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EJECTING PROPERTIES OF A BUCKET ELEVATOR

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Abstract. Air inside the enclosure of a belt elevator may be brought into motion both by moving bucket belt and by spillage flows during loading and unloading of buckets. Initial findings from studies performed to evaluate air motion in ducts with mobile partitions have been published in our earlier monographs [1-3]. Here we'll consider the process of air ejection in bucket elevators from the standpoint of classical laws of change in air mass and momentum. Direction of airflow inside enclosures of the carrying and return runs of a bucket elevator is determined by the drag of buckets and moving conveyor belt as well as ejection head created by a stream of spilled particles when buckets are unloaded. As a result of these forces acting together inside an enclosure, differential pressure arises. This differential pressure is equal to the sum total of ejection heads created by conveyor belt with buckets E_k and flow rate of spilled material E_p minus aerodynamic drag of enclosure walls.

The ejection head E_k created by a bucket-carrying conveyor belt is determined by aerodynamic coefficient c_{ek} (proportional to the number of buckets, their head resistances and squared mid-sectional dimensions) together with an absolute value and the direction of bucket velocity relative to the velocity of airflow inside the enclosure. Ejection head of spilled particles E_p depends on the drag coefficient of particles, their size and flow rate, as well as the enclosure length, enclosure cross-section and relative flow velocity of particles.

When both the carrying and return runs of the conveyor belt are located in a common enclosure, the velocity of forward airflow varies over its length as a result of cross-flows of air through gaps between the conveyor runs and enclosure walls. Cross-flows are caused by a differential pressure between the carrying and return run enclosures and is dependent on the drag of the gap. Cross-flow direction depends on the ratio between p_v and p_u.

Given identical size of elevator enclosures, change in absolute values of longitudinal velocities is identical and depends on absolute values of cross-flow velocities and geometrical dimensions of the gap, as well as enclosure cross-section. The momentum of longitudinal

airflow in this case is determined by variable magnitudes of aerodynamic forces of buckets due to changes in their relative motion velocities.
The flow rate of air in enclosures may be determined by numerically integrating three dimensionless combined differential equations.

1 INTRODUCTION

Let's now consider ejection properties of a bucket-belt elevator. Air inside the enclosure of a belt elevator may be brought into motion both by moving bucket belt and by spillage flows during loading and unloading of buckets. Initial findings from studies performed to evaluate air motion in ducts with mobile partitions have been published in our earlier monographs [1-3]. Here we'll consider the process of air ejection in bucket elevators from the standpoint of classical laws of change in air mass and momentum.

2 THE RESULTS OF THE STUDY

Let's begin with considering airflow in a duct of elevator return line of length dx (Fig. 1). For this area we can a momentum conservation equation can be written in projections onto axis $0x$ directed vertically downwards. We'll formulate a one-dimensional problem assuming that the velocity of ejected air is directed downwards and is equal to the cross-sectional average:

$$u = \frac{Q}{S}; \quad S = ab, \tag{1}$$

where Q is the flow rate of air ejected inside the duct (m^3/s); a, b are cross-sectional dimensions of the duct (m)

The tangential frictional stress on the surface of a moving belt is equal to

$$\tau_l = c_l \frac{(v_e - u)|v_e - u|}{2} \rho, \tag{2}$$

where c_l is a dimensionless resistance coefficient, v_e is the velocity of elevator belt (m/s).

Similarly, tangential frictional stress on the surface of enclosure walls is

$$\tau_w = c_w \frac{u|u|}{2} \rho, \tag{3}$$

where c_w is a dimensionless resistance coefficient of enclosure walls.

It is known that coefficients c_l, c_w are related to friction coefficients in the Darcy-Weisbach equation for determining pressure losses in straight pipe sections

$$c_l = \frac{\lambda_l}{4}; \; c_w = \frac{\lambda_w}{4},$$

where λ_l is the hydraulic friction coefficient of the belt; λ_w is the hydraulic friction coefficient of enclosure walls.

The aerodynamic force of a bucket, expressed similarly to the aerodynamic force of particles, is

$$R_k = c_k F_k \frac{(v_e - u)|v_e - u|}{2} \rho , \tag{4}$$

where F_k is the mid-section area of a bucket $(F_k = A_k B_k)$ (m^2), c_k is the drag coefficient of an empty bucket.

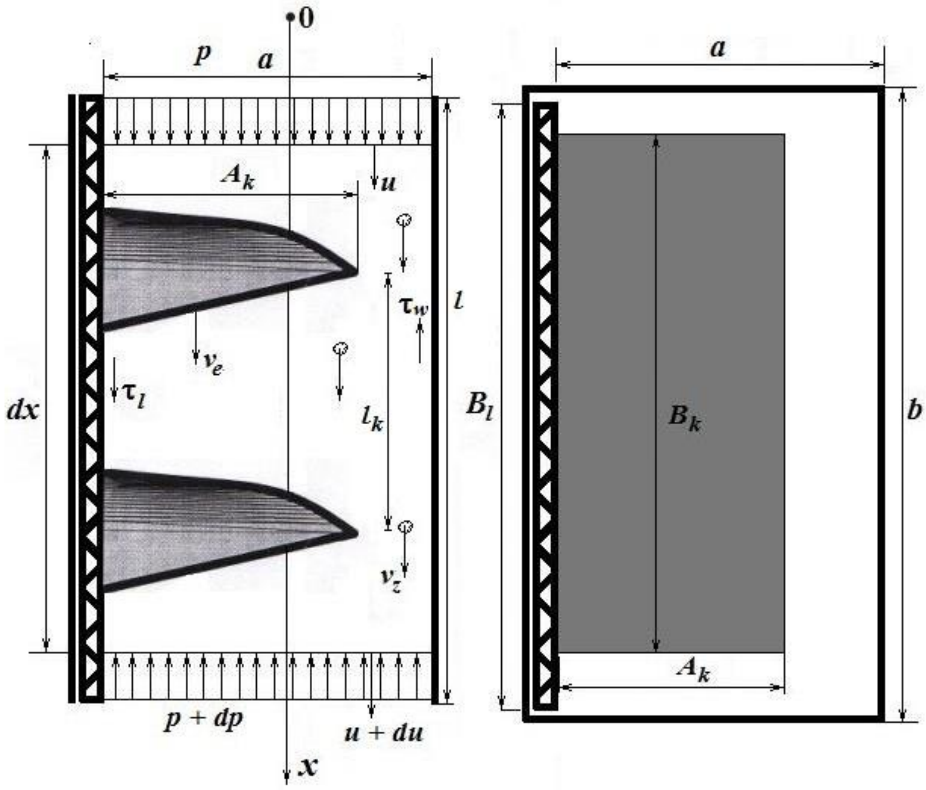

Figure 1: Diagram of forces acting on an element of elevator return run enclosure with a length dx

In this case the equation for change in momentum would appear as:

$$\rho u S(-u) + \rho(u+du) S(u+du) =$$
$$= pS - (p+dp)S - \tau_w (b+2a)dx + \tau_l B_l dx + R_k \frac{dx}{l_k} + R_z \frac{\beta dx S}{V_p} , \tag{5}$$

where l_k is the spacing of buckets on the belt (m); β is volumetric concentration of grain spillage, equal to

$$\beta = \frac{G_p}{\rho_z S v_z} ; \tag{6}$$

G_p is the mass flow rate of grain spillage during bucket unloading (kg/s)

ρ_z is grain density (kg/m^3)

v_z is the fall velocity of grain (m/s)

V_p is the volume of an individual particle (m^3)

R_z is the aerodynamic force of a single particle of spilled material,

$$R_z = \psi_z F_z \frac{|v_z - u|(v_z - u)}{2}\rho\,; \tag{7}$$

F_z is the mid-section area of a particle (m^2)

$$F_z = \frac{\pi d_e^2}{4}\,; \tag{8}$$

d_e is equivalent grain diameter (m).

Considering that, in this case,

$$u = \text{const}\,, \tag{9}$$

after a trivial transformation of equation (5), ignoring infinitesimal second-order terms, we'll obtain the following relation for determining differential pressure inside the enclosure of the return run of the elevator:

$$p(0) - p(l) + E_k + E_p = p_w\,, \tag{10}$$

where $p(0), p(l)$ are static pressures at the inlet/outlet of the enclosure (Pa)

p_w is aerodynamic drag of enclosure walls (Pa)

$$p_w = \int_0^l \lambda_w \frac{2a+b}{4S}\frac{u^2}{2}\rho dx\,, \tag{11}$$

equal at $u = \text{const}$ and $S = \text{const}$

$$p_w = \lambda_w \frac{l}{D_w}\frac{u^2}{2}\rho\,, \tag{12}$$

$$D_w = \frac{4S}{b+2a}\,, \tag{13}$$

l is the total length of elevator enclosure (distance between the axes of driving and return drums along the belt of the bucket elevator) (m); E_k is ejection head created by a conveyor belt with buckets:

$$E_k = \frac{1}{S}\int_0^l \left(c_k \frac{F_k}{l_k} + \frac{\lambda_l}{4} B_l \right) \frac{|v_e - u|(v_e - u)}{2}\rho dx\,, \tag{14}$$

which, at constant relative velocity, is equal to

$$E_k = c_{ek} \frac{|v_e - u|(v_e - u)}{2}\rho\,; \tag{15}$$

c_{ek} is aerodynamic coefficient of the return run of elevator belt (with account of empty buckets and conveyor belt carrying them):

$$c_{ek} = \frac{l}{S}\left(c_k \frac{F_k}{l_k} + \frac{\lambda_l}{4} B_l\right), \tag{16}$$

E_p is the ejection head created by a flow of spilled grain during unloading of elevator buckets:

$$E_p = \frac{1}{S}\int_0^l \psi_z K_m \varepsilon G_p \frac{|v_z - u|(v_z - u)}{2}\frac{dx}{v_z}, \tag{17}$$

which, for a constantly accelerated vertical flow of particles at $\psi_z = \text{const}$, equals:

$$E_p = \frac{\psi_z K_m \varepsilon}{2}\frac{G_p}{Sg}\frac{|v_k - u|^3 - |v_n - u|^3}{3} \tag{18}$$

or, for a uniformly accelerated flow of particles at $v_z = v_e = \text{const}$; $u = \text{const}$,

$$E_p = \frac{\psi_z K_m \varepsilon}{2}\frac{G_p}{S}\frac{l}{v_e}\left[|v_e - u|(v_e - u)\right], \tag{19}$$

where K_m is the ratio of mid-sectional area of a particle to its volume (1/m); ε is the ratio of air density to particle density.

Let's now consider a more complex case with carrying and return runs of a bucket elevator both located in a common enclosure (Fig. 2). In this case air may flow laterally from one part of the enclosure (for example, the one with grain-laden buckets running) to another (with empty buckets running). The velocity of air cross-flow in the gap between the belt and enclosure walls will be designated as ω. Parameters of airflow in the right-hand side of the enclosure (where empty buckets run and spilled particles fall) will be denoted with a subscript u (from the designation of air velocity in the return run) while those in the left-hand side will be denoted respectively with v (from the designation of air velocity in the carrying run of the conveyor).

The cross-flow velocity is determined by differential pressure and aerodynamic drag of gaps:

$$\Delta p = p_v - p_u = \zeta_z \frac{w|w|}{2}\rho, \tag{20}$$

where p_v, p_u are the respective excess static pressures in the left-hand and right-hand sides of the enclosure (Pa); ζ_z is the total local resistance coefficient (LRC) for two gaps between chute walls and the end sides part of carrying and return runs of the conveyor.

The sign of the absolute value in the right-hand side was introduced to ensure universality of the relation which in this case would be just as good for the case of a reverse flow (with $p_u > p_v$). The velocity w in such "vector" case will be negative, that is, velocity vector will be directed oppositely (from right to left). This vector is represented with a dotted line on Fig.2.

Due to the presence of lateral cross-flow of air, velocities u and v will not be constant but rather would change along the height of the elevator enclosure.

Now we'll write an airflow conservation equation while still assuming velocities u and v to be averaged throughout cross-section and the positive direction of these velocities to coincide with bucket traveling direction:

$$uab = (u + du)ab - w(b - B_l)dx\,, \tag{21}$$

$$vab = (v + dv)ab - w(b - B_l)dx\,. \tag{22}$$

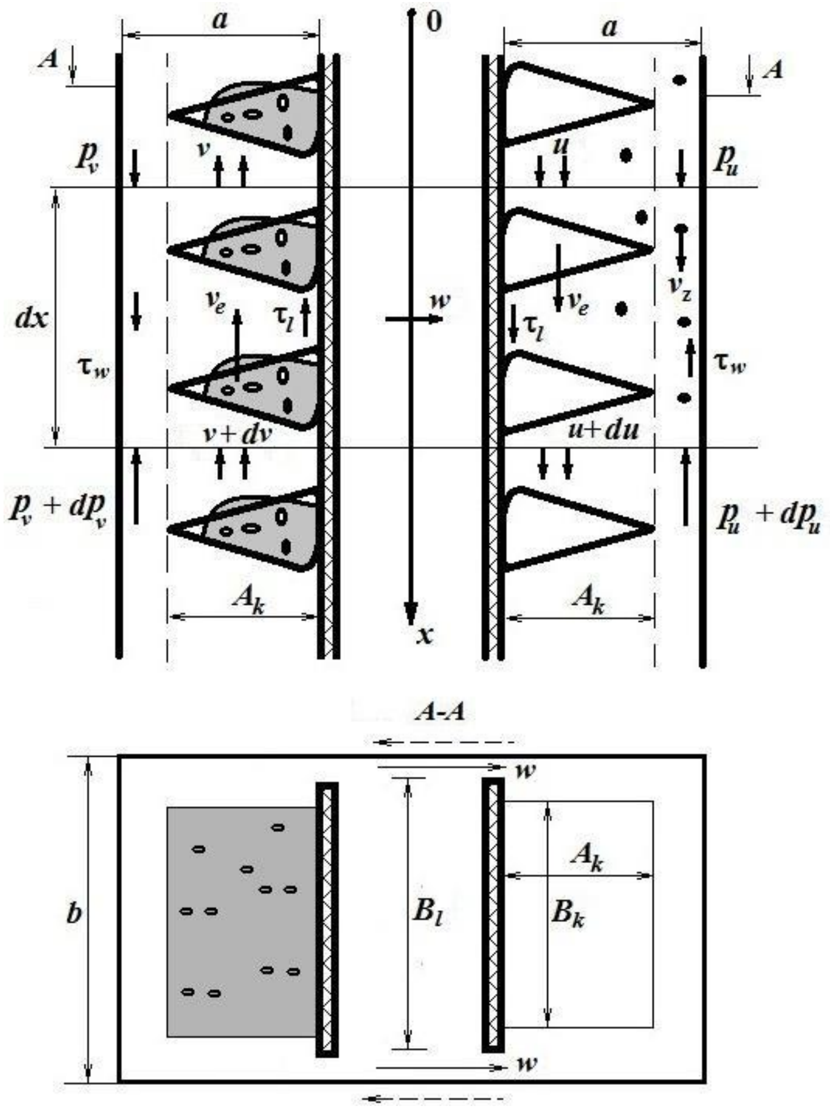

Figure 2: Longitudinal airflow diagram for return and carrying runs both located in a common elevator enclosure

It can be seen from here that change in the absolute value of dilatational velocities is equal:

$$\frac{du}{dx} = w\frac{b - B}{ab}\,, \qquad \frac{dv}{dx} = w\frac{b - B}{ab}\,. \tag{23}$$

And the difference of these velocities does not vary along the enclosure

$$u - v = k = \text{const}\,. \tag{24}$$

Let's now write a motion preservation equation for the chosen element of enclosure of length dx. For the right-hand (downward) airflow the momentum conservation equation in the projection onto axis $0x$ does not differ in any way from equation (5). Trivial transformations would yield the following differential equation for the dynamics of the air current at hand:

$$2\rho u\frac{du}{dx} = -\frac{dp_u}{dx} - \frac{\lambda_w}{D_w}\frac{u^2}{2}\rho + \zeta_u\frac{|v_e - u\,|(v_e - u\,)}{2}\rho + A_n\frac{|v_e - u|(v_e - u\,)}{2v}\rho\,, \tag{25}$$

with the following notational simplification:

$$\zeta_u = \frac{\lambda_l}{4}\frac{B_l}{ab} + c_k\frac{A_k B_k}{abl_k}\,, \tag{26}$$

$$A_n = \frac{1.5}{d_e}\frac{G_p}{\rho_z}\frac{\psi_e}{ab}\,. \tag{27}$$

For the left-hand (upward) airflow we'll first write the momentum preservation equation in differentials:

$$-\rho vvab - \rho(v + dv)ab(-v - dv) = p_v ab - (p_v + dp_v)ab + \tau_w(b + 2a)dx - \tau_l B_l dx - R_k\frac{dx}{l_k}\,, \tag{28}$$

where

$$\tau_w = \frac{\lambda_w}{4}\frac{v|v|}{2}\rho\,; \tag{29}$$

$$\tau_l = \frac{\lambda_l}{4}\frac{(v_e - v)|v_e - v|}{2}\rho\,; \tag{30}$$

$$R_k = c_{kz}A_k B_k\frac{(v_e - v)|v_e - v|}{2}\rho\,; \tag{31}$$

c_{kz} is a dimensionless coefficient of aerodynamic drag of a grain-laden bucket.

After trivial transformations equation (28) would appear as follows:

$$2\rho v\frac{dv}{dx} = -\frac{dp_v}{dx} + \frac{\lambda_w}{D_w}\frac{v^2}{2}\rho - \zeta_v\frac{|v_e - v|(v_e - v)}{2}\rho\,, \tag{32}$$

$$\zeta_v = \frac{1}{ab}\left(\frac{\lambda_l B_l}{4} + c_{kz}\frac{A_k B_k}{l_k}\right). \tag{33}$$

Thus, longitudinal airflow in case of co-location of the carrying and return runs in the same enclosure can be described with combined equations:

$$2\rho u\frac{du}{dx} = -\frac{dp_u}{dx} + f_u\,, \tag{34}$$

$$2\rho v \frac{dv}{dx} = -\frac{dp_v}{dx} + f_v, \tag{35}$$

$$\frac{du}{dx} = w \frac{b - B_l}{ab}; \tag{36}$$

$$v = u - k; \tag{37}$$

$$\Delta p = \zeta_z \frac{w|w|}{2} \rho; \quad \Delta p = p_v - p_u, \tag{38}$$

with the following assignments for brevity:

$$f_u = -\frac{\lambda_w}{D_w} \frac{u|u|}{2} \rho + \zeta_u \frac{|v_e - u|(v_e - u)}{2} \rho + A_n \frac{|v_z - u|(v_z - u)}{2v_z} \rho, \tag{39}$$

$$f_v = \frac{\lambda_w}{D_w} \frac{v|v|}{2} \rho - \zeta_v \frac{|v_e - v|(v_e - v)}{2} \rho. \tag{40}$$

Newly-introduced functions f_u and f_v can be written in a more convenient (symmetric) form. First of all let's assume that spillage velocity is equal to the velocity of the return run (considering that gap between buckets and enclosure walls is rather narrow, particles would first impinge on the bottom of a bucket and then accelerate gravitationally and "catch up" with uniformly moving buckets):

$$v_z \approx v_e. \tag{41}$$

Then,

$$f_u = -\xi_u \frac{\mathrm{T}}{l} \frac{\rho u^2}{2} + \gamma_u \frac{M_u}{l} \frac{\rho (v_e - u)^2}{2}, \tag{42}$$

$$f_v = \xi_v \frac{\mathrm{T}}{l} \frac{\rho v^2}{2} - \gamma_v \frac{M_v}{l} \frac{\rho (v_e - v)^2}{2}, \tag{43}$$

where T, M_u and M_v are dimensionless parameters:

$$\mathrm{T} = \lambda_w \frac{l}{D_w}; \tag{44}$$

$$M_u = \lambda_l \frac{l}{D_l} + c_k \frac{l}{l_k} \frac{A_k B_k}{S} + 1.5\psi \frac{l}{d} \beta_e; \tag{45}$$

$$M_v = \lambda_l \frac{l}{D_l} + c_{kz} \frac{l}{l_k} \frac{A_k B_k}{S}; \tag{46}$$

$$D_w = \frac{4S}{b + 2a};$$

$$D_l = \frac{4S}{B_l}; \tag{47}$$

$$\beta_e = \frac{G_p}{\rho_z v_e S}; \tag{48}$$

$$\xi_u = \text{signum}(u),\ \xi_v = \text{signum}(v) = \text{signum}(u-k); \tag{49}$$

$$\gamma_u = \text{signum}(v_e - u); \quad \gamma_v = \text{signum}(v_e - v) = \text{signum}(v_e + k - u). \tag{50}$$

Combined equations (34 … 38) can be simplified significantly. Cross-flow velocity of a lateral flow can be derived from (38):

$$w = \delta\sqrt{\frac{2|\Delta p|}{\zeta_z \rho}}, \quad \Delta p = p_v - p_u, \tag{51}$$

where

$$\delta = \text{signum}(\Delta p). \tag{52}$$

Substitution of (51) into (36) yields

$$\frac{du}{dx} = \delta \frac{L}{l}\sqrt{\frac{2|\Delta p|}{\rho}}, \tag{53}$$

where

$$L = \frac{l(b-B)}{S\sqrt{\zeta_z}}. \tag{54}$$

By subtracting relation (35) from the equation (34) and considering $\frac{dv}{dx} = \frac{du}{dx}$ in view of (37) we'll get:

$$2\rho k\frac{du}{dx} = \frac{d\Delta p}{dx} - \xi_u \frac{T}{l}\frac{u^2}{2}\rho + \gamma_u \frac{M_u}{l}\frac{(v_e-u)^2}{2}\rho - \xi_v \frac{T}{l}\frac{v^2}{2}\rho + \gamma_v \frac{M_v}{l}\frac{(v_e-v)^2}{2}\rho \tag{55}$$

or, in view of (53),

$$-\frac{d\Delta p}{dx} = -2\delta k\rho \frac{L}{l}\sqrt{\frac{2|\Delta p|}{\rho}} - \xi_x \frac{T}{l}\frac{u^2}{2}\rho + \gamma_u \frac{M_u}{l}\frac{(v_e-u)^2}{2}\rho - \xi_v \frac{T}{l}\frac{v^2}{2}\rho + \gamma_v \frac{M_v}{l}\frac{(v_e-v)^2}{2}\rho. \tag{56}$$

Equation (56) can be supplemented with (34) in view of (42) and (53):

$$\frac{dp_u}{dx} = -2\rho u\delta \frac{L}{l}\sqrt{\frac{2|\Delta p|}{\rho}} - \xi_u \frac{T}{l}\frac{u^2}{2}\rho + \gamma_u \frac{M_u}{l}\frac{(v_e-u)^2}{2}\rho. \tag{57}$$

This, in view of (53), yields a familiar combined set (considering (37)) of three differential equations (57), (56) and (53) describing the process of averaged longitudinal airflows in an enclosure with co-location of the carrying and return runs of bucket elevator inside it.

Changes in the velocity u can be found from (55) using (53) which can be written as follows:

$$\Delta p = A\left(\frac{du}{dx}\right)^2 \frac{\rho}{2}\left(\frac{l}{L}\right)^2, \tag{58}$$

where

$$A = \text{signum}\left(\frac{du}{dx}\right).$$

A substitution of (58) into (55) using the relation (37) results in a 2nd order non-linear equation relative to the sought function u:

$$\begin{gathered} -A\rho\left(\frac{l}{L}\right)^2 \frac{d^2u}{dx^2}\frac{du}{dx} + 2\rho k\frac{du}{dx} = -\xi_u \frac{T}{l}\frac{u^2}{2}\rho + \gamma_u \frac{M_u}{l}\frac{\left(v_e - u\right)^2}{2}\rho - \\ -\xi_v \frac{T}{l}\frac{\left(u-k\right)^2}{2}\rho + \gamma_v \frac{M_v}{l}\frac{\left(v_e + k - u\right)^2}{2}\rho. \end{gathered} \tag{59}$$

We'll use a dimensionless differential equation formula to facilitate our numerical integration of the resulting dimensional equations. It will enable us to reduce the number of constants. The following quantities will be considered as basic values:

– elevator belt velocity v_e

– elevator enclosure length or height l;

– dynamic pressure $\rho v_e^2/2$.

Thus, let

$$p_u = p\frac{v_e^2\rho}{2}; \quad \Delta p = R\frac{v_e^2\rho}{2}; \quad u = u^* v_e; \quad k = m^* v_e; \quad x = zl, \tag{60}$$

then, after we substitute accepted conventions into equations (34 (56) (53) and perform certain trivial transformations, the following system of dimensionless differential equations will result:

$$\frac{dp}{dz} = -\delta u^* 4L\sqrt{|R|} - \xi_u T\left(u^*\right)^2 + \gamma_u M_u\left(1-u^*\right)^2; \tag{61}$$

$$\frac{dR}{dz} = 4\delta m^* L\sqrt{|R|} + \xi_u T\left(u^*\right)^2 - \gamma_u M_u\left(1-u^*\right)^2 + \xi_v T\left(u^* - m^*\right)^2 - \gamma_v M_v\left(1+m^* - u^*\right)^2; \tag{62}$$

$$\frac{du^*}{dz} = \delta L\sqrt{|R|}, \tag{63}$$

where

$$\delta = \text{signum}(R), \quad \xi_u = \text{signum}(u^*), \quad \xi_v = \text{signum}(u^* - m^*), \tag{64}$$

$$\gamma_u = \text{signum}(1-u^*), \quad \gamma_v = \text{signum}(1+m^* - u^*). \tag{65}$$

Similarly, (59) will now appear as:

$$\left(-A_3 \frac{2}{L^2} \frac{d^2 u^*}{dz^2} + 4m^*\right) \frac{du^*}{dz} =$$
$$= -\xi_u T\left(u^*\right)^2 + \gamma_u M_u \left(1-u^*\right)^2 - \xi_v T\left(u^* - m^*\right)^2 + \gamma_v M_v \left(1+m^* - u^*\right)^2, \tag{66}$$

where

$$A_3 = \mathrm{signum}\left(\frac{du^*}{dz}\right). \tag{67}$$

CONCLUSIONS

- Direction of airflow inside enclosures of the carrying and return runs of a bucket elevator is determined by the drag of buckets and moving conveyor belt as well as ejection head created by a stream of spilled particles when buckets are unloaded. As a result of these forces acting together inside an enclosure, differential pressure (10) arises. This differential pressure is equal to the sum total of ejection heads created by conveyor belt with buckets E_k (14) and flow rate of spilled material E_p (17) minus aerodynamic drag of enclosure walls (11).
- The ejection head E_k created by a bucket-carrying conveyor belt is determined by aerodynamic coefficient c_{ek} (16) (proportional to the number of buckets, their head resistances and squared mid-sectional dimensions) together with an absolute value and the direction of bucket velocity relative to the velocity of airflow inside the enclosure.
- Ejection head of spilled particles E_p (19) depends on the drag coefficient of particles, their size and flow rate, as well as the enclosure length, enclosure cross-section and relative flow velocity of particles.
- When both the carrying and return runs of the conveyor belt are located in a common enclosure, the velocity of forward airflow varies over its length as a result of cross-flows of air through gaps between the conveyor runs and enclosure walls. Cross-flows are caused by a differential pressure between the carrying and return run enclosures and is dependent on the drag of the gap (20). Cross-flow direction depends on the ratio between p_v and p_u.
- Given identical size of elevator enclosures, change in absolute values of longitudinal velocities is identical and depends on absolute values of cross-flow velocities and geometrical dimensions of the gap, as well as enclosure cross-section (23, 24). The momentum of longitudinal airflow in this case is determined by variable magnitudes of aerodynamic forces of buckets due to changes in their relative motion velocities.
- The flow rate of air in enclosures may be determined by numerically integrating three dimensionless combined differential equations (61) – (63).
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