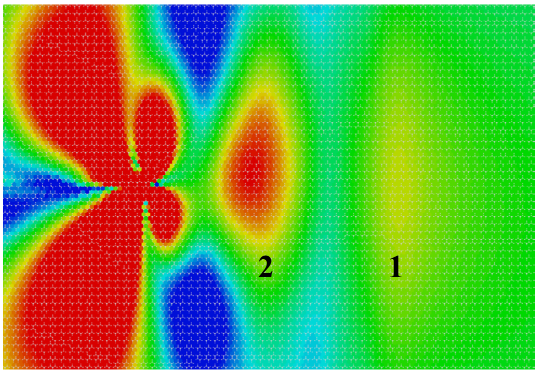

Figure 5: Snapshot of the distribution of equivalent stress near the right tip of growing shear crack after formation of the second vortex. An example shown in Fig. 2 is presented. The first and the second elliptical areas of shear stress concentration concerned with elastic vortices are shown by numbers 1 and 2 correspondingly.

So, mode II crack propagating in conventional sub-Raleigh regime generates a chain of elastic vortices moving ahead of the tip at a shear wave speed V_S. The main feature of these vortices is stress concentration in their frontal parts. In this study the limiting rheological model of material (namely, the elastic-brittle material) was considered. In the framework of this model fracture is not preceded by plastic deformation connected with motion of crystal lattice defects or accumulation of low rank damages. In fracture mechanics models inelastic deformation ahead of the crack tip in real materials is taken into account by means of introduction of a process zone composed of fracture process zone and hardening plasticity zone. Simulation results give grounds to suggest that plastic deformation of material strip (including interface segment) ahead of the tip of unstable growing mode II crack can realize in quasi-periodic manner (by series of elastic vortices).

In conventional analytical models of dynamic propagation of longitudinal shear cracks the steady-state regime of this process is considered as a self-similar [1]. This means that the elastic vortices propagating ahead of the crack tip need to be scale-invariable objects.

This assumption was confirmed by a specially conducted numerical simulations on geometrically similar slabs with initial sections (cracks), obtained by spatial scaling of the sample in Fig. 1 within several orders of magnitude. Scaling was done by means of increasing the size of the distinct elements and corresponding increasing the size of the sample while maintaining the aspect ratio (the initial crack was scaled as well as the slab). Spatial scaling is characterized by dimensionless coefficient which is the ratio of size of scaled distinct element to the original element size. Simulation results showed that the extensive parameters of elastic vortices (including the time from vortex nucleation to separation from the crack, the geometric characteristics of vortex at different stages of development, stress and strain gradients) in scaled samples are scaled along with spatial scaling, while the intensive vortex parameters (values of stresses and velocities in corresponding areas) remain the same in all cases. This demonstrates the scale-invariable nature of elastic vortices propagating ahead of the tip of mode II cracks growing in sub-Rayleigh regime

3.2 Conditions of sub-Rayleigh-to-supershear transition

We have also made an extension to this study. Simulation results showed that stress concentration in frontal part of elastic vortex is determined by the density of elastic strain energy of distortion E_0 accumulated in the slab to the moment of crack propagation beginning. Such a relation is clear because liberation and redistribution of elastic strain energy during dynamic crack propagation is realized at the expense of preliminary accumulated energy. The concentration of elastic energy in elastic vortex is characterized, in particular, by the magnitude of equivalent stress peak at the interface ahead of the crack tip. This stress peak is situated at the central point of the imaginary section of elliptical region of high shear stresses (examples of such a region are shown in Fig. 4 and Fig. 5) by interface line.

Note that an interest to stress peak magnitude is determined by its “responsibility” for crack acceleration towards the supershear propagation regime [2]. The maximum magnitude of the equivalent stress peak $\sigma_{eq}^{\max}$ is achieved to the moment of detaching of the elastic vortex from the crack. The study has shown that $\sigma_{eq}^{\max}$ is proportional to the shear strength τ_0 of the slab with initial crack, or, what is the same, to the root of accumulated elastic strain energy E_0 ($\tau_0 = \sqrt{2E_0G}$, where G is shear modulus of material):

$$\sigma_{eq}^{\max} = a + b(\tau_0 - \tau_\infty), \tag{1}$$

where a and b are constants depending on material properties, τ_∞ is an asymptotic shear strength of the interface with semi-infinite crack. Parameters E_0 and τ_0 are directly connected to geometrical characteristics of initial crack (particularly, E_0 is inversely proportional to its length). It is clear that due to scale invariance of crack and vortex propagation the parameters E_0 and τ_0 (and hence the maximum magnitude of the stress peak $\sigma_{eq}^{\max}$) should be the functions not of individual dimensional geometrical characteristics of initial crack but of dimensionless parameter connecting dimensional geometrical characteristics.

In the partial case of simple shape of initial crack (section or rectangular notch) the most important dimensional characteristics of the crack are the length L_0 and thickness D. Note that for section-like cracks the physical meaning of the parameter D is roughness of crack surfaces (when modelling material with a section by the ensemble of distinct elements the regular artificial roughness of the crack surfaces is determined by distinct element size). For rectangular notch-shaped cracks the parameter D has the meaning of the distance between notch surfaces. Special study has been conducted with the purpose to analyze the dependence of τ_0 on D and L_0 for these two types of cracks (in the case of section-like initial cracks the value of roughness parameter D varied by changing the size of distinct elements simulating the slab). Simulation results showed that the set of two-parameter dependences $\tau_0(D, L_0)$ for initial cracks with various effective thicknesses D can be reduced to a unified (general for all the cracks) dependence of τ_0 on the dimensionless geometrical parameter $P=L_0/D$ [17]. The existence of a dimensionless parameter P, which determines the influence of the geometrical characteristics of a crack on τ_0 (and, consequently, on the equivalent stress peak $\sigma_{eq}^{\max}$) confirms the scale invariant nature of dynamic growth of mode II cracks.

Revealed dependence $\sigma_{eq}^{\max}(P)$ is of fundamental importance for understanding of necessary geometrical condition of shear crack acceleration towards the P-wave speed. As was shown by different authors, the crack accelerates if the stress peak at the interface segment ahead of the crack tip reaches interface strength and a daughter crack is nucleated [2,5]. Simulation results showed that the possibility of reaching a critical value of the stress peak is determined by the value of the parameter P. The initial crack is characterized by the magnitude of dimensionless parameter $P>P_{critical}$ (where $P_{critical}$ is a critical/maximum value providing the conditions for reaching the interface strength σ_{eq}^{is} by the stress peak at the interface ahead of dynamically growing crack), it is able to propagate in conventional sub-Rayleigh regime only. Otherwise ($P<P_{critical}$) it has the potential to overcome Rayleigh wave velocity barrier. This particularly means that only crack whose initial length is $L_0<L_{critical}=D\cdot P_{critical}$ is capable of propagating in intersonic regime.

Simulation results showed that for elastic-brittle materials the quantity $P_{critical}$ varies from 1 to 10. For elastic-plastic materials the quantity $P_{critical}$ should be much smaller.

Thus, the effective thickness of the crack D (roughness of the section surfaces, the distance between the surfaces of the notch and so on) determines the maximum value of the initial crack length $L_{critical}$ such that a crack may potentially propagate at a supershear velocity. Such an effect of the parameter D is defined by the features of displacement and strain distributions around the crack at the stage of stable deformation (prior to unstable crack propagation). In particular, displacement field ahead of the stable shear crack is contorted and the crack is open. The contortion and crack opening both decrease with increase in the height of crack surface asperities (for section-like cracks) or gap between surfaces of notch (i.e. as the parameter D increases). This results in a lower concentration of shear stresses at the interface near the crack tip and consequently in an increase in the value of the density of elastic strain energy of distortion E_0 accumulated in the slab to the moment of crack propagation beginning.

The dependence of the critical value of a dimensionless geometric parameter P on the magnitude of the elastic constants and the density was analyzed for the partial case of elastic-brittle materials. The dynamics of unstable crack propagation is controlled by elastic energy fluxes Therefore from physical point of view stress concentration in the elastic vortex is determined by the following differences between speeds of different elastic waves:

1. Difference between shear wave speed V_S and Rayleigh wave speed V_R (which is an upper limit of crack propagation in sub-Rayleigh regime). The difference (V_S-V_R) characterizes time interval between the moments of elastic vortex nucleation and separation from the crack. The longer time interval the higher stress concentration in the vortex and vice versa (the larger speed difference the smaller stress concentration in the vortex).

2. Difference between longitudinal wave speed V_P and shear wave speed V_S. During the course of elastic vortex propagation a fraction of its elastic strain energy outflows in surrounding areas by longitudinal elastic waves. So, this difference characterizes energy loss by elastic vortex.

From this point of view the critical value of the dimensionless geometrical crack parameter (or what is the same the critical value of shear strength) should depend on the sum of these two differences, namely on the difference between longitudinal and Rayleigh wave speeds (V_P-V_R). Elastic wave speeds depend on the ratios of elastic moduli to material density. Hence they are material parameters.

Fig. 6a shows examples of numerically determined dependences of dimensionless geometrical parameter P on elastic wave difference. Here each set of points connected by line was obtained by varying the density and Young modulus of the slab material while maintaining constant value of Poisson ratio. Different sets correspond to different Poisson ratios. All the sets corresponding to different Poisson ratios are parallel to each over (they are shifted along the vertical axis). The offset is proportional to the Poisson ratio.

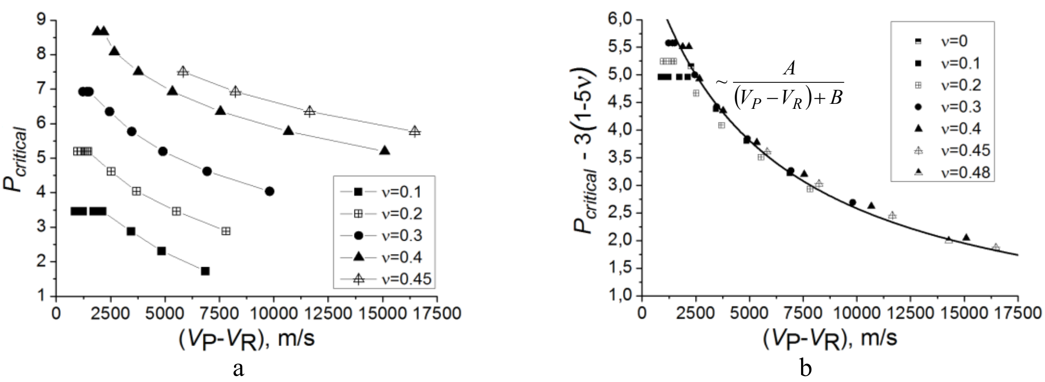

Figure 6: Dependences of the value of critical geometrical parameter $P_{critical}$ on elastic wave difference for slab materials characterized by different ratios E/ρ and Poisson ratios ν (a) and the "master curve" (b).

Therefore these curves can be merged into one so-called "master curve" when using derived geometrical parameter instead of original one (Fig. 6b). Total set of points corresponding to different values of elastic constants (E, ν) and densities ρ is approximated well by the following empirical equation, which is inverse proportion to elastic wave difference:

$$P_{critical} \approx \frac{A}{(V_P - V_R) + B} + 3(5\nu - 1), \tag{2}$$

where A and B are constants. Expression (2) determines critical value of dimensionless geometrical crack parameter and hence the critical shear strength via elastic wave velocity difference and Poisson's ratio. The relation is valid in the very wide ranges of elastic constants and densities except the region of very small values of velocity difference. This region corresponds to extremely heavy or soft material. Here the dependence $P_{critical}(V_P\text{-}V_R)$ tends to saturation.

4 CONCLUSIONS

- The paper nicely complements numerous numerical and laboratory studies of dynamic mode II fracture and explains the concentration of high shear stresses at some distance from the crack tip by formation of a collective circular motion (elastic vortex) ahead of the shear crack. Elastic vortex in solid is a scale-invariable dynamic object. This feature explains the generality of regularities of longitudinal shear crack propagation at different spatial scales and, in particular, the well-known fact that supershear regime of shear crack propagation is observed at all scales. Moreover the importance of the results is related to their predictive ability. Numerically derived dependence (2) makes it possible to estimate critical value $P_{critical}$ for the considered brittle material. This allows one to forecast an ability of pre-existing crack (characterized by geometrical parameter P) in the considered material to propagate in supershear regime under shear-dominated loading.
- Note that elastic vortex related mechanism of stress redistribution in solids can determine not only the regularities of dynamic crack growth but peculiarities of deformation of materials under dynamic loading as well. For example the papers [18,19] describe the results of experimental observation of the formation of "a transient, short range periodicity in the direction of shear band growth in the form of an array of intense ''hot spots'' reminiscent of the well-known, shear-induced hydrodynamic instabilities in fluids". The results of the present study suggest that experimentally revealed small-scale localized shear regions may arise in the areas involved in intensive (high-speed) elastic vortex-like motion. Moreover the vortex related mechanism can be responsible for grain boundary migration under high-rate shear loading [20,21].

5 ACKNOWLEDGEMENTS

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