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INVESTIGATION OF MASS DISCHARGE RATE AND SEGREGATION FROM HOPPER BY DISCRETE ELEMENT METHOD

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Abstract. Hoppers of different shape and angle are widely used in different industries particularly in handling of solids as storage units and in unit operations, e.g. mixing, tableting, etc. It is a challenge to choose a right hopper to achieve desired flow and insignificant segregation due to difference in material properties. General approach for the selection of optimum hopper for a given unit operation is based on the trial-and-error experimental approach. To address this optimum hopper selection, combined experimental and numerical approach is presented in this study. The objective of this study is to analyze the effect of mixture composition and hopper angle on the flow rate and segregation behavior. The numerical simulation of granular flow out of various conical hoppers was also performed using the discrete element method (DEM). The materials considered include different particle size glass bead particles in different proportions by mass. The experimental study is done to validate the DEM results, particularly, mass flow rate. The results analyzed include temporal development of mass fraction of a given particle size during discharge. In addition, the mass flow rate is also computed. The results indicate that fines percentage in the mixture, ratio of smallest particle size to largest in the mixture, and hopper angle plays significant role in determining the segregation and mass flow rate. The flow pattern found to be influenced by the hopper angle and mean particle size of mixture. The results of discharge rate from DEM are also compared with existing empirical correlations and finite element method based elastoplastic model. The DEM prediction shows a good agreement with the existing correlations for a wide range of hopper angles, and with the experimental data.

1 INTRODUCTION

The behavior of granular materials during discharge from hoppers or bins plays a crucial role in many industrial processes such as chemical, agricultural, food technology and pharmaceutical industries [1]. Granular flows can be extremely complex and in general are not well understood [2, 3]. The granular material cannot be considered as solid, liquid or gas to fully characterize their flow state. In fact they are characterized as the fourth state of matter due to its puzzling nature and behaviour [3]. The gravity flow of bulk solids from bins and hoppers is a subject of considerable practical and theoretical interest. The key understanding towards the behavior of granular material in a hopper not only aids in obtaining the correct flow, but also to ensure that the hopper is properly designed to deliver quality product during processing time. The discharge dynamics from hopper depends upon several factors which include material properties, processing conditions and hopper geometry. The flow dynamics gets completely changed due to change in any one of these parameters, which may results in non-uniformity in product stream termed as segregation. The degree of segregation is greatly affected by factors such as particle size distribution width (span), mass fraction of individual components, mean particle diameter as well as the hopper angle. The different experimental and modeling studies were conducted in past owing to the importance of discharge dynamics of hopper. It includes flow mode identification i.e. funnel and mass flow as well as analysis of discharge rate and segregation behavior [4-8]. Apart from these, flows rate prediction from hopper attracted wide attention. The numbers of correlations have been proposed over many decades and are used to predict discharge rates of granular materials from hoppers of different angles. Some of them were based on the dimensional analysis and others on experimental results. From earlier investigations, a dimensional analysis suggests that flow rate is directly proportional to the bulk density of the system, gravitational acceleration, and outlet width of hopper. Beverloo et al. [4] performed experiments on mono-disperse particles. They presented the following empirical correlation for estimating the discharge rate out of a flat bottom hopper: $\dot{m} = C\rho\sqrt{g}(D - kd)^{2.5}$, where $\dot{m}$ is the mass flow rate, C is a constant (~ 0.58), D is the outlet diameter, k is shape factor (= 1.5 for spherical particles), and d is mean particle diameter [4]. Rose and Tanaka (RT) [5] introduced a multiplicative correction factor (F_{corr}) for the application of Beverloo correlation to the conical shape hoppers, and given as follow:

$$F_{corr} = \begin{cases} 1; & \tan\theta\tan\chi \geq 1 \\ (\tan\theta\tan\chi)^{-0.35}; & \tan\theta\tan\chi < 1 \end{cases}$$

where, θ is hopper angle from the vertical and χ is angle from the horizontal of the stagnant-flowing boundary. It is suggested that when no information about χ is available, it may assumed to be 45° [5]. Kurz and Munz investigated the flow properties of limestone and concluded that the mean particle size alone cannot be the used in order to determine granular flow behavior [6]. There have been several attempts to improve the Beverloo correlation to include the effects such as fill height [7], friction between granular material and hopper wall [9, 10,11], hopper outlet diameter [12], mean particle diameter [4], material density, and hopper angle [13]. The effect of hopper half angle, particle-wall friction and material properties on Beverloo constant C, while calculating the discharge rate has always been a matter of debate for many decades [4,14]. In the latest study, Zheng et al. [14] used elastoplastic model to predict the flow rate in conical hopper using finite element method (FEM). Their study introduced the dependency of internal friction angle (φ), hopper half angle (α), wall friction coefficient (μ) and dilatancy angle (ψ) to predict the mass flow rate constant C in Beverloo equation [14]. The equation takes the form by FEM results as follows:

$$C = 0.58\left(\frac{1-\cos^{m}\alpha}{\sin^{5/2}\alpha}\right)(1.08 - 0.0164\,\psi)$$

$$\text{where, } \mathrm{m} = 4.25\, e^{-0.1\varphi} + 0.44 e^{-8.5\mu} + 1.31$$

With increasing computacional resources, various simulation methods have been used to understand the flow dynamics inside hopper in addition to estimate the discharge rate. In this category, DEM [15] has become the widely adopted method for simulating particle behavior and seems to be an alternative method for use in the modeling the behaviour of bulk solids. DEM has been well established for simple flows for many years and was widely employed to study granular flow from hoppers [18, 19]. These investigations helped in the study of flow behavior, velocity distribution, segregation etc.during Hopper discharge.

2 MATERIALS AND METHODS

This section presents all the details about the materials and experimental setup. All the experiments are conducted at least in quintuple, unless otherwise mentioned, to provide mean and relative standard deviation (RSD). The details about numerical simulation are also given at the end of this section.

2.1 Experimental Setup

The experiments were performed using the spherical shaped glass beads. The glass beads particles with different sizes were grouped into three sets through sieving: (i) 0.71 mm < d_1< 0.85 mm (small size particles or fines), (ii) 1.0 < d_2< 1.7 mm (medium size particles), and (iii) 1.7 mm < d_3< 2 mm (large size particles), and the mean particle diameters of these sets were 0.78 mm, 1.35 mm, and 1.85 mm respectively. These sets were used to prepare binary and ternary mixture samples. The mixtures with different fines mass fraction were prepared by increasing fines amount by 10% successively, whereby coarse or medium size particles proportion was adjusted accordingly to achieve 100% mixture mass. Starting with binary mixture, which consists of coarse particles (mean diameter = 1.85 mm) and fines (mean diameter = 0.78 mm) as described above, was prepared by varying the fines amount by 10%, i.e. first binary mixture consisted of 10% fines and 90% coarse particles by mass (sample A, see Table 1). Similarly other samples were prepared as tabulated in the Table1.

Table 1: Detail of sample preparation for binary mixture.

Sample	Glass beads diameter (mm)		Mass percentage (%)	
	(d_1)	(d_3)	(d_1)	(d_2)
A	0.78	1.85	10	90
B	0.78	1.85	20	80
C	0.78	1.85	30	70
D	0.78	1.85	40	60

Similarly, the ternary mixtures were prepared by increasing fines mass percentage by 10% from one sample to other whereas remaining proportion is filled with equal mass percentage of coarse and medium size particles. For example, for ternary mixture 1, the fines mass percentage was 10 % whereas coarse and medium size particles mass % were adjusted to 45%

Table 2: Detail of sample preparation for ternary mixture

Sample	Glass beads diameter (mm)	Mass percentage (%)

	(d_1)	(d_2)	(d_3)	(d_1)	(d_2)	(d_3)
E	0.78	1.35	1.85	10	45	45
F	0.78	1.35	1.85	20	40	40
G	0.78	1.35	1.85	30	35	35
H	0.78	1.35	1.85	40	30	30

individually (sample E, see Table 2). The details about all the ternary mixture samples were presented in Table 2. The schematic of the experimental setups are given in Figure. 1. The setup consists of two conical shaped (at the bottom) hoppers which have hopper angle of 60° and 80°. These hoppers were custom built and made up of acrylic glass (see Figure. 1). The outlet diameter is same for both the hoppers and is equal to 10 mm. The outlet of these hoppers were designed in such manner that outlet can be closed while filling material, which can be easily opened during discharge rate measurement experiments without disturbing the hopper and filled material.

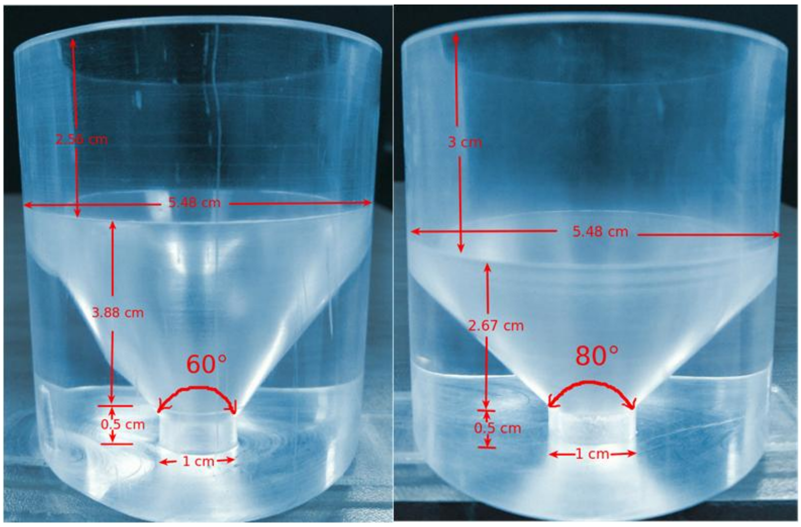

Figure 1: Schematic of the experimental hoppers having different internal angle.

A series of experiments using different mixture samples as shown in the Table 1 and 2, were carried out to study the mass discharge rate of the glass beads from the custom built hoppers. In all experiments, the same amount of sample (100 grams) was weighed and carefully filled into the hopper using consistent filling procedure to obtain the same degree of consolidation by closing the opening lid at the outlet. Once desired amount with consistency is filled into the selected hopper, the discharge rate measurement is performed by removing the outlet lid slowly and then recording the time for complete material discharge using a stopwatch. This procedure was repeated at least five times for selected hopper and selected material (Table 1 and 2), and a mean RSD of the discharge rate is calculated. To reduce electrostatic charge during filling and discharging, static energy discharge gun (Milty Zerostat 3 Anti-Static Gun, China) was used. This device emits positive and negative ions to effectively neutralize static charge that may be built up on surface.

2.2 Numerical simulations using DEM

The simulations are carried out using the three-dimensional DEM, which computes the trajectories of each and every particle using Newton's second law of motion. This is a well-established approach to study the granular material flow dynamics and the details about this method and theory can be found in [17]. Hertz and Mindlin & Deresiewicz theories are used to compute the normal and tangential forces, respectively. An open source DEM software known as LIGGGHTS (LAMMPS Improved for General Granular and Granular Heat Transfer Simulations) version 3.5.0 has been used to perform the simulations [16]. Refer to [16], to understand the implementation of different models in LIGGGHTS along with computational algorithms, and their validation. The material properties used in numerical simulations are given in Table 1, which corresponds to spherical glass-bead particles [18].

Table 3:Parameter considered in simulations.

Properties	Glass beads	Hopper wall (acrylic)	Glass beads – Hopper wall
Density (kg/cm^3)	2500	1800	-
Young'smodulus,Y (GPa)	70	3	-
Coefficient of restitution, e(−)	0.94	-	0.9
Poisson's ratio,v(−)	0.22	0.35	-
Coefficient of friction,μ (−)	0.2	-	0.5

3 RESULTS AND DISCUSSION

The results and discussion are separated into the following three sections. At first the discharge rate of single component or pure component material in both the hoppers (60° and 80°) were presented followed by the binary and ternary mixture's results. Next, a comparison of experimental discharge data with that of DEM numerical results was presented. Towards the end, the comparison of numerical simulations with that of empirical correlations such as Beverloo, Rose and Tanaka, FEM, etc. was established.

3.1 DISCHARGE RATE RESULTS

The main focus of this study to evaluate the discharge rate of pure component materials as well as the binary and ternary mixtures listed in Table 1 and 2, respectively. The experiments were performed as described in section 2.1 to capture the total time to discharge a constant amount of selected material in a given hopper. The results are presented in the following sub-sections.

3.1.1 Pure Component Material

The pure component material i.e., glass beads which were classified into three categories (refer section 2.1) were first analyzed to find the influence of particle size on the discharge rate for mono-sized particles (fines: 0.78 mm, medium-sized: 1.35 mm, coarse: 1.85 mm).The mass flow rate of each size glass beads having total mass of 100g was evaluated by recording the complete discharge time from hoppers. The mean mass discharge rate of these three mono-sized particles for both type of hoppers were depicted in Figure. 2.

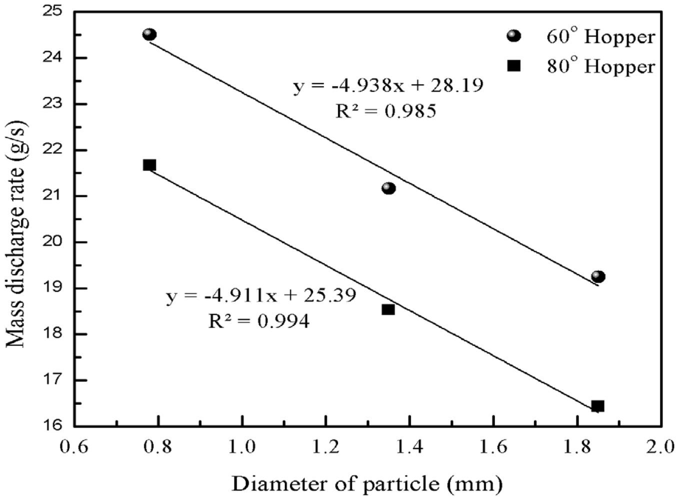

Figure 2: Mass flow rate as a function of particle diameter (mm) in different angle hopper.

The results are also tabulated along with RSD in Table 4 for these experiments. First of all, a low RSD (< 1%) indicates the reproducibility and reliability of these experiments. For both type of hoppers, the discharge rate decreases with increasing particle size as shown in the Figure 2 and Table 4, which is in agreement with the results reported in reference [6].

Table4: Mean discharge rate for single component in different angle hopper.

Particle diameter (mm)	Hopper angle (°)	Mean discharge rate (g/s)	Relative Standard Deviation (RSD) (%)
0.78	60	24.51	0.92
0.78	80	21.67	0.95
1.35	60	21.16	0.95
1.35	80	18.53	0.51
1.85	60	19.25	0.80
1.85	80	16.43	0.71

With 60° hopper, the discharge rate decreases from 24.51 g/s to 21.16 g/s, when the particle size was increased from 0.78 mm to 1.35 mm. The discharge rate further decreased to 19.25 g /s when the particle size was increased to 1.85 mm from 1.35 mm. Similar trend was observed for 80° hopper, where mass flow rate was decreased from 21.67 g/s to 18.53 g/s and further to 16.43 g/s with increase in diameter from 0.78 mm to 1.35 mm and from 1.35 mm to 1.85 mm respectively. Comparison of 60° with that of 80° revealed that the steeper the hopper (or in other words the lower the hopper angle) higher the flow rate, which is independent of the particle size (see Figure 2 and Table 4), which is also in agreement with literature data [5]. The correlation between discharge rate and mean particle size used in these experiments were also shown in Figure 2 with very good correlation coefficient. Thus, for a fixed outlet diameter of hoppers, the flow rate exhibits a linear relationship with respect to particle diameter. Furthermore, the effect of hopper angle on flow rate was also observed in the Figure 2, showing decrease in mass flow rate by 10% as hopper angle was changed from 60° to 80°. These results indicate that small particle diameter (0.78 mm) granular material, particles exhibits very little resistance and behave more like a fluid, while large particle diameter experiences substantial amount of resistance and it becomes more difficult to get the particles to flow as the ratio between particle diameter to hopper outlet diameter increases.

3.1.2 Discharge Behaviour of Multicomponent Mixture Samples

The real particulate solids are composed of various sizes of particles and are more commonly encountered in industrial applications unlike single component granular materials. For this reason, binary (sample A to D, Table 1) and ternary (sample E to H, Table 2) mixtures were prepared, and the flow rates were measured as explained in section 2.1. The particle diameter of the mixtures was represented by the mean particle diameter. First, the mass flow rate of binary mixture having different percentage of fines (sample A to D, Table 1) was measured. The mass discharge rate of binary mixtures from both the 60° and 80° hoppers as a function of fine percentage was shown in the Figure. 3.

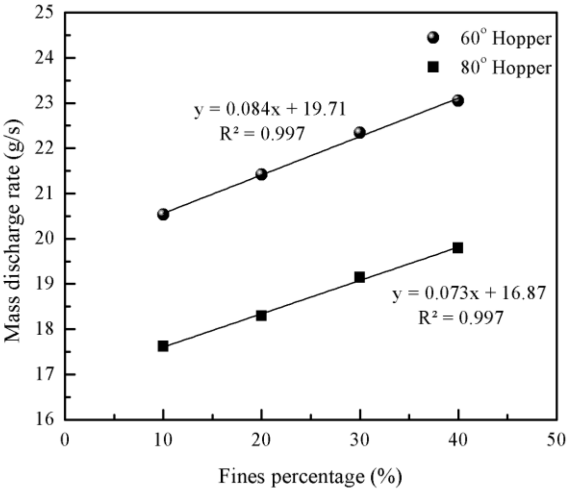

Figure 3: Mass flow rate as a function of fine percentage (binary mixture) in different angle hopper.

The flow rate of granular material increases linearly as the fines mass percentage increases for both types of hoppers as seen in the Figure 3. The good correlation between the flow rate and fines percentage with the correlation coefficient R^2 as high as 0.99 was observed. In addition to fine percentage, the effect of hopper angle on flow rate was also observed clearly. For every 10% increase in fine particles amount, the flow rate was increased by a factor of 0.8 and 0.7 for 60° and 80° hopper respectively (Figure 3).The increase in flow rate in 80° hopper is less than that for 60° hopper, which could be due to steeper hopper walls in 60° hopper to ensure better particle movement compared to 80° hopper. Anand et al. [19] reported that with increasing mass fraction of fines in granular system mass discharge rate tend to increase. This increase in flow rate was attributed to the increase in flowing density during discharge [20]. It is evident that decrease in flow rate with increasing fines % was due to the particle distribution generated by changing fines mass %. A similar behavior was reported, wherein increase in mean diameter resulted in decrease in the flow rate [21]. Ternary mixtures were prepared by addition of a particles of intermediate size (as described in section 2.1) to study the effect of multicomponent mixture on the flow rate. Similar to the binary mixture the effect of mass % of fines on discharge rate was investigated by changing the fine % (samples E to H, Table 2). The effects of fines mass percentage on discharge rate is shown in the Figure 4.

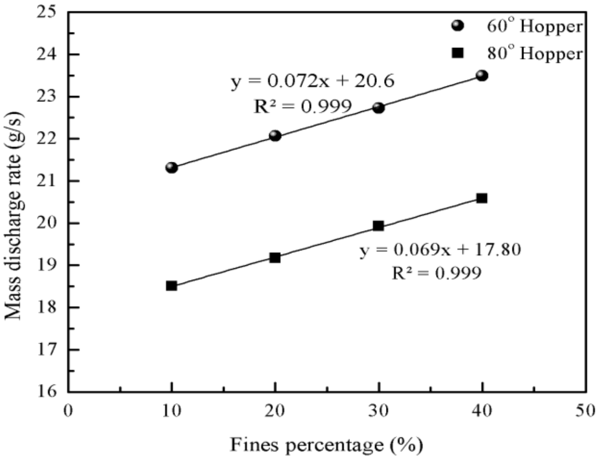

Figure 4: Mass flow rate as a function of fine percentage (ternary mixture) in different angle hopper.

In comparison to the binary mixture, the mass flow rate of the ternary mixture shows an improvement, which is attributed to the decrease of mean particle diameter due to addition of intermediate particles to binary mixture. Also, as the fine percentage was increased, the maximum mass flow rate corresponds to ternary mixture for same percentage of fines. This is explained based on the fact that better particles packing could be achieved when mixture was changed from binary to ternary results into increased flowing density during discharge. The effect of increase in fine content on discharge rate was reduced in ternary mixture compared to binary mixture, which can be clearly seen by comparing the slopes of Figure. 3 and Figure. 4. The mass flow rate increased by factor of about 0.07 with ternary mixtures whereas for binary mixtures the mass flow rate increases by a factor of about 0.08 for 60° hopper.

3.2 Mass Flow Rates Comparison with Different Correlations

The discharge flow rates obtained by experiments were validated using well-established correlations reported in literature as well as with DEM simulations. The basic equation to calculate the theoretical flow rate was published by Beverloo et al. [4]. This formula is for cylindrical orifices with flat bottom and thus does not consider the hopper angle. A correction factor based on the Rose and Tanaka [5] equation can be applied to improve Beverloo equation, which is a function of the hopper angle as described in section 1. In the latest study, Zheng et al. [14], developed an empirical correlation for predicting the mass discharge rate of conical hoppers based on FEM with elastoplastic model (refer to section 1). In this context, the measured data of flow rates for binary (samples A to D, Table 1) and ternary (samples E-F, Table 2) mixtures with both the hoppers (60°and 80°) were calculated using Rose and Tanaka (RT) correlation, DEM, and FEM based empirical correlation. The Figures 5(a) and 5(b) show the comparative prediction of these correlations along with DEM simulations and experimental data for binary mixture (samples A to D) in 60° and 80° hopper, respectively.

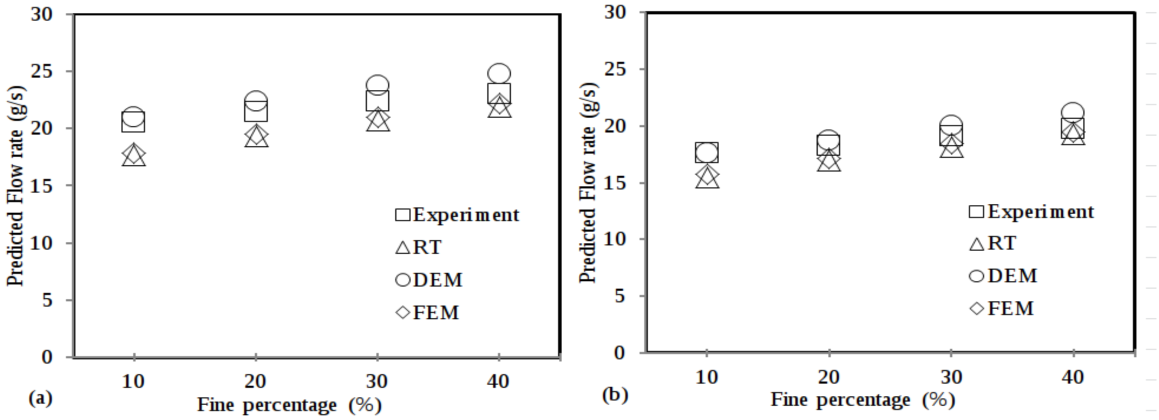

Figure 5: Comparison of mass flow rate between experiment, Rose and Tanaka, DEM and FEM based elastoplastic model for binary mixtures (samples A to D) in (a) 60° hopper (b) 80° hopper.

The results indicate that the flow rate increases with increasing fines content and this profile is very well captured by all the correlations and DEM results. The good agreement between the results obtained by DEM and experiments was observed for cases with lower fines content (< 30% by mass), whereas the empirical correlations (RT and FEM) underestimate the flow rates in these cases. One of the possible reasons for overprediction in DEM results for cases with fines content above 30 % may be due to the size difference between particles in simulations and experiments. Basically, simulations takes into account the exact sizes of particles whereas the actual glass beads used in experiments had a distribution in the sizes and hence were grouped into three sizes as described in Section 1. On the other hand, RT and FEM correlations underpredicts the experimental results by less than 10%. It was also found that the RT and FEM deviation with experimental flow rate were increase with decrease in fines content. Thus, it may be concluded that the fines fraction in mixture probably had a greater impact on predicting discharge rate. Furthermore, with the increase in internal angle of the hopper, the predicted discharge rates by FEM and those by RT correlations were improved with the experimental discharge rates.

3.3 Segregation Results

Figure.6 shows segregation results for 60° and 80° hoppers for sample D (x_f- 40%). The first 10% discharged material contain a lesser amount of fines due to filling process. Generally, larger particles roll down to the bottom of the Hopper during filling and this effect seems to be greater for 80° Hopper due to wide angle at bottom. For both 60° and 80° hoppers, almost equal proportion of fines were discharged until $M/M_{total} \approx 0.55$.

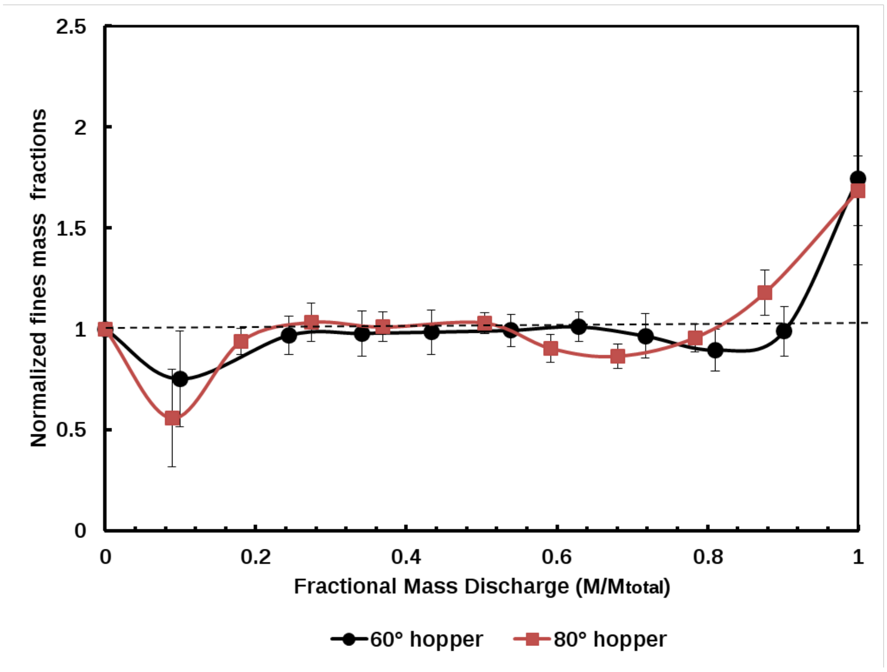

Figure 6: Experimental segregation results for sample D (x_f- 40%) in different angle hopper.

At approximately 60% mass discharged (M/M_{total} = 0.6), the behavior of granular material inside both hoppers vary significantly. For 80° hopper, the discharged mass was depleted in fines between 60 % to 80 % discharged mass. But after 80 %, discharge mass collected contain excess of fines due to segregation. Generally, after $M/M_{total} \approx 0.5$, coarse particles tend to occupy most of the centerline channel resulting in accumulation of fines at side walls of hopper and hence discharged material was depleted in fines approximately between $0.55 < M/M_{total} < 0.8$. During end discharge, the resulting accumulated fines were discharged. On the other hand, 60° Hopper attained fairly uniform fines mass fractions discharge as compared to 80° Hopper during initial discharge mass until $M/M_{total} \approx 0.7$. This might be due to increase in wall steepness as the Hopper wall angle was decreased to θ=60°. After $M/M_{total} \approx 0.7$, the discharge material was depleted in fines and during end discharge, material rich in fines was collected. Ketterhagen et al. [18] in their work also observed a similar behavior and concluded that as the internal angle of Hopper increases, the segregation of the granular material increases.

CONCLUSIONS

Firstly, the hopper angle is found to be significantly impacting the flow rate where it is shown that the steeper the hopper the better the flow rate out of the hopper. Next, with pure component materials, it is reported that while keeping the outlet diameter constant, the larger the particle diameter the lesser the flow rate. The experimental data of these mixtures shows that presence fine particle in addition to coarse improves the overall flow rate where the fines particle imparts fluidity effect thereby increasing the flow rate. The observations made in this study are in agreement with the literature data. The comparison of experimental data with that of the well-known correlations and DEM results indicate that overall DEM results are better predicting the flow rates than the empirical correlations or FEM based correlations. However, it found that the DEM results are slightly over predicting especially in the higher fines content mixtures which attributed to the fact that there is slight difference between glass-bead particle sizes used in experiments and the ones used in DEM simulations. With this study it is possible to provide a first estimate of the discharge rates out of a given hopper without actually performing experiments. The focus of the future study is to investigate segregation behavior as mass is discharging which is currently ongoing. Some of these results will be presented at the Particles 2017 meeting.

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