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HOMOGENIZATION AND MODELING OF FIBER STRUCTURED MATERIALS

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Abstract. For the mechanical modeling and simulation of the heterogeneous composition of a fiber structured material, the material properties at the micro level and the contact between the fibers have to be taken into account. The material behavior is strongly influenced by the material properties of the fibers, but also by their geometrical arrangement. In consideration of the different length scales the problem involves, it is necessary to introduce a multi scale approach based on the concept of a representative volume element (RVE). For planar structures like technical textiles the macromodel is discretized by shell elements. In contrast the microscopic RVE is modeled with three dimensional elements to account for the contact between the fibers. The macro-micro scale transition requires a method to impose the deformation at a macroscopic point onto the RVE by suited boundary conditions. The reversing scale transition, based on the Hill-Mandel condition, requires the equality of the macroscopic average of the variation of work on the RVE and the local variation of the work on the macroscale. For the micro-macro transition the averaged forces and the resulting moments have to be extracted by a homogenization scheme. From these results an effective constitutive law can be derived.

1 INTRODUCTION

The sector of technical textiles is expanding because of a variety of applications of recently developed fiber shapes, materials and structures. Innovative potential and economic growth make technical textiles to an important research area. Examining textiles it is obvious that on the macro scale textiles show inhomogeneous material properties, which are different from the underlying fiber material. These nonlinearities rely on interactions in between the fibers on the micro level and the friction in the contact areas. The determination of phenomenological constitutive laws for this material group by classical material testing procedures is very time and cost intensive. Therefore numerical methods to determine the material characteristics of textiles are developed. Apparently discretization of the whole macro structures modeled in micro scale element dimensions leads to systems with too many degrees of freedom. So for the macro- and microscopic consideration multi scale methods, so called FE^2-methods are introduced. Hence each macroscopic material point is assigned a microscopic, heterogeneous Representative Volume Element (RVE). For a simultaneous calculation a relation between microscopic and macroscopic scale has to be derived. From the deformations on macro scale boundary conditions for the micro calculation were developed. For the reversing scale transition a homogenization scheme is introduced.

2 TOWARDS FIBER STRUCTURED MATERIALS

Technical textiles are considered to be all textile products with an application in the technical sector. Thereby they can be used in architecture, agriculture or engineering. Textiles are flexible materials, which consist of particular fibers where a fiber has a high length to diameter ratio. In this paper, without loss of generality, a fiber is assumed to consist of homogeneous material and to be characterized by a circular cross section.

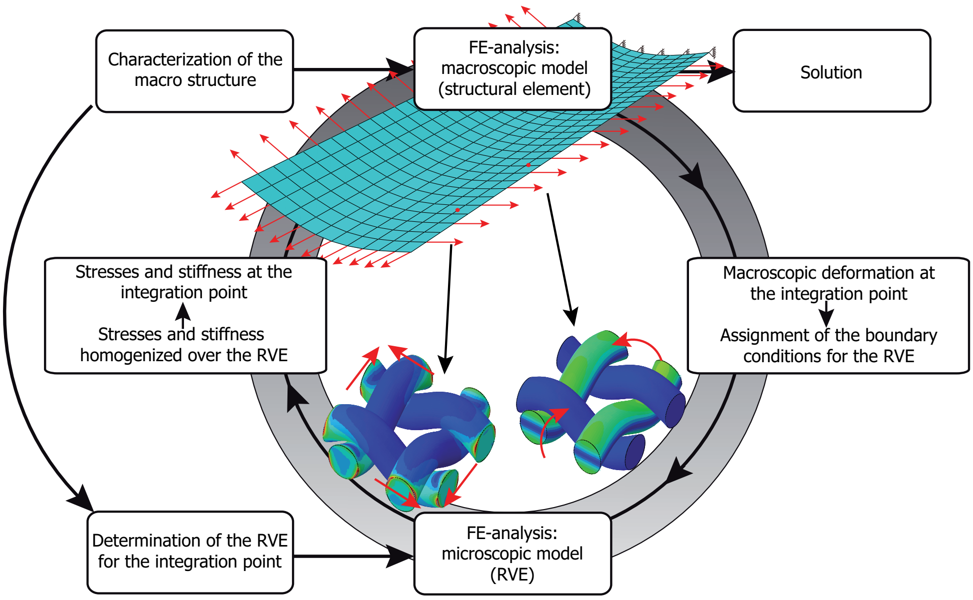

Figure 1: Towards a FE^2-scheme

The mechanical properties are closely linked to the manufactured materials. Generally one can differentiate between natural and chemical fibers. Polymers as well as glass, ceramic, carbon and metal fibers belong to the last-mentioned ones. Because of growing ecological awareness there is a trend towards natural fibers like hemp and jute. They find application in architecture and as geological and agricultural textiles. For textile behavior material properties and volume fraction are as important as the assembly of the fibers [1]. On the one hand there are periodic assemblies like woven or knitted structures, on the other hand there are random ones like felt.

In the following, technical textiles are examined in a multi scale homogenization scheme, which is different to the classical first order approach. A requirement of the first order homogenization is that the macroscopic and the microscopic length scales differ in dimension [2]. This is achieved for textiles for the in plane direction, but in thickness direction the lengths are considered the like. For that reason no classical homogenization can be accomplished in this direction. Textiles are rather assumed to be shells on the macroscopic level. Intrinsically, shell problems are second order homogenization problems because beside stretching and shearing, bending and twisting of the shell are considered.

3 SHELL-KINEMATICS

For the description of a shell continuum $\mathcal{B}_0$ in the material configuration and $\mathcal{B}_t$ in the spatial configuration in a three dimensional space, two coordinate systems are introduced [5]. One cartesian system x_i with the orthonormal vector $\boldsymbol{E}_i$ and one curvilinear, convective system θ_i that is connected to the middle surface $\mathcal{M}_0$ and $\mathcal{M}_t$ of the shell. For the notations it is introduced that Latin indices range from 1 to 3 and Greek indices range from 1 to 2. A description of a finite deformation shell is given by the equations

$$\begin{aligned} \boldsymbol{X}(\theta^i) &= \bar{\boldsymbol{X}}(\theta^\alpha) + \theta^3 \boldsymbol{D}(\theta^\alpha) \quad \text{and} \\ \boldsymbol{\varphi}(\theta^i) &= \bar{\boldsymbol{\varphi}}(\theta^\alpha) + \theta^3 \lambda(\theta^\alpha) \boldsymbol{d}(\theta^\alpha) \ , \end{aligned} \tag{1}$$

where $\boldsymbol{X}$ is the position vector of a material point in the undeformed shell and $\boldsymbol{\varphi}$ is the position vector of a material point in the deformed shell. Streching in thickness direction is neglected $\lambda(\theta^\alpha) = 1$.

The vectors $\bar{\boldsymbol{X}}$ and $\bar{\boldsymbol{\varphi}}$ provide a parametric representation of the middle surface of the shell in the reference and the current state. The parameter $\theta^3 \in [\frac{-h_0}{2}, \frac{h_0}{2}]$ determines the position of a point normal to the middle surface in the undeformed state.

All kinematic values can be calculated, if the shell geometry in the material and the spatial configuration are known. For this the covariant basis vectors on the middle surface of the shell can be derived from the partial derivative of the material vector $\bar{\boldsymbol{X}}$ and the spatial vector $\bar{\boldsymbol{\varphi}}$ with respect to the curvilinear coordinates

$$\begin{aligned} \boldsymbol{A}_\alpha &= \frac{\partial \bar{\boldsymbol{X}}}{\partial \theta^\alpha} = \bar{\boldsymbol{X}}_{,\alpha} \quad , \qquad \boldsymbol{A}_3 = \boldsymbol{D} \ , \\ \boldsymbol{a}_\alpha &= \frac{\partial \bar{\boldsymbol{\varphi}}}{\partial \theta^\alpha} = \bar{\boldsymbol{\varphi}}_{,\alpha} \qquad \text{and} \qquad \boldsymbol{a}_3 = \boldsymbol{d} \ . \end{aligned} \tag{2}$$

The unit director $\boldsymbol{D}$, which is normal to the shell in the material configuration is given by

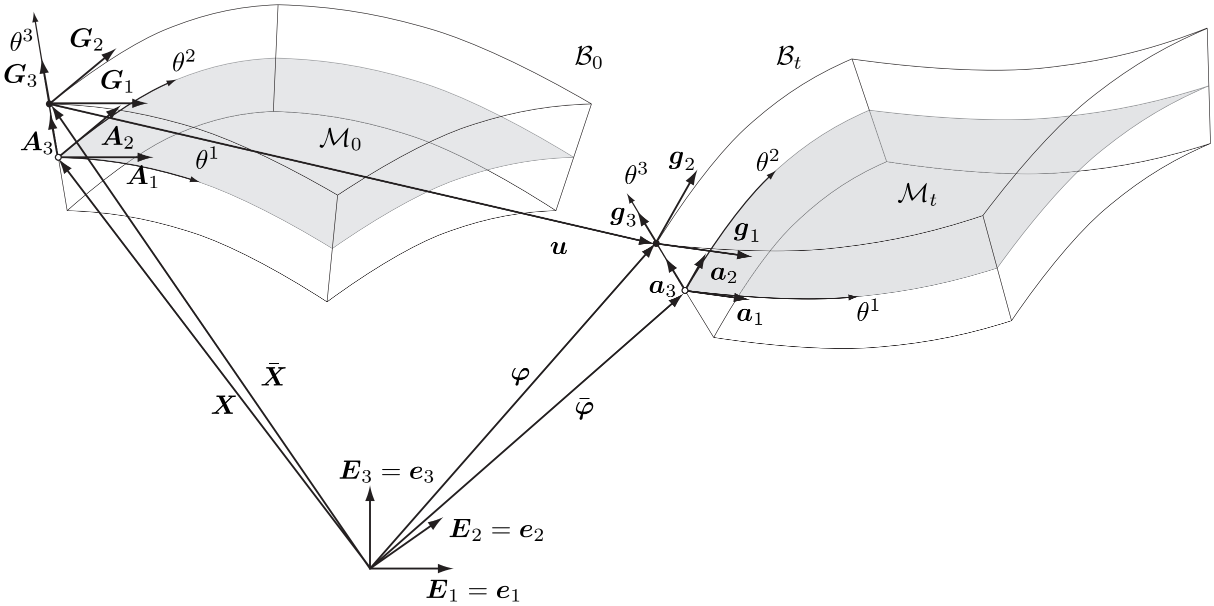

Figure 2: Towards a FE^2-scheme

$$\boldsymbol{D} = \frac{\boldsymbol{A}_1 \times \boldsymbol{A}_2}{|\boldsymbol{A}_1 \times \boldsymbol{A}_2|} \ . \tag{3}$$

For the displacement $\boldsymbol{u}(\theta^i)$ of a point from the reference to the current state it reads

$$\boldsymbol{u}(\theta^i) = \bar{\boldsymbol{\varphi}}(\theta^\alpha) - \bar{\boldsymbol{X}}(\theta^\alpha) + \theta^3(\boldsymbol{a}_3 - \boldsymbol{A}_3). \tag{4}$$

For the macro to micro scale transition the shell formulation has to be extended to a three dimensional formulation [6]. From equation (1) covariant basis vectors for the description of the shell body can be derived to

$$\boldsymbol{G}_i = \frac{\partial \boldsymbol{X}}{\partial \theta^i} \qquad \text{and} \qquad \boldsymbol{g}_i = \frac{\partial \boldsymbol{x}}{\partial \theta^i} \ , \tag{5}$$

with the connection to the covariant shell basis vectors

$$\begin{aligned}
\boldsymbol{G}_\alpha &= \frac{\partial \boldsymbol{X}}{\partial \theta^\alpha} = \frac{\partial \bar{\boldsymbol{X}}}{\partial \theta^\alpha} + \theta^3 \frac{\partial \boldsymbol{D}}{\partial \theta^\alpha} = \boldsymbol{A}_\alpha + \theta^3 \boldsymbol{A}_{3,\alpha} \ , \\
\boldsymbol{G}_3 &= \frac{\partial \boldsymbol{X}}{\partial \theta^3} = \boldsymbol{A}_3 \ , \\
\boldsymbol{g}_\alpha &= \frac{\partial \boldsymbol{\varphi}}{\partial \theta^\alpha} = \frac{\partial \bar{\boldsymbol{\varphi}}}{\partial \theta^\alpha} + \theta^3 \frac{\partial \boldsymbol{a}_3}{\partial \theta^\alpha} = \boldsymbol{a}_\alpha + \theta^3 \boldsymbol{a}_{3,\alpha} \ , \\
\boldsymbol{g}_3 &= \frac{\partial \boldsymbol{\varphi}}{\partial \theta^3} = \boldsymbol{a}_3 \ .
\end{aligned} \tag{6}$$

The contravariant basis vectors result from the relation

$$\boldsymbol{G}^i \cdot \boldsymbol{G}_j = \delta^i_j \qquad \text{and} \qquad \boldsymbol{g}^i \cdot \boldsymbol{g}_j = \delta^i_j \;, \tag{7}$$

with the Kronecker delta δ^i_j. For the deformation map the deformation gradient $\boldsymbol{F}$ is introduced

$$\boldsymbol{F} = \frac{\partial \boldsymbol{\varphi}}{\partial \boldsymbol{X}} = \frac{\partial \boldsymbol{\varphi}}{\partial \theta^i} \otimes \frac{\partial \theta^i}{\partial \boldsymbol{X}} = \boldsymbol{g}_i \otimes \boldsymbol{G}^i \;. \tag{8}$$

Due to the shell kinematics in equation (1) reads

$$\boldsymbol{F} = [\boldsymbol{a}_\alpha + \theta^3 \boldsymbol{a}_{3,\alpha}] \otimes \boldsymbol{G}^\alpha + \boldsymbol{a}_3 \otimes \boldsymbol{G}^3 \;. \tag{9}$$

For a Kirchhoff-Love shell the director $\boldsymbol{d}$ in the current state is also normal to the middle surface

$$\boldsymbol{d} = \frac{\boldsymbol{a}_1 \times \boldsymbol{a}_2}{|\boldsymbol{a}_1 \times \boldsymbol{a}_2|} \;. \tag{10}$$

Therefore, the contribution of the transverse shear is neglected. Based on that assumptions only the in plane components of the deformation gradient are necessary to be taken into account, with the projection on the middle surface

$$\widehat{\boldsymbol{a}} = \boldsymbol{a} \cdot (\boldsymbol{I} - \boldsymbol{a}_3 \otimes \boldsymbol{a}_3) \;, \tag{11}$$

with the second order unit tensor $\boldsymbol{I}$. With that the in plane deformation gradient $\widehat{\boldsymbol{F}}$ reads

$$\widehat{\boldsymbol{F}} = \widehat{\boldsymbol{H}} + \theta^3 \widehat{\boldsymbol{K}} \;, \tag{12}$$

with

$$\widehat{\boldsymbol{H}} = \boldsymbol{a}_\alpha \otimes \boldsymbol{G}^\alpha \qquad \text{and} \qquad \widehat{\boldsymbol{K}} = \boldsymbol{a}_{3,\alpha} \otimes \boldsymbol{G}^\alpha \;. \tag{13}$$

4 SCALE-TRANSITIONS

The approach to create a boundary value problem for a mircostructural RVE requires to consider the shell deformation gradient derived in equation (12) to be the macroscopic gradient $\widehat{\boldsymbol{F}}_M$ of the multi scale analysis [3]. Furthermore the macroscopic deformation gradient has to be equal to the volume average of the microscopic deformation gradient $\widehat{\boldsymbol{F}}_m$. The index $\boldsymbol{X}_m$ is connected to quantities on the micro structure and the index $\boldsymbol{X}_M$ is assigned to quantities on the macro structure. For the homogenization the position of a point in the microscopic RVE is given by

$$\widehat{\boldsymbol{\varphi}}_m = \widehat{\boldsymbol{F}}_M \cdot \boldsymbol{X}_m + \widehat{\boldsymbol{\omega}}(\boldsymbol{X}_m) \;, \tag{14}$$

where

$$\widehat{\boldsymbol{F}}_M = \frac{1}{V_0} \int\limits_{\mathcal{B}_{0m}} \widehat{\boldsymbol{F}}_m dV_m = \frac{1}{V_0} \int\limits_{\partial\mathcal{B}_{0m}} \widehat{\boldsymbol{\varphi}}_m \otimes \boldsymbol{N}_m dA_m \ , \tag{15}$$

with the micro fluctuation $\widehat{\boldsymbol{\omega}}(\boldsymbol{X}_m)$ and the outward normal in the material configuration $\boldsymbol{N}_m$. Inserting equation (14) in (15) leads to the condition for the micro fluctuation field

$$\int\limits_{\partial\mathcal{B}_{0m}} \widehat{\boldsymbol{\omega}}(\boldsymbol{X}_m) \otimes \boldsymbol{N}_m dA_m = \boldsymbol{0} \ . \tag{16}$$

This equation is a requirement for the boundary conditions of the RVE and a possibility for the realization are periodic ones. For this the boundary has to be split in three parts $\partial\mathcal{B}_0 = \partial\mathcal{B}_0^+ \cup \partial\mathcal{B}_0^- \cup \partial\mathcal{B}_{0t}$. The parts $\partial\mathcal{B}_0^+$ and $\partial\mathcal{B}_0^-$ are on opposite sides on the RVE faces normal to the plane direction. On this faces periodic boundary conditions are applied with the condition to the fluctuation

$$\widehat{\boldsymbol{\omega}}^+ = \widehat{\boldsymbol{\omega}}^- \ , \tag{17}$$

as pointed out in [2] and antiperiodic tractions

$$\widehat{\boldsymbol{t}}_0^+ = \widehat{\boldsymbol{t}}_0^- \ . \tag{18}$$

On the faces $\partial\mathcal{B}_{0t}$, parallel to the textile plane zero traction boundary conditions are applied. The reversing scale transition is based on an averaging of the microscopic stresses. An energy averaging theorem, which requires the equality of the microscopic average of the virtual work δW_m on the RVE and the virtual work on the macro scale δW_M is the Hill-Mandel condition. The Hill-Mandel condition can be expressed for a volume RVE

$$\delta W_m = \frac{1}{A_{0m}} \int\limits_{\mathcal{B}_{0m}} \boldsymbol{P}_m : \delta\boldsymbol{F}_m dV_m = \widehat{\boldsymbol{N}}_M : \delta\widehat{\boldsymbol{H}}_M + \delta\widehat{\boldsymbol{M}}_M : \delta\widehat{\boldsymbol{K}}_M = \delta W_M \ , \tag{19}$$

where δa is the variation of a and the coefficients of the stress resultants are given by

$$\begin{aligned}
\widehat{\boldsymbol{N}}_M &= \int\limits_H \left[\frac{1}{A_{0m}} \int\limits_{\mathcal{M}_{0m}} \widehat{\boldsymbol{P}}_m dA_{0m} \right] d\theta^3 = \frac{1}{A_{0m}} \int\limits_{\partial\mathcal{B}_{0m}^+ \cup \partial\mathcal{B}_{0m}^-} \boldsymbol{t}_{0m} \boldsymbol{X}_m d\partial\mathcal{B}_{0m} \ , \\
\widehat{\boldsymbol{M}}_M &= \int\limits_H \left[\frac{1}{A_{0m}} \int\limits_{\mathcal{M}_{0m}} \theta^3 \widehat{\boldsymbol{P}}_m dA_{0m} \right] d\theta^3 = \frac{1}{A_{0m}} \int\limits_{\partial\mathcal{B}_{0m}^+ \cup \partial\mathcal{B}_{0m}^-} \theta^3 \boldsymbol{t}_{0m} \boldsymbol{X}_m d\partial\mathcal{B}_{0m} \ ,
\end{aligned} \tag{20}$$

for heterogeneous RVE sections [4]. Therewith the coefficients of the generalized forces are

$$\begin{aligned} \widehat{\boldsymbol{N}}^{i}_{M} &= \widehat{\boldsymbol{N}}_{M} \cdot \boldsymbol{G}^{i}_{M} \ , \\ \widehat{\boldsymbol{M}}^{\alpha}_{M} &= \widehat{\boldsymbol{M}}_{M} \cdot \boldsymbol{G}^{\alpha}_{M} \ . \end{aligned} \tag{21}$$

5 CHARACTERIZATION OF THE MICRO LEVEL AND EXAMPLES

For the exemplary application of the introduced methods a periodic woven RVE is considered. With a size of $4 \times 4 \times 2 \ \text{mm}^3$ the RVE is composed of 20-node hexahedral elements. The constitutive law of the fiber material is chosen to be isotropic, linear elastic, so the stress-strain relation can expressed by Hooke's Law

$$\boldsymbol{\sigma} = \frac{E}{1+\nu}\left[\frac{\nu}{1-2\nu} tr(\boldsymbol{\varepsilon})\boldsymbol{I} + \boldsymbol{\varepsilon}\right] \ , \tag{22}$$

where $\boldsymbol{I}$ is the identity matrix and $tr(\boldsymbol{\varepsilon})$ is the trace of the linearized strain tensor $\boldsymbol{\varepsilon}$. E is the Young's modulus and ν is the Poisson ratio. After characterization of the material behavior also the interaction between the fibers has to be considered. Contact is a unilateral, nonlinear coupling condition where forces are transferred in the common contact zone.

Thereby stresses are acting between the contact partners, which can be classified in stresses $\boldsymbol{t}_n$ normal to the contact plane and tangential stresses $\boldsymbol{t}_t$ in the contact plane with the contact stresses

$$\boldsymbol{t}_c = \boldsymbol{t}_n + \boldsymbol{t}_t. \tag{23}$$

Further the relation between normal and tangential stress is introduced

$$|\boldsymbol{t}_n| = \mu \, |\boldsymbol{t}_t| \ , \tag{24}$$

known as Coulomb friction with the friction coefficient μ chosen in this exemplary model to $\mu = 0.5$, realized by a Penalty method.

For the testing of the introduced methods calculations on the micro level were accomplished to evaluate the nonlinear textile behavior. Considering two typical deformations like bending (Fig. 3(a)) and membrane shearing (Fig. 3(b)) the deformed shape and the von Mises stress is plotted. Further the homogenized, macroscopic response over deformation for bending κ_{22} and shearing γ_{12} is shown.

6 CONCLUSIONS

This paper treats the basic principles to consider technical textiles in a multi scale scheme, connecting macroscopic shells with 3D-modeled fiber structures. It allows to

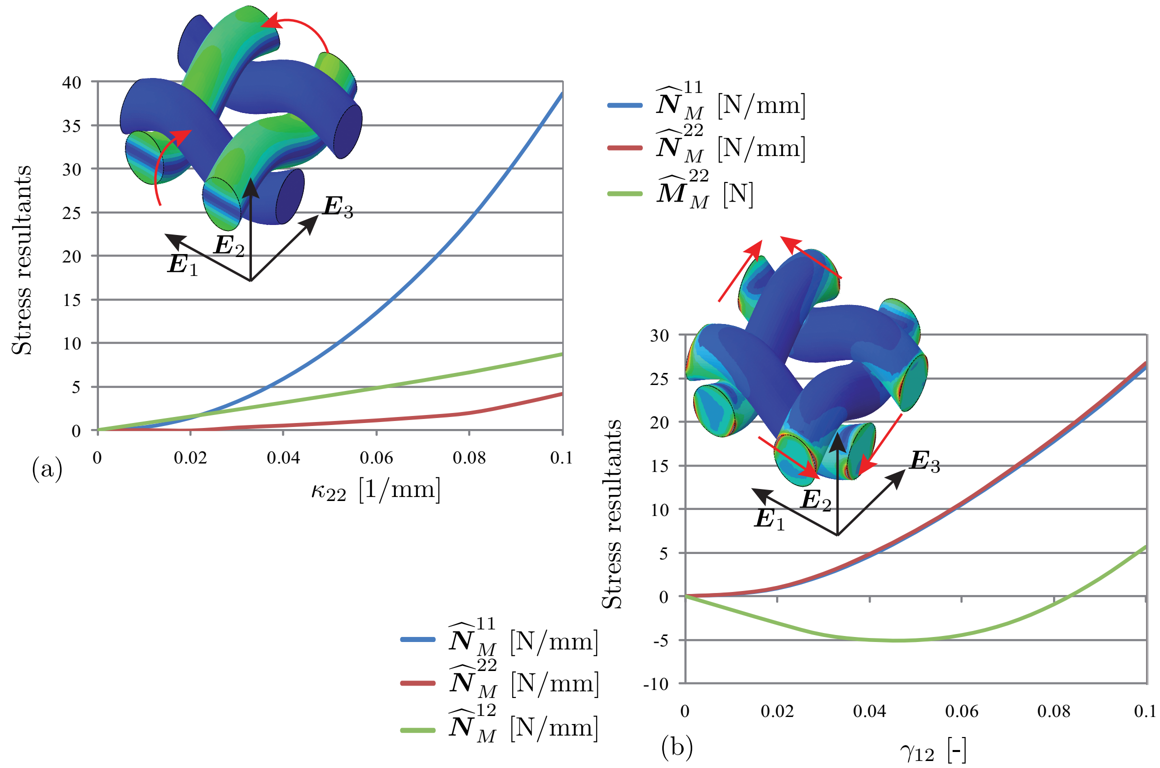

Figure 3: Macroscopic homogenized response for different RVE deformations

account for the coupling between structural heterogeneous shells and the underlying microstructural features that cause this behavior. Main focus was put on the extraction of the macroscopic phenomenological constitutive laws of the textile. A analysis of different 3D deformations was shown within the context of a shell response.

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