

## PERFORMANCE OF LARGE SCALED TSUNAMI RUN-UP ANALYSIS USING EXPLICIT ISPH METHOD

Keita Ogasawara<sup>1</sup>, Mitsuteru Asai<sup>2</sup>, Mikito Furuichi<sup>3</sup> and Daisuke Nishiura<sup>4</sup>

<sup>1</sup> Department of Civil Engineering, Graduate School of Engineering Kyushu University  
744 Motooka, Nishi-ku, Fukuoka, Japan  
ogasawara@doc.kyushu-u.ac.jp

<sup>2</sup> Department of Civil Engineering, Graduate School of Engineering Kyushu University  
744 Motooka, Nishi-ku, Fukuoka, Japan  
asai@doc.kyushu-u.ac.jp

<sup>3</sup> Department of Mathematical Science and Advanced Technology, Japan Agency for  
Marine-Earth Science and Technology  
3173-25, Showa-machi, Kanazawa-ku, Yokohama, 236-0001, Japan  
m-furuic@jamstec.go.jp

<sup>4</sup> Department of Mathematical Science and Advanced Technology, Japan Agency for  
Marine-Earth Science and Technology  
3173-25, Showa-machi, Kanazawa-ku, Yokohama, 236-0001, Japan  
nishiura@jamstec.go.jp

**Key words:** stabilized EISPH, Tsunami run-up, parallel computing

**Abstract.** The tsunami run-up simulation by the particle method at city level needs to huge number of particle at least 1 billion particles. The conventional particle simulation method is not easy to solve these huge problem even on the premise of using supercomputer. Then, a new particle method 'fully explicit Incompressible SPH' is developed that takes into consideration both calculation efficiency and accuracy. Finally, we demonstrate the future plan how to use our simulation results for a practical 'Soft' disaster mitigation method through the evacuation education with the Virtual Reality(VR) system.

### 1 Introduction

On March 11, 2011, the huge tsunami caused by the great east Japan earthquake devastated many infrastructures in pacific coast of north eastern Japan. Particularly, the damage of outflow of bridge girders caused a traffic disorder and these collapse behaviours led to delay of recovery after the disaster. After 2011 tsunami, disaster prevention and mitigation techniques are actively developing in coastal infrastructures and establishing

prediction method for tsunami disaster is one of the severe issues toward the next millennium tsunami. After Tohoku earthquake 2011 Japan, our research group has been developed a three-dimensional tsunami run-up analysis tool using the stabilized ISPH[1] which is one of the semi-implicit Lagrangian particle method. The target site of simulation is Kochi city where huge tsunami disaster is anticipated with the next big Nankai Trough Earthquake. Tsunami run-up simulation in the urban area requires high resolution at least 2m in order to resolve the complicated tsunami flow through buildings and the other structures. It leads to the large scale problem, and the tsunami run-up simulation at Kochi city needs to at least 1 billion particles. It is difficult to simulate the conventional ISPH method even on the premise of using supercomputer. In this research, a new particle method 'fully explicit Incompressible SPH' is developed that takes into consideration both calculation efficiency and accuracy. Finally, we demonstrate the future plan how to use our simulation results for a practical 'Soft' disaster mitigation method through the evacuation education with the Virtual Reality(VR) system.

## 2 Analysis method

In the past, we adopted the stabilized ISPH method, but this method is a kind of implicit method and it is not suitable for large scale parallel calculation. Therefore, we have established a method corresponding to ultra large scale parallel computation by explicitizing the stabilized ISPH method.

### 2.1 Governing Equations of Fluid Analysis

To solve incompressible fluid problems, independent variables, flow velocity  $\mathbf{u}$  and pressure  $p$ , should be obtained while satisfying the conservation law of mass and momentum. The governing equations in the Lagrange description are given by

$$\frac{D\rho}{Dt} + \rho \nabla \cdot \mathbf{u} = 0 \quad (1)$$

$$\frac{D\mathbf{u}}{Dt} = -\frac{1}{\rho} \nabla p + v \nabla^2 \mathbf{u} + \mathbf{g} \quad (2)$$

where  $v$  is the coefficient of kinetic viscosity and  $\mathbf{g}$  is the acceleration of gravity.

Based on the assumption that the density of water is constant, the mass conservation law eq.(1) is rewritten as

$$\nabla \cdot \mathbf{u} = 0 \quad (3)$$

Applying Eqs. (2) and (3) to particle  $i$ , we obtain

$$\frac{D\mathbf{u}_i}{Dt} \approx -\frac{1}{\rho_i} \langle \nabla p_i \rangle + v \langle \nabla^2 \mathbf{u}_i \rangle + \mathbf{g}_i \quad (4)$$

$$\langle \nabla \cdot \mathbf{u} \rangle \approx 0 \quad (5)$$

The symbol of  $\langle \cdot \rangle$  is the approximate value referring to the values of neighbor particles in the vicinity on the basis of the concept of the SPH method.

## 2.2 ISPH Method

In the particle method, density is calculated through the distribution of particles, therefor it leads to the problem of the conservation of volume. To solve this problem, the ISPH method, in which the velocity and pressure fields of the equations of motion are separated using the projection method to evaluate the pressure field implicitly and the velocity field explicitly, was proposed [2, 3]. Using the projection method, the velocity and pressure fields can be separated by defining a tentative state without considering the pressure gradient term in the Navier-Stokes equations of motion. In this section, the procedure for updating variables from time step  $n$  to  $n + 1$  is explained with the aim of applying the projection method to the SPH method. First, the time-derivative term in eq. (2) is subjected to forward difference approximation and the intermediate velocity  $\mathbf{u}^*$  is defined in an intermediate state to separate the velocity as

$$\frac{D\mathbf{u}}{Dt} = \frac{\mathbf{u}^{n+1} - \mathbf{u}^n}{\Delta t} = \frac{\mathbf{u}^{n+1} - \mathbf{u}^*}{\Delta t} + \frac{\mathbf{u}^* - \mathbf{u}^n}{\Delta t} \quad (6)$$

Among the components of acceleration after separation, the intermediate velocity  $\mathbf{u}^*$  is evaluated as follows assuming that the first and second terms on the right side of eq. (6) correspond to the pressure gradient term and other terms of eq. (2), respectively.

$$\begin{aligned} \frac{\mathbf{u}_i^* - \mathbf{u}_i^n}{\Delta t} &= v \langle \nabla^2 \mathbf{u}_i^n \rangle + \mathbf{g}_i \\ \rightarrow (\text{predictor}) \mathbf{u}_i^* &= \mathbf{u}_i^n + \Delta t (v \langle \nabla^2 \mathbf{u}_i^n \rangle + \mathbf{g}_i) \end{aligned} \quad (7)$$

The velocity from the intermediate state to the next time step is updated as follows assuming that the pressure is evaluated based on an appropriate method.

$$\begin{aligned} \frac{\mathbf{u}_i^{n+1} - \mathbf{u}_i^*}{\Delta t} &= -\frac{1}{\rho} \langle \nabla p_i^{n+1} \rangle \\ \rightarrow (\text{corrector}) \mathbf{u}_i^{n+1} &= \mathbf{u}_i^* + \Delta \mathbf{u}^* \end{aligned} \quad (8)$$

$$\Delta \mathbf{u}^* = -\Delta t \left( \frac{1}{\rho} \langle \nabla p_i^{n+1} \rangle \right) \quad (9)$$

As shown above, two-separated processes are implemented to update the state of velocity in ISPH method. The pressure is evaluated by solving the pressure Poisson equation given by

$$\begin{aligned} \langle \nabla^2 p_i^{n+1} \rangle &= -\frac{\rho^0}{\Delta t} \langle \nabla \cdot \Delta \mathbf{u}_i^* \rangle \\ &= \frac{\rho^0}{\Delta t} \langle \nabla \cdot \mathbf{u}_i^* \rangle \end{aligned} \quad (10)$$

### 2.3 A Modified Source Term to Relax the Incompressible Conditions

The original ISPH and our proposed stabilized ISPH are applied the same projection method in order to split velocity and pressure updates. In the projection method, an intermediate state is defined by excluding pressure gradient term in Navier-Stokes eqn. in (11). The difference between the original ISPH and the stabilized ISPH is the source term in the pressure Poisson equation (see [1]) for the detailed formulation of equations).

In the SPH method, the numerical density is evaluated by counting the number of neighbor particles. During the fluid dynamics simulation using the ISPH, the numerical density is changing in time. In other words, it is difficult to keep the constant numerical density value because of non-uniform particle distributions. Therefore, the modified source term has an important role to decrease the numerical error in the numerical density value.

$$\langle \nabla^2 p_i^{n+1} \rangle \approx \frac{\rho^0}{\Delta t} \langle \nabla \cdot \mathbf{u}_i^* \rangle + \alpha \frac{\rho_i^0 - \langle \rho_i^n \rangle}{\Delta t^2} \quad (11)$$

The pressure Poisson equation given by eq. (11) completely conforms to the formulation under a velocity divergence-free condition assuming that the relaxation parameter is zero. In addition, when the density instantaneously agrees with the initial density (or the density is negligibly small), the pressure Poisson equation and formulation are considered to be the same because the second term of the source term of the pressure Poisson equation is negligible. According to the formulation, the accumulated error related to the density produced during analysis is gradually canceled by the term of density difference, leading to a scheme with a good conservation of volume as a result of an almost constant density even for a long-term calculation.

### 2.4 Explicit pressure assessment scheme

Here, the stabilized ISPH method, which utilizes a semi-implicit time integration, is converted to an explicit version. The most time consuming part in the ISPH is the linear equation solver for the pressure Poisson eqn.. In addition, the linear equation solver is not so easy to get faster and to get a good parallel efficient in the supercomputers. The main purpose of the explicit version of the stabilized ISPH is to decreasing the total computational time in the supercomputer environments. The idea of the totally explicit ISPH method is proposed by Barcarolo[4] et al.. They just showed a couple of numerical examples without verification and validation. Therefore, we developed a modified EISPH method that satisfies accuracy and stability by explicitly solving the stabilized ISPH method that realized long term volume storability by referring the stabilized ISPH. In Eq.(11), it can be rewritten in a discrete approximation form for the left side, the following equation is obtained.

$$\langle \nabla^2 P_i^{n+1} \rangle = \sum_j m_j \frac{2}{\rho^0} \frac{r_{ij} \cdot \nabla W(r_{ij}, h)}{r_{ij}^2 + \eta^2} (P_i^{n+1} - P_j^{n+1}) \quad (12)$$

When Equation (12) is transformed, the following equation is obtained.

$$P_i^{n+1} = \frac{B_i + \sum_j A_{ij} P_j^{n+1}}{\sum_j A_{ij}} \quad (13)$$

For convenience of notation, the following section is used here.

$$A_{ij} = m_j \frac{2 r_{ij} \cdot \nabla W(r_{ij}, h)}{\rho^0 (r_{ij}^2 + \eta^2)}, B_i = \frac{\rho^0}{\Delta t} + \alpha \frac{\rho_i^0 - \rho_i^n}{\Delta t^2}$$

In the equation (13), assuming that the pressure of the neighboring particles, which are originally unknown quantities, can be approximated by the pressure of the present time at minute time intervals from  $n$  steps to  $n+1$  steps, Can be rewritten as follows.

$$P_i^{n+1} = \frac{B_i + \sum_j A_{ij} P_j^n}{\sum_j A_{ij}} \quad (14)$$

By using the expression (14) it is possible to make an explicit pressure assessment. The proposed method changes only the pressure evaluation in the conventional stabilized ISPH method, and does not need to change the other schemes. In this study, we call the above scheme a stabilized Explicit ISPH (EISPH) method.

### 3 Verification test

#### 3.1 Simple dam break test

By using the stabilized EISPH method, it is possible to perform highly parallel calculation as compared with the calculation by the stabilized ISPH method. However, verification was conducted because verification of validity about calculation accuracy was insufficient. The verification problem was verified with the dam break problem shown in the Figure 1. We analyze by both EISPH method and ISPH method referred to as conventional method and verify the proposed method by comparing both analysis results. Other conditions related to analysis are as shown in Table 1, and both analyzes were carried out under the same conditions. Regarding time increment, both methods need to satisfy the CFL condition, so we decided to use the same value (0.0002 sec). However, with the EISPH method, which is a fully explicit method, the relationship between accuracy and time increment should be investigated separately in addition to the CFL condition. In order to verify focusing only on the accuracy of the EISPH method, the same conditions as the conventional method as semi-implicit method were used. The Figure. 2 shows the result at 6000 Step, and the color means the pressure contour, indicating that it shows a qualitatively good agreement. Figure. 3 is the time history of the pressure on the red dot in Figure. 1. This result also agrees well.

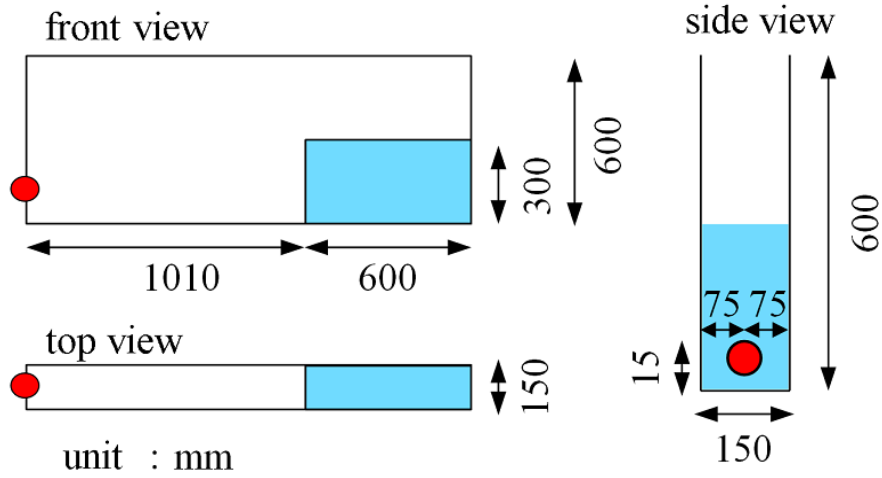


Figure 1: dam break model

Table 1: analysis conditions

particle diameter	number of particles	time increasement
10mm	214932	0.0002sec/step

### 3.2 Tsunami run-up simulation in a real city

A tsunami run-up simulation in a real city, Japan is conducted here. Figure. 4 shows the numerical model. The analysis conditions are as shown in Table 2, and the same value is used for time increment here as well. The color in the figure indicates the y-directional velocity. It also shows that this also shows a good agreement.

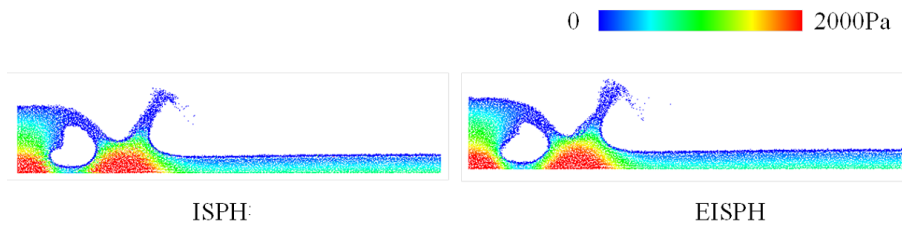


Figure 2: result(6000Step)

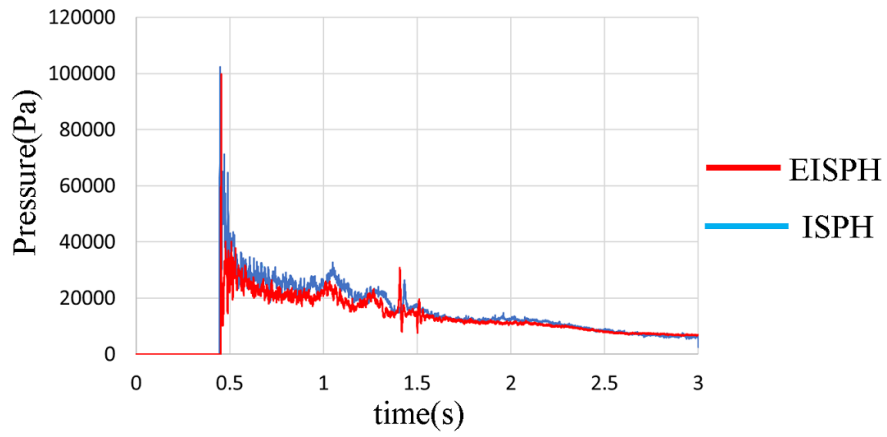


Figure 3: time history of th pressure

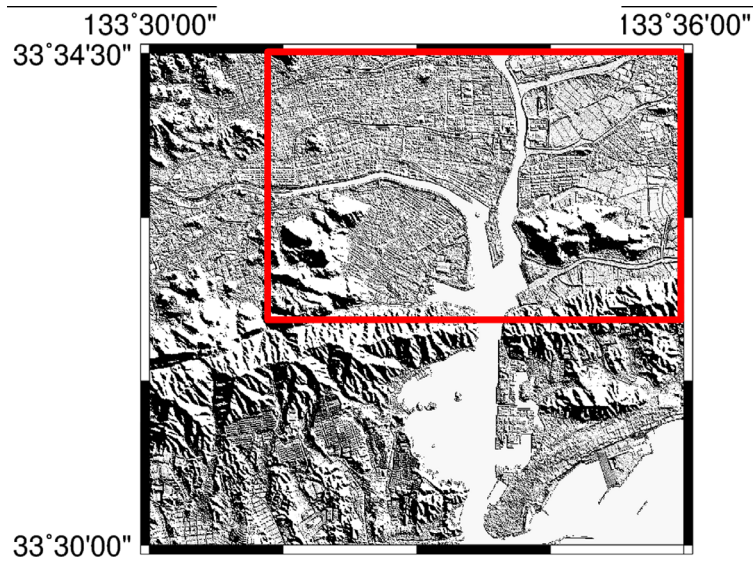
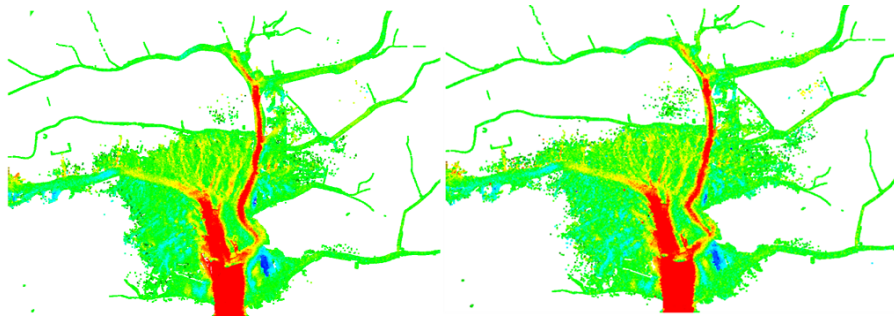


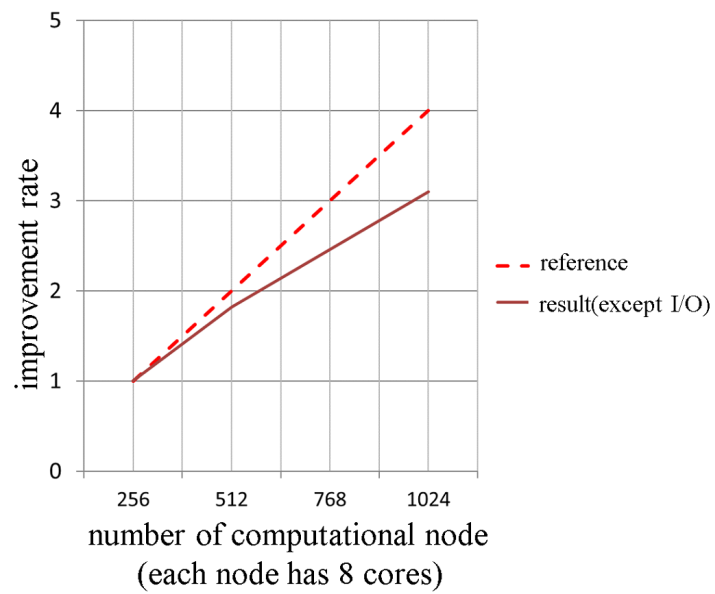
Figure 4: tsunami run-up area at Kochi city

Table 2: Analysis conditions

particle diameter	number of particles	time increasement
4m	33591333	0.05sec/step

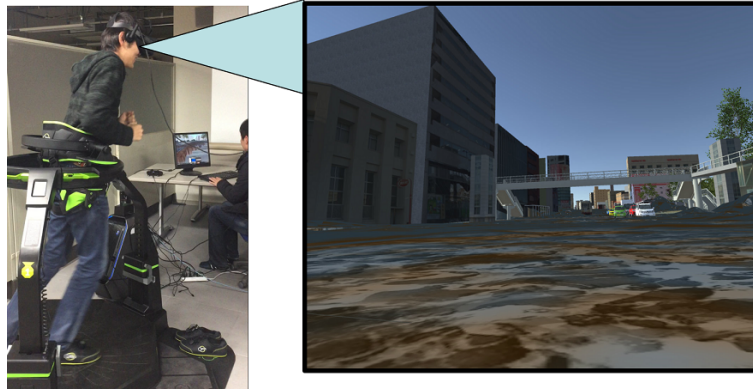


**Figure 5:** Results in EISPH method (left figure) and ISPH method (right figure)



**Figure 6:** Parallelization efficiency





**Figure 7:** Virtual Reality usage example

### 3.3 Parallelization efficiency

By explicitization, it is not necessary to solve the pressure Poisson equation, which is the most time consuming part in the implicit version, and it is relatively easy to perform massively parallel calculations. The figure-6 is the calculation speed improvement rate in which parallel numbers are executed in 256 parallel, 512 parallel, 1024 parallel in the above-mentioned tsunami run-up analysis, respectively. For 256 parallels to 512 parallels, 91% of the ideal value shows good parallelization efficiency. In the 1024 parallel, 78% is not bad, but it tends to decline slightly, which seems to be overpowering as the number of parallels is excessive for this calculation.

### 3.4 Large scale parallelism and VR visualization

By enabling large-scale parallel computation, it is possible to calculate with resolution of several tens of centimeters in the tsunami run-up analysis. As a result of this, it becomes possible to accurately resolve the tsunami that moves up the block as described above, so that accurate calculation becomes possible, Even in visualization it makes it possible to draw waves through the road and it has big meaning. The authors made VR visualization and constructed a system for enlightenment etc. of the tsunami damage. Figure. 7 is a picture walking in the image with a walking controller with visualization of VR by computing only a specific area experimentally at 50 cm resolution. It is possible to walk not only inside the city block where the tsunami arrives but also in the VR image like the one on the right.

## 4 Conclusions

In this study, a stabilized EISPH method is proposed for the massively parallel computations for tsunami run-up simulation. We are trying to utilize the 3D tsunami run-up simulation results for the tsunami evacuation education through the VR systems. The

VR system is composed by the head mount display and walking controller. In the future, we plan to calculate the same tsunami run-up simulation with 50 cm resolution at the same city model.

## REFERENCES

- [1] M. Asai, A.M.Aly, Y.Sonoda, Y.Sakai, A Stabilized Incompressible SPH method by relaxing the Density invariance condition. *International Journal for Applied Mathematics*, No. 20130011, (2012)
- [2] S.J.Cummins, M.Rudman, An SPH project method, *Journal Computational Physics*, 152.(2), (1990)
- [3] A.Khayyer, H.Gotoh, S.Shao, Corrected incompressible SPH method for accurate water-surface tracking in breaking waves, *Coastal Engineering*, 55, 236–250, (2008)
- [4] D.A.Barcarolo, A.le Touze, F. de Vuyst, VALIDATION OF A NEW FULLY-EXPLICIT INCOMPRESSIBLE SMOOTHED PARTICLE HYDRODYNAMICS METHOD. *Blucher Mechanical Engineering Proceedings*, vol.1,num.1, (2014)