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DIFFICULTIES IN IMPLEMENTATION OF VISCOSITY MODELS IN THE FRAGMENTON-BASED VORTEX METHODS

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Abstract. Fragmenton-based lagrangian vortex methods showed their effectiveness for inviscid fluid dynamics problems. However the attempts to extend these methods to viscous flows simulation meet difficulties resulting from nonfulfillment of the Helmholtz theorems of vorticity motion. Direct implementation of the viscosity models used in particle-based vortex methods leads to fragmenton "splitting" problem and accumulation of numerical errors. In this paper we discuss in details the essence of splitting problem on the examples of a classical Particle Strength Exchange (PSE) method and a hybrid DVM-PSE scheme, adapted to a fragmenton-based vortex method.

1 INTRODUCTION

Simulation of viscous fluid with vortex methods has been intensively investigated over the last 40 years. Wide range of approaches has been created to account for the diffusion term in the vorticity evolution equation, starting from the stochastic "random-walk" model of Chorin, particle strength exchange (PSE) [1], diffusion velocity method (DVM) [2], hybrid DVM-PSE schemes [3] and ending with hybrid particle-mesh methods, where the diffusion term is discretized on the mesh [4].

In all mentioned approaches vorticity is discretized over pointwise singular or regularized vortex particles (vortons), somehow distributed in the flow [5]. From the physical point of view this sight is not "natural". Vorticity field is solenoidal and it is best represented with the notion of vortex tubes, which intensity must be conserved in any flow, both viscous and inviscid. Vortex particles are mostly mathematical objects than physical ones and do not constitute a solenoidal field. This may lead to accumulation of approximation errors during the simulation. Though using hybrid particle-mesh approaches one can resolve this issue, the method itself stops being pure lagrangian anymore.

Instead of pointwise particles we consider vortex line fragments (fragmentons) [6]. They can be used either independently or connected into close filaments, making fragmentons also the basis of vortex filament method [7, 8]. Fragmenton-based vortex methods showed to be more effective in some specific inviscid cases in terms of the amount of vortex elements needed for a stable simulation [9]. Pure lagrangian vortex fragmenton methods also showed their effectiveness for the inviscid fluid-structure interaction (FSI) problems [10], where the use of a mesh is undesirable or leads to extensive computational costs.

The good performance of fragmenton-based methods for the simulation of inviscid flows motivates to expand them also to viscous flows. However the direct application of the particle-based viscosity approaches (like PSE or DVM-PSE) to the fragmenton-based methods leads to the problem of "vorticity splitting". This problem appears due to nonfulfilment of the Helmholtz theorems that state that the material lines, initially chosen as vortex lines, stay vortex lines during their evolution in an inviscid flow, that is generally not true for the viscous flows.

It the following sections we give a brief review of the fragmenton vortex method and discuss the implementation of the classical PSE model, DVM and the hybrid DVM-PSE approach suggested by Mycek et al. [3], for the fragmenton-based vortex methods. We explain in detail the essence of the splitting problem and show that the both viscosity concepts lead to excessive splitting in the test problem of vortex oval evolution in viscous fluid.

2 FRAGMENTON-BASED VORTEX METHODS

Consider vorticity evolution equation for a tree-dimensional incompressible viscous fluid without bodies and frontiers

$$\frac{\partial \boldsymbol{\omega}}{\partial t} + (\boldsymbol{V} \cdot \nabla)\,\boldsymbol{\omega} = (\boldsymbol{\omega} \cdot \nabla)\,\boldsymbol{V} + \nu \Delta \boldsymbol{\omega}, \tag{1}$$

where $\boldsymbol{V} = \boldsymbol{V}(\boldsymbol{x}, t)$, $\boldsymbol{\omega} = \nabla \times \boldsymbol{V}$ are velocity and vorticity fields ν – kinematic viscosity.

The classical vortex methods suggest approximate the continuous vorticity field $\boldsymbol{\omega}$ with a set of pointwise vortex particles (vortons):

$$\boldsymbol{\omega}(\boldsymbol{x}, t) \approx \sum_{k=1}^{N} \boldsymbol{\alpha}_k(t) \delta(\boldsymbol{x} - \boldsymbol{x}_k),$$

where δ is the Dirac delta function and $\boldsymbol{\alpha_k}$ is the intensity of k-th vorton, which can be treated as the amount of vorticity concentrated in a point with the position $\boldsymbol{x}_k$.

Fragmenton-based vortex methods approximate the vorticity field with a set of fragments of vortex lines [6]. Each fragment (fragmenton) can be mathematically interpreted as integrals of delta function over the material vector $2\boldsymbol{h}_k$ (fig. 1):

$$\boldsymbol{\omega}(\boldsymbol{x}, t) \approx \sum_{k=1}^{N} \boldsymbol{\gamma}_k \int_{-1}^{1} \delta\big(\boldsymbol{x} - (\boldsymbol{x}_k + s\boldsymbol{h}_k)\big) ds, \tag{2}$$

where $\boldsymbol{\gamma}_k = \gamma_k \boldsymbol{h}_k$, γ_k is the k-th fragmenton scalar intensity, which can be treated as circulation of a vortex filament; $\boldsymbol{x}_k$ is fragmenton's center position (marker). It should be emphasized that $\boldsymbol{\gamma}_k$ must stay collinear to material vector $\boldsymbol{h}_k$ in the simulation. Violating this requirement may lead to stability problems caused by breakdown of the solenoidity of the vorticity field and accumulation of numerical errors.

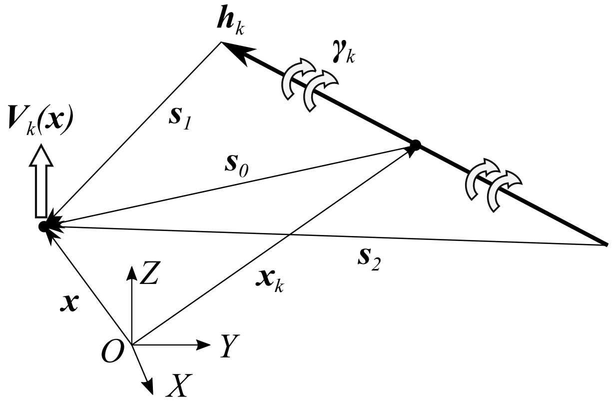

Figure 1: Fragmenton model

Velocity field, induced with the set of fragmentons (2), can be recovered by integrating (2) with the Biot-Savart kernel, that gives [6]

$$\boldsymbol{V}(\boldsymbol{x}, t) \approx \sum_{k=1}^{N} \frac{\gamma_k}{4\pi} \frac{\boldsymbol{h}_k \times \boldsymbol{s}_0}{|\boldsymbol{h}_k \times \boldsymbol{s}_0|^2} \left[\left(\frac{\boldsymbol{s}_2}{|\boldsymbol{s}_2|} - \frac{\boldsymbol{s}_1}{|\boldsymbol{s}_1|} \right) \cdot \boldsymbol{h}_k \right],$$

which is also only true when $\boldsymbol{\gamma}_k$ and $\boldsymbol{h}_k$ are collinear. This formula gives singularities upon reaching the axis of a fragmenton and can be regularized as described in [6].

For the case of inviscid fluid ($\nu = 0$) substitution of (2) into the vorticity evolution equation (1) gives the system of the ODEs for the fragmenton parameters: $\boldsymbol{x}_k$, $\boldsymbol{h}_k$ and $\boldsymbol{\gamma}_k$:

$$\begin{cases} \dfrac{d\boldsymbol{x}_k}{dt} = \boldsymbol{V}(\boldsymbol{x}_k), \\ \dfrac{d\boldsymbol{h}_k}{dt} = \boldsymbol{h}_k \cdot \nabla \boldsymbol{V}(\boldsymbol{x}_k), \\ \dfrac{d\boldsymbol{\gamma}_k}{dt} = \boldsymbol{\gamma}_k \cdot \nabla \boldsymbol{V}(\boldsymbol{x}_k). \end{cases} \tag{3}$$

The last equation of (3) has the same form as the equation for $\boldsymbol{h}_k$ and therefore can equivalently be written as $\frac{d\gamma_k}{dt} = 0$, and $\boldsymbol{\gamma}_k = \gamma_k \boldsymbol{h}_k$.

Analysis of (3) shows that fragmenton markers $\boldsymbol{x}_k$ follow the velocity field $\boldsymbol{V}$. Material vector $\boldsymbol{h}_k$ and intensity vector $\boldsymbol{\gamma}_k$ change their length and direction with the velocity gradient tensor $\nabla \boldsymbol{V}$, while the fragmenton intensities γ_k stay constant in time. This behavior is aligned with the Helmholtz theorems of motion of vortex filaments in inviscid fluid, that state that

- vortex filaments conserve their strength in time;
- vortex filaments move and deform with the material lines that carry them.

The latter statement in terms of fragmenton method means that $\boldsymbol{\gamma}_k$ and $\boldsymbol{h}_k$ always stay collinear in time, which is particularly important to correctly reconstitute velocity field.

Nonzero viscosity ν results in a nonzero diffusion term in the vorticity evolution equation (1) and in nonfulfilment of the Helmholtz theorems. Physically it means that vortex filaments generally do not move and deform as the material lines, that initially carried them. This fact does not cause any problem for the hybrid particle-mesh methods, where the velocity field is found as the solution of Poisson equation [4]. However for pure lagrangian fragmenton-based methods it becomes a challenge in as much as $\boldsymbol{\gamma}_k$, which we associate with vorticity vector, and $\boldsymbol{h}_k$, which we associate with material vector, should stay collinear.

In what follows we call *splitting* the fact of misalignment of the vectors $\boldsymbol{\gamma}_k$ and $\boldsymbol{h}_k$ that potentially may occur in the simulations.

3 VISCOSITY MODELS

The variety of viscosity models created for particle-based vortex methods are all referred to the means to simplify or approximate the diffusion term $\nu\Delta\boldsymbol{\omega}$ [5, 3]. Direct implementation of these approaches to the fragmenton-based methods results in modification of the evolution equations for $\boldsymbol{h}_k$ and $\boldsymbol{\gamma}_k$ of (3) that may cause splitting, as these equations loose their symmetrical form.

Here we discuss three approaches to account for the diffusion term in particle-based vortex methods, applying them for fragmenton-based methods: Particle Strength Exchange, Diffusion Velocity Method and hybrid DVM-PSE scheme proposed by Mycek et al. [3].

3.1 Particle strength exchange

The PSE method was suggested by Degond & Mas-Gallic in [1] who gave its profound analysis in application to numerical solution of advection-diffusion equations with particle methods. The idea of the PSE method is approximation of the diffusion term $\nu\Delta\boldsymbol{\omega}$ with the integral operator of the form

$$Q^{\epsilon}(\boldsymbol{\omega}) = \frac{\nu}{\epsilon^2}\int_{\mathbb{R}^3} \eta_{\epsilon}(\boldsymbol{x}-\boldsymbol{y})(\boldsymbol{\omega}(\boldsymbol{y},t)-\boldsymbol{\omega}(\boldsymbol{x},t))dy, \tag{4}$$

where $\eta_{\epsilon}(\boldsymbol{x}) = \frac{1}{\epsilon^3}\eta\left(\frac{\boldsymbol{x}}{\epsilon}\right)$; $\eta(\boldsymbol{x}) \in L^1(\mathbb{R}^3)$ is kernel function that must satisfy several moment conditions in order to make $Q^{\epsilon}(\boldsymbol{\omega})$ converge towards $\nu\Delta\boldsymbol{\omega}$ in certain norms when $\epsilon \to 0$. Details on the kernel function choice can be found in [1].

Replacing $\nu\Delta\boldsymbol{\omega}$ with the integral operator (4) and applying the fragmenton approximation (see [11] for details) we get the following set of the ODEs

$$\begin{cases} \dfrac{d\boldsymbol{x}_k}{dt} = \boldsymbol{V}(\boldsymbol{x}_k), \\ \dfrac{d\boldsymbol{h}_k}{dt} = \boldsymbol{h}_k \cdot \nabla\boldsymbol{V}(\boldsymbol{x}_k), \\ \dfrac{d\boldsymbol{\gamma}_k}{dt} = \boldsymbol{\gamma}_k \cdot \nabla\boldsymbol{V}(\boldsymbol{x}_k) + \dfrac{\nu}{\epsilon^2}\displaystyle\sum_{q=1}^{N} G_{kq}(\boldsymbol{\gamma}_q S_k - \boldsymbol{\gamma}_k S_q), \end{cases} \tag{5}$$

where S_k is the cross-section area of the vortex tube, associated to the k-th fragmenton at initial time; $S_k = \sigma_k/|2\boldsymbol{h}_k|$, where σ_k is fragmenton's volume that stays constant in time; G_{kq} is the exchange coefficient between the k-th and q-th fragmentons, which depends on their length and mutual orientation:

$$G_{kq} = \int_{-1}^{1}\int_{-1}^{1} \eta_\epsilon(x_k + \tau\boldsymbol{h}_k - \boldsymbol{x}_q - s\boldsymbol{h}_q)dsd\tau.$$

Comparison of (5) and (3) shows that the equations for $\boldsymbol{x}_k$ and $\boldsymbol{h}_k$ stay without change comparing to the equations for $\boldsymbol{x}_k$ and $\boldsymbol{h}_k$ for ideal fluid. The equation for $\boldsymbol{\gamma}_k$ obtained the additional term, responsible for the exchange of intensities between k-th and q-th fragmentons. In two-dimensional problems this additional term causes no splitting as $\boldsymbol{h}_k$ and $\boldsymbol{\gamma}_k$ have the only nonzero component in the direction normal to the symmetry plane, thus they always stay collinear. However for an arbitrary three-dimensional flow this is not true.

3.2 Diffusion velocity method

The general idea of the diffusion velocity method is to find such vector field $\boldsymbol{U}$ that can be considered as the "virtual" velocity field that transfers vortex tubes in a viscous fluid in a way that the Helmholtz theorems are valid. According to the Fridman theorem [12] such $\boldsymbol{U}$ must satisfy the equation

$$\frac{\partial\boldsymbol{\omega}}{\partial t} + (\boldsymbol{U}\cdot\nabla)\,\boldsymbol{\omega} - (\boldsymbol{\omega}\cdot\nabla)\,\boldsymbol{U} + \boldsymbol{\omega}\nabla\cdot\boldsymbol{U} = 0 \tag{6}$$

or, which is the same,

$$\frac{\partial\boldsymbol{\omega}}{\partial t} + \nabla\times(\boldsymbol{\omega}\times\boldsymbol{U}) = 0.$$

In this view there arise two principle questions:

1. Does such vector field $\boldsymbol{U}$ exist for an arbitrary three-dimensional viscous flow?

2. If yes, are there any practical ways to find this field? In that case the splitting problem would be totally fixed.

Markov and Sizykh proved in [13] that for any elementary fragment of a vortex tube there always exist such field $\boldsymbol{U}$, called *Fridman velocity*. Moreover, this field is not unique and is given by

$$\boldsymbol{U} = \boldsymbol{V} + \frac{\boldsymbol{\omega}\times(-\nu\nabla\times\boldsymbol{\omega} - \nabla f + \nabla W)}{\boldsymbol{\omega}^2} + \gamma\boldsymbol{\omega},$$

where W is arbitrary scalar field, constant along vortex filaments, γ – arbitrary scalar field, f is any scalar function satisfying the condition

$$\boldsymbol{\omega}\cdot\nabla f = -\nu\boldsymbol{\omega}\cdot(\nabla\times\boldsymbol{\omega}). \tag{7}$$

Determination of function f from the condition (7) requires integration along the vortex filaments, thus making the numerical algorithm of search of $\boldsymbol{U}$ very impractical. Another contribution in the problem of the practical ways to find Fridman velocity is given in [14]. It is demonstrated on several examples of swirling flows, that there exist such flows for which there are no local expressions of Fridman velocity $\boldsymbol{U}$, i.e. at any point $\boldsymbol{U}$ cannot be expressed through the flow parameters in the infinitesimal neighborhood of this point. This result, conceivably, concludes the discussions and gives the negative answer on the second posed question for an arbitrary tree-dimensional viscous flow.

For two-dimensional or axisymmetrical flows without swirling determination of Fridman velocity does not require integration along the vortex filaments. For the 2D-flows it reduces to a simple expression

$$\boldsymbol{U} = \boldsymbol{V} + \boldsymbol{V}_d,$$

where

$$\boldsymbol{V}_d = -\nu \frac{\nabla |\boldsymbol{\omega}|}{\boldsymbol{\omega}^2}$$

is traditionally called *diffusion velocity.*

Therefore for two-dimensional viscous flows, vorticity evolution equation (1) can be reformulated in terms of diffusion velocity $\boldsymbol{V}_d$ as follows

$$\frac{\partial \boldsymbol{\omega}}{\partial t} + \left((\boldsymbol{V} + \boldsymbol{V}_d) \cdot \nabla \right) \boldsymbol{\omega} = 0. \tag{8}$$

The fragmenton approximation of (8) gives the following system of ODEs:

$$\begin{cases} \dfrac{d\boldsymbol{x}_k}{dt} = (\boldsymbol{V} + \boldsymbol{V}_d)(\boldsymbol{x}_k), \\ \dfrac{d\boldsymbol{h}_k}{dt} = 0, \\ \dfrac{d\boldsymbol{\gamma}_k}{dt} = 0. \end{cases}$$

Here fragmenton markers $\boldsymbol{x}_k$ follow the Fridman velocity field $\boldsymbol{V} + \boldsymbol{V}_d$, while their intensities $\boldsymbol{\gamma}$ stay constant unlike in the PSE approach, where they constantly change. The DVM became primary viscosity model used in 2D vortex methods like VVD [15, 16] and 2D-codes based on it [17].

3.3 Hybrid DVM-PSE scheme

Although the idea of diffusion velocity is not directly applicable for an arbitrary 3D-flow, one can use it in a hybrid approach where in some sense "dominating" part of the diffusion term is simulated with the DVM, while the resting part is treated with the PSE method. This decomposition can be done in different ways.

Mycek et al. [3] suggested decompose the diffusion tensor $\nu \nabla \boldsymbol{\omega}$ into convective tensor $-\boldsymbol{V}_d \otimes \boldsymbol{\omega}$ and residual tensor $\hat{B}$:

$$\nu \nabla \boldsymbol{\omega} = -\boldsymbol{V}_d \otimes \boldsymbol{\omega} + \hat{B}. \tag{9}$$

Substitution of (9) into (1) gives

$$\frac{\partial \boldsymbol{\omega}}{\partial t} + \nabla \cdot ((\boldsymbol{V} + \boldsymbol{V}_d) \otimes \boldsymbol{\omega}) = (\boldsymbol{\omega} \cdot \nabla) \boldsymbol{V} + \nabla \cdot \hat{B}$$

or

$$\frac{\partial \boldsymbol{\omega}}{\partial t} + ((\boldsymbol{V} + \boldsymbol{V}_d) \cdot \nabla) \boldsymbol{\omega} = (\boldsymbol{\omega} \cdot \nabla) \boldsymbol{V} - \boldsymbol{\omega} (\nabla \cdot (\boldsymbol{V} + \boldsymbol{V}_d)) + \nabla \cdot \hat{B}. \tag{10}$$

$\boldsymbol{V}_d$ is chosen in a way to minimize the components of $\hat{B}$ in a least-squares manner (see [3] for details) and is found to be

$$\boldsymbol{V}_d = -\nu \frac{\nabla |\boldsymbol{\omega}|}{|\boldsymbol{\omega}|},$$

and the tensor $\hat{B}$ takes the form convenient to be approximated with the PSE integral operator:

$$\hat{B} = \nu |\boldsymbol{\omega}| \left(\nabla \frac{\boldsymbol{\omega}}{|\boldsymbol{\omega}|} \right).$$

Though the authors of [3] call $\boldsymbol{V}_d$ *diffusion velocity*, strictly speaking, it is unfortunate naming, because vorticity deformation term $(\boldsymbol{\omega} \cdot \nabla)\boldsymbol{V}$ in (10) does not have additional $\boldsymbol{V}_d$ that is required according to the Fridman's theorem (6). For the particle-based vortex methods this is not a principle problem, since the main aim of such decomposition is transformation of the diffusion term $\nu \Delta \boldsymbol{\omega}$ in a convenient way and not the treatment of the vorticity splitting problem. Applying fragmenton approximation to (10) we get the following ODE system

$$\begin{cases} \dfrac{d\boldsymbol{x}_k}{dt} = (\boldsymbol{V} + \boldsymbol{V}_d)(\boldsymbol{x}_k), \\ \dfrac{d\boldsymbol{h}_k}{dt} = \boldsymbol{h}_k \cdot \nabla (\boldsymbol{V} + \boldsymbol{V}_d)(\boldsymbol{x}_k), \\ \dfrac{d\sigma_k}{dt} = \sigma_k \nabla \cdot (\boldsymbol{V} + \boldsymbol{V}_d)(\boldsymbol{x}_k), \\ \dfrac{d\boldsymbol{\gamma}_k}{dt} = \boldsymbol{\gamma}_k \cdot \nabla \boldsymbol{V}(\boldsymbol{x}_k) + \dfrac{\nu}{\epsilon^2} \displaystyle\sum_{q=1}^{N} G_{kq} \dfrac{|\boldsymbol{\gamma}_q| S_k + |\boldsymbol{\gamma}_k| S_q}{2} \left[\dfrac{\boldsymbol{\gamma}_q}{|\boldsymbol{\gamma}_q|} - \dfrac{\boldsymbol{\gamma}_k}{|\boldsymbol{\gamma}_k|} \right], \end{cases} \tag{11}$$

where $\sigma_k = |2\boldsymbol{h}_k| S_k$ is the k-th fragmenton's volume.

As for the PSE scheme, the equations for $\boldsymbol{h}_k$ and $\boldsymbol{\gamma}_k$ in (11) loose their symmetrical form and cause splitting. It should be also emphasized that $\boldsymbol{h}_k$ and $\boldsymbol{\gamma}_k$ have different deformation tensors: $\nabla(\boldsymbol{V} + \boldsymbol{V}_d)$ and $\nabla \boldsymbol{V}$ respectively.

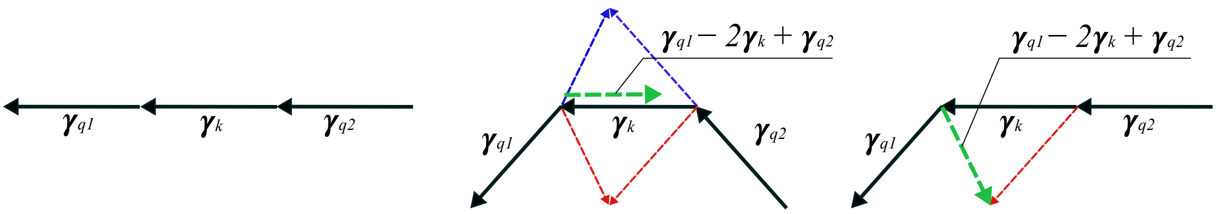

Figure 2: Three fragmenton models and splitting problem

4 "SPLITTING" PROBLEM AND SIMULATION EXAMPLES

The splitting problem that occurs with the PSE and DVM-PSE viscosity models, adapted to the fragmenton-based vortex methods, can be exposed on a simple model. Consider three fragmentons, connected to each other in three different configurations (fig. 2). Every configuration may come up in a real simulation with the vortex filament method, where these fragmentons form a part of a filament. The first case on fig. 2 is associated with the plane-parallel motion (rectilinear vortex tube), the second one — with the axisymmetrical motion (vortex ring) and the third case can be associated with the more general case of 3D-motion, where a rectilinear part merges into a curvilinear part (p.e. a part of a vortex oval).

Let us examine each of suggested configurations in terms of splitting of the vectors $\boldsymbol{h}_k$ and $\boldsymbol{\gamma}_k$ in the PSE and DVM-PSE models. For every fragmenton configuration on fig. 2 we consider $|\boldsymbol{h}_k| = |\boldsymbol{h}_{q1}| = |\boldsymbol{h}_{q2}| = h$, $|\boldsymbol{\gamma}_k| = |\boldsymbol{\gamma}_{q1}| = |\boldsymbol{\gamma}_{q2}| = \gamma$, $S_k = S_{q1} = S_{q2} = S$ and $G_{kq1} = G_{kq2} = G$[1] so that the terms responsible for splitting for k-th fragmenton in the PSE (5) and DVM-PSE (11) models simplify as follows:

$$R_P = \sum_{q=\{q1,q2\}} G_{kq}(\boldsymbol{\gamma}_q S_k - \boldsymbol{\gamma}_k S_q) = GS(\boldsymbol{\gamma}_{q1} - 2\boldsymbol{\gamma}_k + \boldsymbol{\gamma}_{q2}),$$

$$R_{DP} = \sum_{q=\{q1,q2\}} G_{kq}\frac{|\boldsymbol{\gamma}_q|S_k + |\boldsymbol{\gamma}_k|S_q}{2}\left[\frac{\boldsymbol{\gamma}_q}{|\boldsymbol{\gamma}_q|} - \frac{\boldsymbol{\gamma}_k}{|\boldsymbol{\gamma}_k|}\right] = GS(\boldsymbol{\gamma}_{q1} - 2\boldsymbol{\gamma}_k + \boldsymbol{\gamma}_{q2}).$$

As we see, for these particular fragmenton configurations both PSE and DVM-PSE models give the same result for the term $R = R_P = R_{DP}$, responsible for splitting of the central k-th fragmenton.

Analyzing each case separately we see that for the first configuration splitting does not occur as long as $\boldsymbol{\gamma}_{q1} - 2\boldsymbol{\gamma}_k + \boldsymbol{\gamma}_{q2} = 0$ for the k-th fragmenton. For the second configuration on fig. 2 R is nonzero, but it is collinear to $\boldsymbol{\gamma}_k$ and causes decrease of $|\boldsymbol{\gamma}_k|$ without change of its direction (i.e. no splitting). For the third case R tends to change not only the length of $\boldsymbol{\gamma}_k$, but also its orientation, causing splitting.

The first and the second layouts on fig. 2 are the simplified prototypes of plane-parallel and axisymmetrical motions for which both PSE and DVM-PSE models work well, cause no splitting and can be used without problems [18]. But for more general 3D-motion the possible influence of splitting should be analyzed in more proper simulations.

[1]Strictly speaking, for the third case $G_{kq1} \neq G_{kq2}$, but we can consider here $G_{kq1} \approx G_{kq2}$ without loss of generality

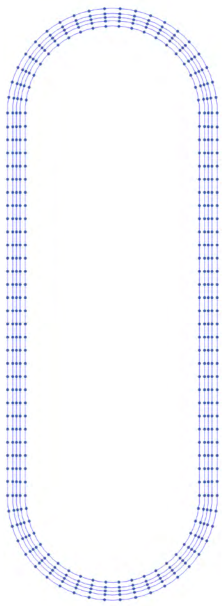

Figure 3: Vortex oval

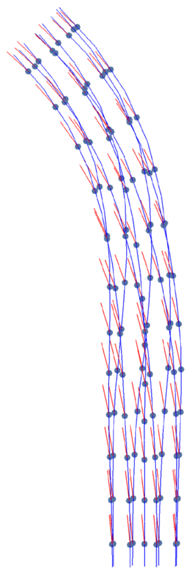

Figure 4: Splitting with the PSE model

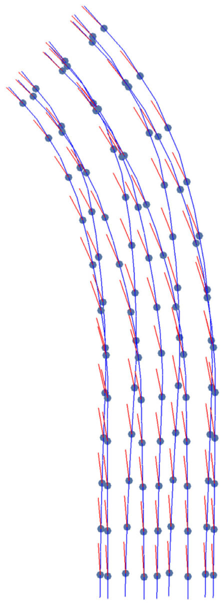

Figure 5: Splitting with the DVM-PSE model

Consider vortex oval with length to width ratio $L/D = 3$ with $D = 2$ (fig. 3). Oval consists in 9 filaments, each divided into 100 fragmentons. Filaments form the oval's core of radius 0.1 and have same intensity so that the oval's overall circulation $\Gamma_0 = 1$. Viscosity $\nu = 0.01$ and Reynolds number is $Re = \Gamma_0/\nu = 100$, PSE cut-off parameter $\epsilon = 0.1$. The problem was simulated with the PSE and DVM-PSE viscosity models, using explicit first order Euler scheme for integrating ODEs (5) and (11) with time step $\Delta t = 0.001$ up to $T = 1$.

At the time T the oval is only on its initial development stage where the circular parts begin rolling over out of the oval's plane, but the fragmentons' splitting is already clearly seen for both models (figures 4 and 5). Fragmentons, that initially formed continuous filaments are now completely separated, though material vectors $\boldsymbol{h}_k$ still form closed structures. This leads to incorrect velocity reconstitution and to imminent breakdown during the following simulation of the oval due to accumulated numerical errors.

5 CONCLUSIONS

Fragmenton-based vortex methods showed to be promising in the simulations of inviscid flows, especially when the use of mesh is inconvenient. Intentions to extend these methods to viscous fluid simulation encouraged the development and adaptation of viscosity models from existing particle-based approaches to the fragmenton-based ones.

The main principal difficulty has physical reasoning that consists in nonfulfillment of the Helmholtz laws for viscous fluid with vorticity, which are crucial for fragmenton-based methods. Every existing viscosity model aimed onto the approximation and discretization between the particles of the diffusion term $\nu\Delta\boldsymbol{\omega}$, faces the problem of misalignment of vorticity vector and material vector that we call "splitting".

Numerical analysis and simulation examples of a vortex oval demonstrate the splitting problem for the classical PSE approach and the DVM-PSE scheme proposed by Mycek et al. Though 3D diffusion velocity could be a solution for the stated problem, its search is an ill-posed problem and appears to be very impractical for the fragmenton-based methods.

It should also be mentioned about the problems with the general use of PSE-approximation with fragmenton-based methods, as PSE technics needs particles (or fragmentons) to be positioned uniformly, i.e. on the mesh, to maintain approximation accuracy. It is rather difficult to achieve with fragmentons, as they must always form closed structures in a flow.

One should admit that accurate simulation of viscous flow without any assumptions with the fragmenton-based vortex methods seems to be improbable. However, in the sight of the advantageous use of these methods for complicated inviscid FSI problems [10], efforts should be made to the search and elaboration of hybrid DVM-PSE viscosity models that minimize splitting. The second way is to analyze the "no-splitting" assumption, where splitting is suppressed. In this case the DVM part would play the dominating role, and it should be chosen wisely. These problems are the questions of the following authors' research.

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