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Membrane restrained columns

HOLGER ALPERMANN*, CHRISTOPH GENGNAGEL*

* Chair of Structural Design and Technology, School of Architecture
University of the Arts Berlin
Hardenbergstrasse 33, 10623 Berlin, Germany
e-mail: alpermann@udk-berlin.de, web page: http://www.arch.udk-berlin.de/gengnagel

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Summary. This paper describes the structural behaviour of membrane restrained columns. Initial experiments on tensile restrained columns are presented and discussed. FE-analysis has been carried out and the structural behaviour of the hybrid systems is analysed. The influence of several design parameters is research.

1 INTRODUCTION

The use of restraining systems is a very effective way to stabilise structures that are under axial forces and/or bending moments. The restraining system reduces the deformation and the bending stress of the structure. For structural elements under axial stress the buckling length is reduced by the restraining system. As restraining system cables as linear elements or membranes as two-dimensional elements can be used. As cables have the advantage of high stiffness and strength, membranes offer the opportunity of a continuous contact between the stabilised structure and the restraining system. This leads to a further reduction of the buckling-length and avoids peaks in the internal forces as it occurs with cable restrained structures at the connection-node.

Compression elements are endangered by buckling. If compression elements are very slender the strength of the material can not be used fully, because the buckling load becomes determining for the dimensioning. By use of a restraining system the buckling load can be increased so much, that the material of the compression element can be used to its yield limit.

The use of membrane restrained columns is efficient for very slender columns under small loads. They can be used in mobile and rapid deployable constructions as well as for roof and façade systems ranging from small to moderate size.

2 TEST OF TENSILE RESTRAINED COLUMNS

First ideas for tensile restrained columns have been tested by the LSU [1] in physical models. The tests were thought to get an idea of the potential and structural behaviour of tensile restrained columns. Except for the vertical load no deformations, strains or stresses have been measured. Figure 1 shows the four different systems that have been tested. All systems were flat and had a height of about 1m, the systems I, II and III were built of rectangular balsa-stripes and cord, system IV was build of circular GRP profiles and paper in-between.

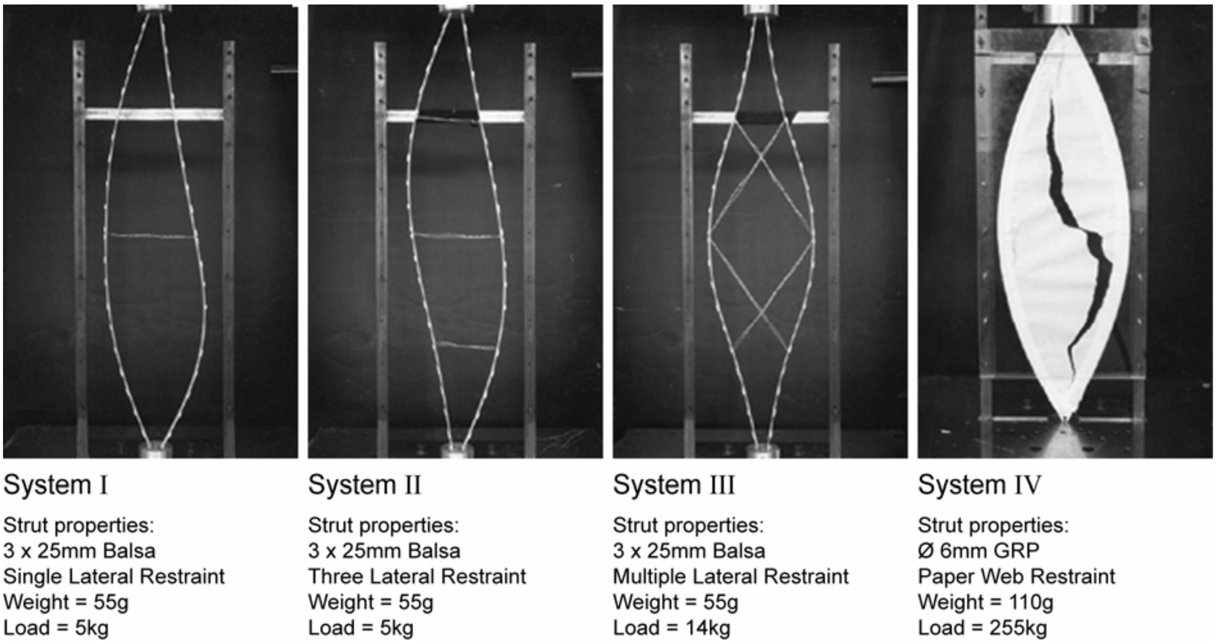

Figure 1: tests of flat, tensile restraint struts, deformed systems I - IV under vertical load [1]

The systems I and II had the same buckling load, the two additional lateral restrains do therefore not increase the buckling capacity. System III is restrained by diagonal cords, and the buckling load is nearly three times higher compared to system I and II. This result was quite surprising, as the horizontal connections were expected to be most important due to the deformation of the stripes to the outer side. The reason for a higher buckling load of the diagonal restrained column results from the existing imperfections. This subject will be explained more in detail in chapter 4. System IV is the stiffest system, nevertheless it needs to be stated, that the stiffness of the bars in system IV is higher compared to the systems I, II and III. The failure shape of system IV shows that the paper is torn, which indicates a material failure and the buckling load is not reached yet. However the paper membrane in-between seems to be very potent, as the bars are restrained continuously and the membrane forces can set up freely in the membrane surface.

3 GEOMETRY AND FE-MODELL

To get a more detailed understanding of the structural behaviour of the membrane restrained columns FEM–analysis was used. The analysed structure is 3m high, the width varied between 0.1m and 0.8m. The column consists of three bars, between which a membrane runs. The bars are rotated by 120° related to the middle axis. The bars are orientated in that way, that the local z-axis of the cross-section is in the plane which is defined by the profile axis and the middle axis. The bars are single curved with a constant curvature. The bars are bent warm so they are free of stress. The geometry of the membrane is double curved, although the curvature of the membrane between the bars is very small.

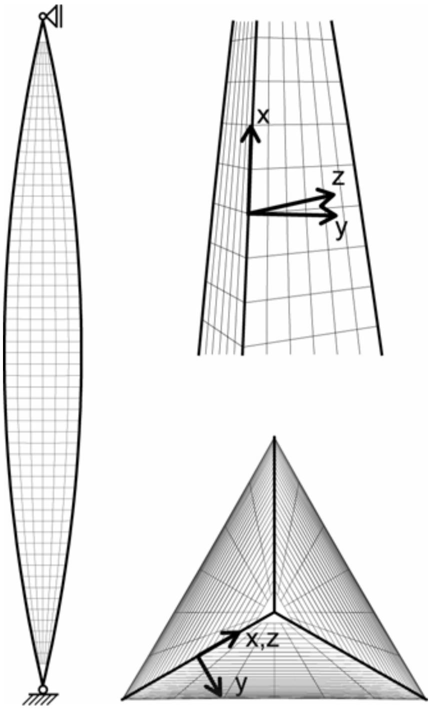

Figure 2: structural system of the column and local coordinate system of one bar in side view (left), plan view (lower right) and perspective

The bars are connected to each other in the following way:

- related to the local y- and x-axis with a hinge
- related to the local z-axis fixed.

The lower end of the column is simple supported and fixed against torsion, at the upper end the column is simple supported in both horizontal directions but free in vertical direction and fixed against torsion as well. The connection between the membrane and the bars is solved by membrane pockets, where the bars can be pushed in. This procedure would allow an easy and quick assembling-process.

For the bars the cross-sections were defined by 20mm x 20mm hollow steel-sections with a thickness of 1.2mm. The membrane was calculated with a similar E-Modulus in warp- and weft direction of 600 kN/m and a G-Modulus of 25 kN/m. The calculations were performed with the FE-program SOFISTIK. Geometric Nonlinearities were taken into account. For the membrane the orthotropic, linear-elastic material model from Münsch-Reinhard was used. Only tension stiffness was considered for the membrane, whereas for the bars linear material behaviour was set without considering the yield-point of the material.

4 STRUCTURAL BEHAVIOUR

In the FE-calculations the structure has been analysed with perfect and imperfect geometry. To find the most unpropitious imperfections the eigenforms of the column were examined. There are two basic kinds of eigenforms for the columns: The first is a twisted one (symmetric to the middle-axis), the second is a bended one looking like an “S”. Both eigenforms have been used independently from each other as imperfection. The eigenforms were scaled to a maximum deformation of 15mm. This equates a preflexure of 1/200 as requested in the German steel-code DIN 18800-1.

The structural behaviour of the columns is decisive depending on the curvature of the bars and therefore of the width in the middle of the columns. In the figures 3, 4 and 5 the load-displacement curves are presented for different column-geometries under a vertical load. As displacement the vertical deformation of the upper support is used. For a small column with a width of 10cm the perfect system shows a linear load-displacement curve (figure 3). Two different membrane orientations have been analysed, 0° (warp or weft-direction is parallel to the middle-axis) and 45° (membrane is rotated by 45°). For the structural behaviour of the perfect system the membrane orientation has no influence. If the imperfection of the twisted eigenform is applied, the structural behaviour is identical with the perfect system for small loads and slightly weaker for higher loads. In contrast to that an imperfection of the bended eigenform shows a different and much weaker behaviour. The buckling load is only about one third compared to the system with an imperfection of the twisted eigenform. The load-displacement curves are identical for both imperfect systems; however the buckling load is smaller with a membrane orientation of 0°.

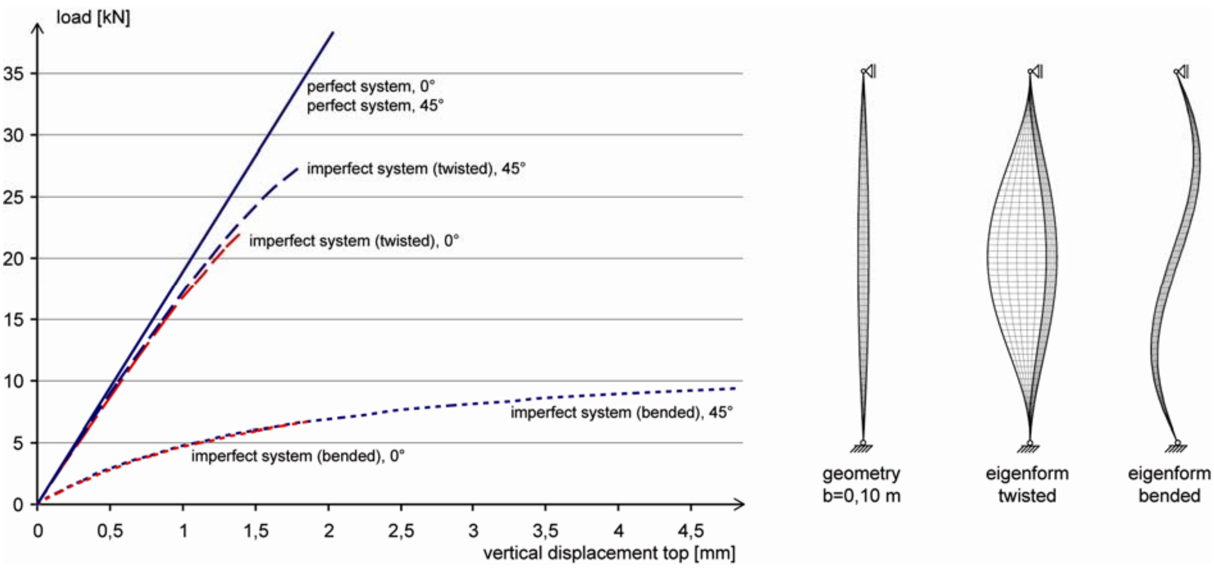

Figure 3: load-displacement-curves for a column 0,10m wide, right: perfect geometry and eigenforms, the deformation of the eigenforms have been scaled and applied as imperfections.

Looking at a column with a width of 30cm (figure 4) the perfect system shows again a linear structural behaviour. In contrast to the smaller column the load-displacement curve varies depending on the membrane orientation and an orientation of 0° is stiffer than an orientation of 45°. This results from the fact that in the perfect system the membrane is only stressed in horizontal direction and the membrane stiffness is higher in warp- and weft direction than in any other direction. This effect gets more and more significant when the columns are wider, as the length of the membrane between the bars increases. When the imperfections of the twisted eigenform are applied, again the structural behaviour is similar to the perfect system for small loads and weaker for higher loads. Whereas for the perfect structure a membrane orientation of 0° results in a stiffer behaviour, for the imperfect system the system with a membrane orientation of 45° is stiffer. For the column with an imperfection of the bended eigenform the load-displacement curve is much weaker compared to the perfect and first imperfect system. However there is a significant difference in the structural behaviour depending on the membrane orientation. The column with an orientation of 45° is stiffer, so apparently the asymmetric deformations caused by the imperfections can be restrained more efficiently by a membrane orientated at 45°.

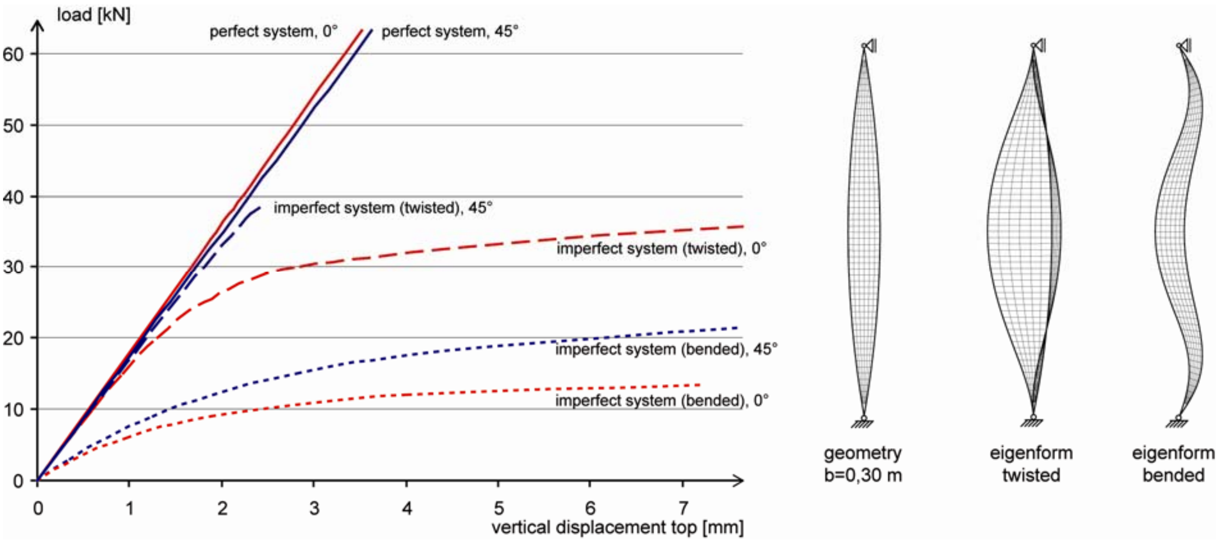

Figure 4: load-displacement-curves for a column 0,30m wide, right: perfect geometry and eigenforms.

For a wide column with a width of 60cm the structural behaviour of the perfect system is still linear. The difference in the load displacement curves depending on the membrane orientation is more significant than for smaller columns. Applying the imperfection of the twisted eigenform the structural behaviour is similar to smaller columns: For small loads the behaviour is like the perfect system, for higher loads the imperfect system is weaker. With an increasing load the influence of the membrane orientation plays a more and more important role: The load-displacement curves are likely the same for loads up to 25kN, but as the system with an orientation of 0° has only a buckling load of 32kN, the system with an orientation of

45° has a more than the double sized buckling capacity. Applying an imperfection of the bended eigenform to the system the structural behaviour is still the weakest system, nevertheless the difference to the perfect system and the “twisted” imperfect system is lesser compared to smaller columns. The system with a membrane orientation of 45° is again much stiffer compared to an orientation of 0°.

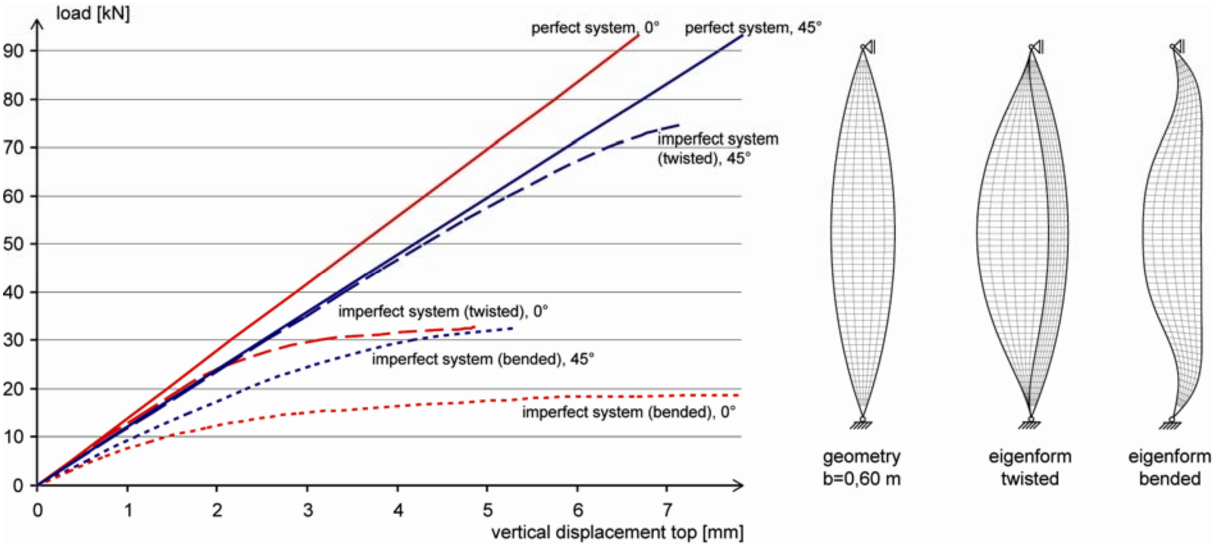

Figure 5: load-displacement-curves for a column 0,60m wide, right: perfect geometry and eigenforms.

The comparison of membrane restrained columns shows, that for perfect systems the structural behaviour gets weaker when the columns are wider (figure 6). Taking imperfection (“bended eigenform”) into account it is the other way round and the structural behaviour gets stiffer when the columns are wider. The buckling capacity, which is predicted by the load-displacement curve of the imperfect, “bended” system is increasing with the width of the columns. As the load-displacement curves of perfect and imperfect system appear each other with the increase of the width, this means, that the structures get less prone to imperfections.

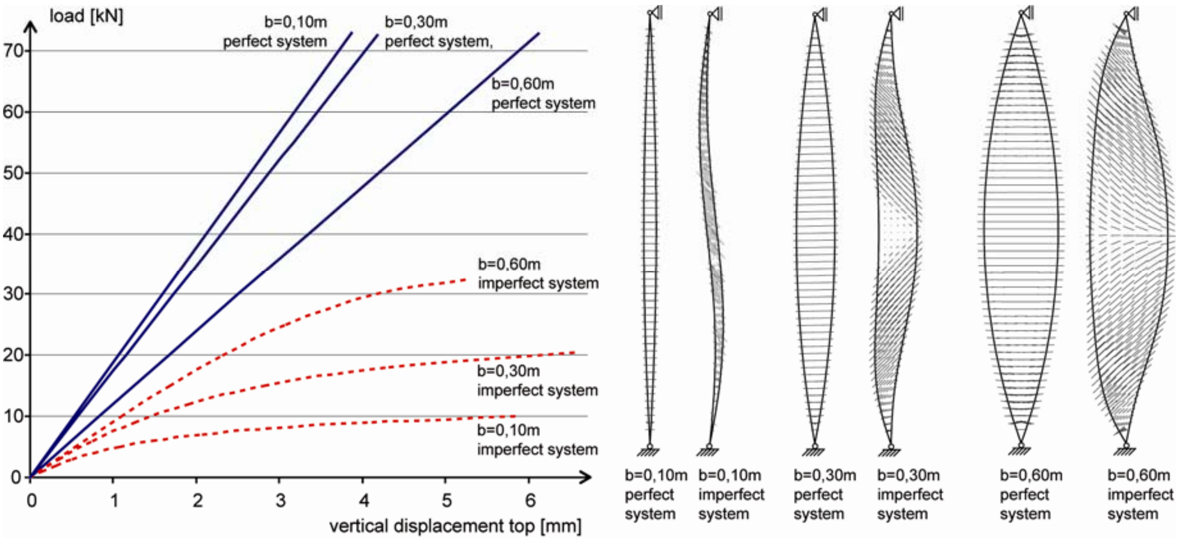

Figure 6: left: load-displacement-curves for a different columns (all membrane orientation 45°), right: vectors of principle membrane forces

Figure 6 shows as well the principle m embrane forces under load. In the perfect system s the membrane forces run horizontal between the bars, in the imperfect system they mostly run diagonal.

In figure 7 the buckling capacity is shown in dependency on the width of the colum ns and the membrane orientation. For a m embrane orientation of 0° the buckling capacity is increasing quite linear with the width and the buckling capacity reaches a maximum of 24 kN. If the m embrane is orientated 45°, the buckli ng capacity is s imilar to the 0° o rientated membrane for the two smallest columns. As the load-displacement curves were already nearly identical for these two colum ns the membrane orientation plays obviously only a subordinate role for the structural behaviour. The reason for this behaviour is in the small curvature of the bars, so that under the imperfection of the bended eigenform the bars deform quite parallel to each other. Therefore the membrane can not res train one bar to the o ther, as both bars have analogue deformations. With an increase of the width of the columns the deformations of the bars differ more (figures 3-5) and this enables the membrane to restrain one bar to another. The restraining effect can be described by th e principle stresses in the membrane. For all imperfect columns the princ iple stresses do no t run hor izontal between the ba rs but mostly rotated between 30° and 60°. A m embrane orientation of 45° offe rs therefore a higher restraining effect and leads to a stiffer structural behaviour and higher buckling capacity.

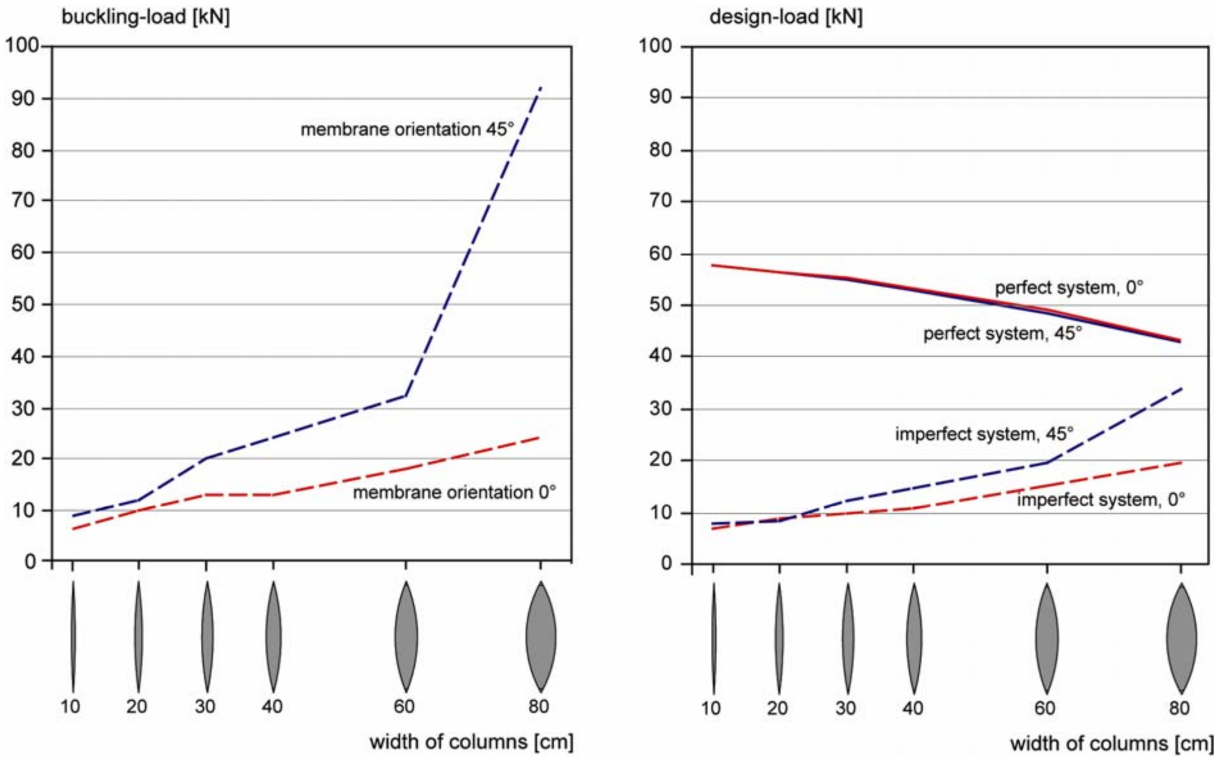

Figure 7: buckling-load (left) and design loads (right) for maximum stress of 214 N/mm² in the bars

Whereas the buckling capacity is determined independently from the stresses in the bar, for the design of the structure the stresses need to be taken into account. In figure 7 the design-loads for a stress of 214 N/ mm² (S 235, safety-factor 1.1) are shown. For the perfect system the design loads decrease when the width of the columns increases. This results from the fact that with an increasing width the distance between the middle of the bars and the middle axis gets higher. This distance causes higher bending moments in the bars and subsequently higher stresses. The design load for a column with a membrane orientation of 0° is minimal higher compared to a membrane orientation of 45°. It needs to be mentioned, that the design-loads for the perfect system are only theoretical as they lie above the buckling capacity.

Surprisingly the design loads for the imperfect system increase with the width of the system. Although for the imperfect system the same effect occurs, that a higher width causes higher bending moments, a second effect is more significant. This second effect results from the imperfections applied to the structure. The stresses in the imperfect structure are by far higher than the stresses in the perfect system. That means for the membrane, that it is not only her task to restrain the bars horizontal to each other but even more to restrain the bars against the asymmetric deformations caused by the imperfections. The load-displacement curves showed already, that this restraining effect increases with the width of the columns. This increase of the restraining effect is higher than the increase of the bending moment caused by the width and therefore the design loads increase with the width. The design-loads for the imperfect columns with a width of 10 or 20cm are equal for both membrane orientations.

Regarding the wider columns a membrane orientation of 45° offers higher design loads compared to a membrane orientation of 0° - for the same reason as described for the buckling load. For the small columns the design load is only slightly under the buckling load, but with an increasing width of the columns the distance between buckling load and design load gets higher. This makes the structures more redundant and allows the use of higher-quality steel.

5 COMPARISON WITH A SINGLE COLUMN

Of course the question comes up under which static conditions a membrane restrained column is more effective compared to a simple strut with a circular hollow cross section. In figure 8 the buckling- and design-load of a membrane restrained column with a width of 30cm and a membrane orientation of 45° is shown for different cross-sections. The buckling- and design loads increase with the cross section area, but the increase gets smaller as the cross section gets "closer". This results from the stresses in the bars, which are dominantly caused by bending moments and only little by axial forces and therefore hollow sections are more efficient than full profiles.

Beside quadratic also rectangular cross sections have been analysed, but as the profiles usually are stressed by two-axial bending, the structural behaviour is very similar. Regarding the buckling- and design-load they offer only a small option of optimisation.

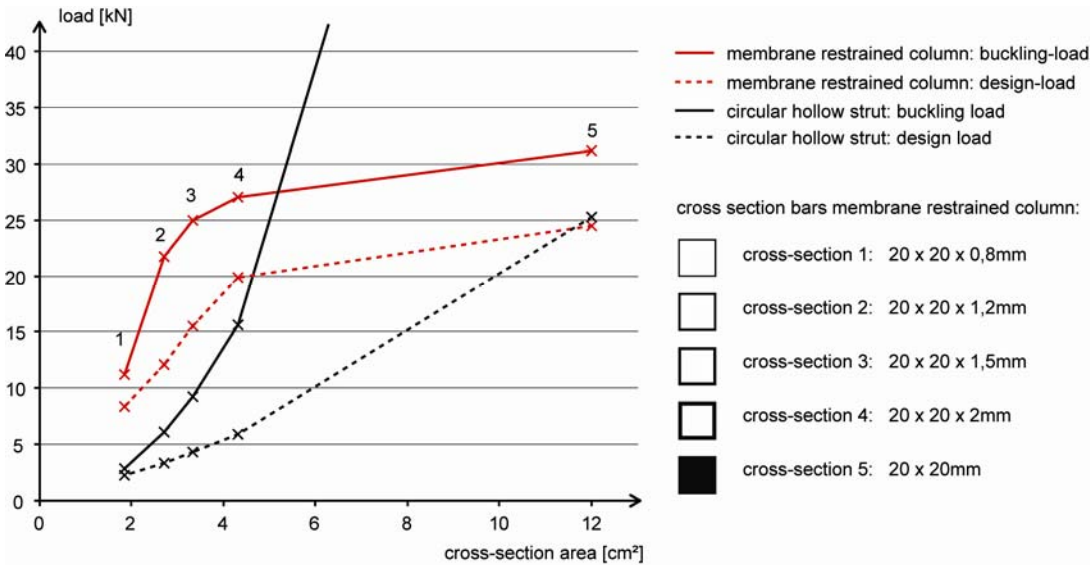

Figure 8: buckling- and design-loads for a membrane restrained column (b=0,30m) and a circular hollow

The compared circular hollow strut is defined by the same mass as the sum of the three bars of the membrane restrained column. The thickness is set to one tenth of the diameter and as material steel is used as well. With the increase of the cross section area the diameter of the circular hollow section increases as well and consequently the buckling- and design load increases progressively. For small cross-section areas the membrane restrained columns have higher buckling- and design loads, but using full cross sections for the bars is not very efficient and has less potential as a simple strut. The slenderness of the circular strut is 144 for the cross section area equivalent to cross section 5. As the cross section area decreases the slenderness increases from 240 (equivalent cross section 4) to 367 (equivalent cross section 1).

6 CONCLUSIONS

The use of membrane restrained columns is recommend for structures with a slenderness higher than 200. To use the full potential of membrane restrained columns the width should be at least 10% of the height and the membrane should be orientated by 45°. Hollow cross sections for the bars are more effective in relation to the weight than full profiles. As materials steel and aluminium can be used.

As the use of the columns is especially thought for temporary structures, the transport of the columns becomes an important issue: The bars could be easily divided in two or three (or even more) sections, which are connected by a plug-in connection or a screwed joint. The membrane can be folded and so both elements can be transported easily. As the connected bars only need to be pushed into membrane pockets, the column can as well be assembled and disassembled very fast.

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