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SHAPE ANALYSIS FOR INFLATABLE STRUCTURES WITH WATER PRESSURE BY THE SIMULTANEOUS CONTROL

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Summary.

Simultaneous control is an incremental technique that can be applied to shape analysis with soap film elements, when the shape has large volume and high rise. Furthermore, the tangent stiffness method is a clear and strict analytical theory to be able to solve the problems with large deformational behavior. The simultaneous control brings out the best performance for form-finding of isotonic surfaces in combination with the tangent stiffness method. This study proposes a significant modification of the simultaneous control. The modification realizes a wide application range regarding load conditions and boundary conditions for soap film shape analyses by calculating the average of surface tension. Results are presented as some computational examples in which the behavior of soap film structures under air pressure and /or water pressure becomes evident.

Key words: Shape analysis, soap film surface, simultaneous control, tension averaging, approximation of surface, polyhedral equilibrium solution, tangent stiffness method.

1 INTRODUCTION

Recently, membrane structure are used not only for roof of stadiums, but also underground structures such as geo-membrane or coastal membrane structures. However, compared to the huge application of this type of structure, studies and examples are significantly few for form finding analysis, such as embedded oil tank in coastal area by Ishii[1], rubber dam by Ogiwara etc.[2]. Furthermore, the field of studies is only limited to the issues of axial symmetrical or two dimensional problems, it is not sufficient for the cases of complicated boundary shape or loading condition such as follower load.

Until recently, authors had executed form finding for soap film surface (Ref.[3 to 5]), the application of the tension force as a constant value for triangular shape soap film element using tangent stiffness method algorithm which solves geometrically non linear problems with high convergence. In Ref.[4], to analyze the shape of soap film inflated by internal air pressure up to extra large volume, it is suggested to apply compulsory displacement on a node by incremental method using simultaneous control. In addition, when observing the behavior of the volume consisted in soap film, an unstable condition occurs which has the similarity to elastic buckling. Furthermore, according to Ref.[5], it was clear that the soap film surface satisfy the isotonic condition not only in loading condition of the internal pressure which is distributed uniformly, but also in case of the location dependency loading such as water pressure.

In this paper, based on the average internal pressure suggested by the previous authors, average tension by simultaneous control is proposed and in this concept, it is focused on the physical characteristic of soap film which is 'constant value of surface tension'. In this method, the application of location dependency loading in shape analysis for soap film is widely executed, as for example, it was able to perform as a simulation of a volume expansion phase in a soap film surface by water pressure which works from the inside of the structure. In addition, the input of loading condition (air, water or earth load), boundary condition and control point coordinate could produce output results which are surface morphology and tension of soap film, and by this result, the rational design for real membrane structure could be carried out by achieving the surface prototype and initial tension simultaneously.

2 SOAP FILM SHAPE ANALYSIS USING TANGENT STIFFNESS METHOD

TANGENT STIFFNESS AND TANGENT GEOMETRICAL STIFFNESS EQUATION FOR TRIANGULAR MEMBRANE ELEMENT

An element constituted by two edges with element edge force and the force vector for both edges is considered as S, displayed as external force for the nodal force vector, U, in a three dimensional coordinate system, and the equilibrium matrix is J, the equilibrium relation is shown as the following equation;

$$\mathbf{U} = \boldsymbol{JS} \tag{1}$$

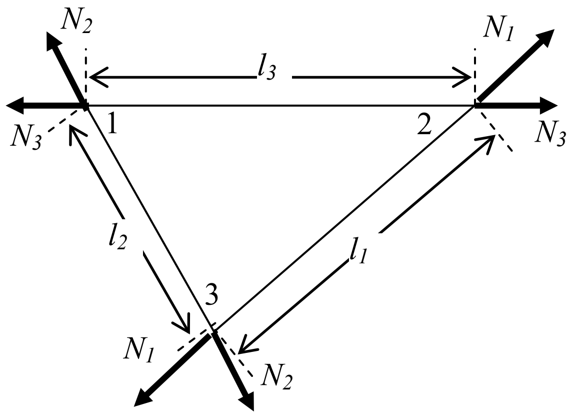

Figure-1 : Triangular membrane element and element edge force along the element direction

With the differentiation of the equation above, the tangent stiffness equation could be represented as;

$$\boldsymbol{\delta}\mathbf{U} = \mathbf{J}\boldsymbol{\delta}\mathbf{S} + \boldsymbol{\delta}\mathbf{J}\mathbf{S} = (\mathbf{K_O} + \mathbf{K_G}) \tag{2}$$

Here, $\mathbf{K_O}$ represents the element behavior**,** correspondent to the element stiffness in the coordinate system while $\mathbf{K_G}$, represents the element displacement originated by the tangent geometrical stiffness. In addition, **u** is the nodal displacement vector displayed in the coordinate system. With the definition of the statement above, the tangent stiffness method is able to evaluate the geometrically non-linear factor caused by the rigid body displacement strictly.

Therefore, we can apply the same geometrical stiffness to soap film elements as real elements with material stiffness. Furthermore, in the case that an element edge force vector which constituents have the direction of triangular sides, the $\mathbf{K_G}$ in Eq. (2) has the same form as a triangular truss block.

Furthermore, **e** is a 3 x 3 unit matrices, $\boldsymbol{\alpha}_\mathrm{i}$ is the cosine vectors for the respective node of the element.

$$\mathbf{K_G} = \begin{bmatrix} \mathbf{k_{G2}} + \mathbf{k_{G3}} & -\mathbf{k_{G3}} & -\mathbf{k_{G2}} \\ -\mathbf{k_{G3}} & \mathbf{k_{G3}} + \mathbf{k_{G1}} & -\mathbf{k_{G1}} \\ -\mathbf{k_{G2}} & -\mathbf{k_{G1}} & \mathbf{k_{G1}} + \mathbf{k_{G2}} \end{bmatrix} \tag{3}$$

$$\mathbf{k_{Gi}} = \frac{N_i}{l_i}\left(\mathbf{e} - \boldsymbol{\alpha}_\mathbf{i}\boldsymbol{\alpha}_\mathbf{i}^\mathrm{T}\right) \quad (i = 1,2,3) \tag{4}$$

ELEMENT FORCE VECTOR FOR SOAP FILM ELEMENT

It is convenient to apply element force equation, achieved from the differentiation of the element measurement potential P, in the element measurement. As for performing shape analysis of an element without any material stiffness such as soap film element. Here, assume the area of a triangular element as A and tension of soap film as σt (constant), the proportion of element potential to the cross section could be defined as;

$$P = \sigma t A \tag{5}$$

The element measurement potential of soap film elements, which is the function of element area, gives the minimal surface of an isotonic surface, while the stationary of the potential energy gives an equilibrium state. It showed that the element measurement potential of soap film elements is equivalent to the strain energy of real members. If the direction of force is parallel to sides of a triangle and defined as element edge force, the treatment of geometrical stiffness becomes more simple as shown in Figure-1 and if the element constituents are grouped into a triangular element, the element edge force;

$$N_1 = \frac{\partial P}{\partial l_1} = \sigma t \frac{l_1(l_2^2 + l_3^2 + l_1^2)}{8A} \tag{6}$$

$$N_2 = \frac{\partial P}{\partial l_2} = \sigma t \frac{l_2(l_3^2 + l_1^2 + l_2^2)}{8A} \tag{7}$$

$$N_3 = \frac{\partial P}{\partial l_3} = \sigma t \frac{l_3(l_1^2 + l_2^2 + l_3^2)}{8A} \tag{8}$$

could be achieved by differentiating the element potential in the length and the element edge force vector could be defined as the following equation;

$$\mathbf{S} = [N_1 \quad N_2 \quad N_3]^T \tag{9}$$

Therefore, the tangent stiffness method to perform form-finding for soap film analysis calculates the nodal force vector from a constant value of air pressure or static water pressure and NR method produces the perfect equilibrium shape by convergence of the unbalanced forces.

In addition, based on Ref.[3], the element stiffness $\mathbf{K_O}$, is relatively small compared to the tangent geometrical stiffness $\mathbf{K_G}$ for soap film element and $\mathbf{K_O} = 0$ is substituted into Eq. (2) of the calculation. Moreover, based in Ref.[3], the modification to one degree of freedom analysis has been made and the displacement of the node is to be set in the normal direction of the surface.

INCREMENTAL PROCESS BY SIMULTANEOUS CONTROL

REVIEW OF THE SIMULTANEOUS CONTROL

The idea of 'the simultaneous control averaging internal pressure' is explained specifically in Ref.[4,5], and this study described 'the simultaneous control averaging tension'.

In order to obtain a soap film surface using a mechanical approach, the following determinations have to be prepared other than the boundary condition as the input information;

1) The tension of soap film σt, which is stress multiplied by thickness of membrane, is constant along the surface.
2) The internal pressure p, which is the air pressure or static water pressure.

However, assuming an isotonic surface which involves air or water, if the value p is larger than the value of σt relatively, convergence result could not be achieved. This suggests that the maximum value exists on the P-V curve which represents the relation between pressure and volume, and additional geometrical restriction is required in order to get the equilibrium solutions.

'The simultaneous control averaging internal pressure' is to get the equilibrium shape and the internal pressure of a constant value along the surface simultaneously from input of tension force and the displacement of a control point. However, in the case of the water pressure acts on the soap film, the tension distributed on surface, of course, should be a constant value, but the magnitude of the pressure is depending on the position. Therefore, it is more rational to inverse the input and the output to obtain equilibrium shape with large volume.

In this study, the procedure of form-finding is as follows;

1) A control point is displaced with small incremental step.

2) 'The converted tension', which is balanced with the sum total of 'the element edge forces' gathering at a node, is calculated at every node.
3) 'The converted tension' is averaged out to all nodes, and adopted to tension of soap film for next iteration step. We call this, 'the converted average of tension'.
4) 'The converted average of tension' is renewed in every iteration step, and finally the unbalanced forces of all nodes and the reaction force at the control point are converged to zero. Furthermore, the tensions of all elements are equalized on the obtained equilibrium shape.

According to this procedure, the pressure acting on the soap film surface do not have to be distributed uniformly. Therefore, for example, it became possible to search a shape of soap bubble involving liquid with extremely large volume. In this paper, this incremental procedure is called 'the simultaneous control averaging tension'.

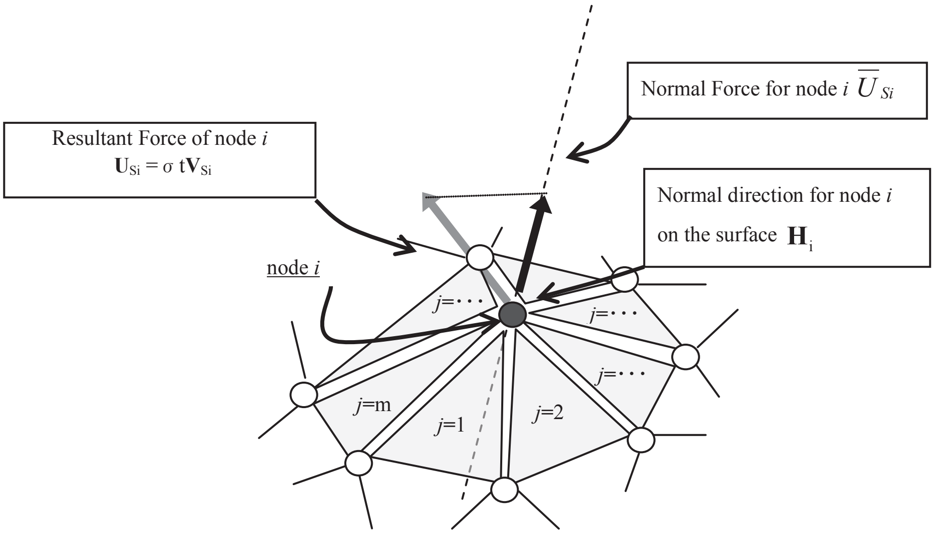

Figure-2 : Elements connected to node i and force in the normal direction

CONVERTED AVERAGE OF TENSION

Assume that the total number of element, m are connected to node i as shown in Figure-2. In the case of the total area vector for element j, the pressure loading that subjected on node i is assumed as 1/3, the corresponding total area vector for each element are shown in the following equation.

$$\mathbf{A}_{ij} = \frac{\mathbf{a}_{ij}}{3} \tag{10}$$

Furthermore, the normal cosine vector for node i on the surface could be expressed as;

$$\mathbf{H_i} = \frac{\sum_{j=1}^{m} \mathbf{A_{ij}}}{\left|\sum_{j=1}^{m} \mathbf{A_{ij}}\right|} \tag{11}$$

Meanwhile, if the element edge force for edge *i* in element *j* is assumed as $\mathbf{S_{ij}}$, the conversion to the standard coordinate system is expressed in equilibrium matrix $\mathbf{J_{ij}}$ which can be expressed as the following equation.

$$\mathbf{U_{S_{ij}}} = \mathbf{J_{ij}S_{ij}} \tag{12}$$

Therefore, the resultant force for the element edge force for node *i* is;

$$\mathbf{U_{S_i}} = \sum_{j=1}^{m} \mathbf{U_{S_{ij}}} \tag{13}$$

Since the stiffness is only applied in the normal direction of the soap film surface, the component of resultant force for the element edge force in the normal direction is;

$$\bar{U}_{S_i} = \mathbf{H}_i^T \mathbf{U}_{S_i} \tag{14}$$

Then, the equilibrium equation at node *i* is expressed as follows;

$$p_i(\mathbf{u_i}) \left| \sum_{j=1}^{m} \mathbf{A_{ij}} \right| - \bar{U}_{S_i} = 0 \tag{15}$$

where, $p_i(\mathbf{u_i})$ is the pressure at node *i*, which is depending on the position of the node.

Referring to Eq. (6) until Eq. (8), element edge force $\mathbf{S_{ij}}$ and tension of soap film σt have a linear relation, with that Eq. (14) can be rewritten as following;

$$\bar{U}_{S_i} = \sigma t \mathbf{H_i^T V_i} \tag{16}$$

Therefore, Eq. (15) can also be rewritten as;

$$\sigma t_{C_i} = p_i(\mathbf{u_i}) \frac{\left| \sum_{j=1}^{m} \mathbf{A_{ij}} \right|}{\mathbf{H_i^T V_i}} \tag{17}$$

Eq. (17) provides 'the converted tension' σt_{Ci} at node *i*, which is balanced with the pressure referring to the current geometry.

By averaging the converted tension σt_{Ci} will establish the tension of soap film to become constant along the whole surface. However, as the following calculation example (Figure-3) clearly shows, the tension of soap film can have an infinite value when the shape of surface is close to plane under the designation of non-zero pressure. Therefore, by applying the average value of inverse number for converted membrane stress, 'the converted average of tension' σt_{AV} could be determined and be renewed in each iteration step.

$$\frac{1}{\sigma t_{AV}} = \frac{1}{n} \sum_{i=1}^{n} \frac{1}{\sigma t_{C_i}} = \frac{1}{n} \sum_{i=1}^{n} \frac{\mathbf{H_i^T V_i}}{p_i(\mathbf{u_i}) \left| \sum_{j=1}^{m} \mathbf{A_{ij}} \right|} \tag{18}$$

The unbalanced force for all nodes and the reaction force for the control point converge simultaneously, then the morphology of the soap film surface including the control point and the equalized tension of soap film could be determined.

3 NUMERICAL ANALYSIS EXAMPLE

CASE 1 : STATIC WATER PRESSURE SUBJECTED ON SOAP FILM

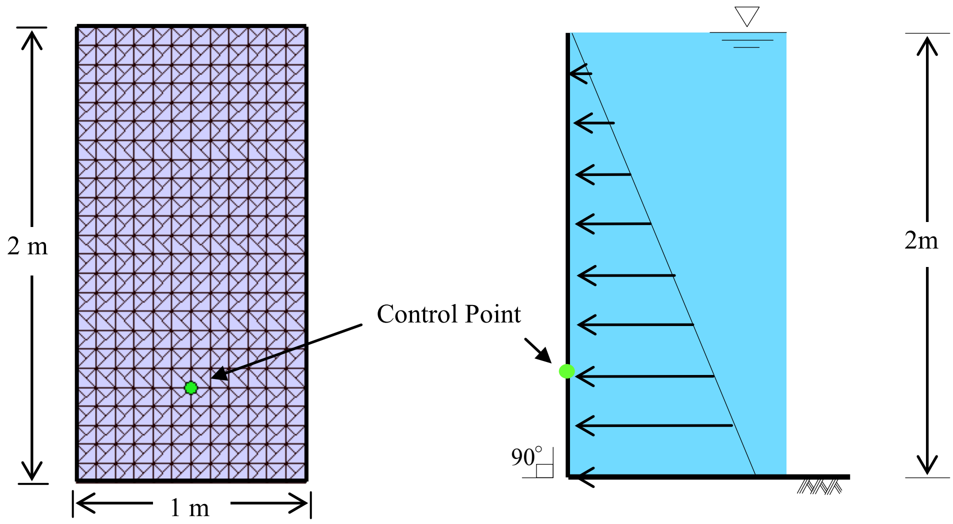

Figure-3 : Front view and side view for Case 1

Here is an example of a soap film subjected by water pressure. As shown in Figure-3, the orientation of the rectangular shape boundary frame forms a 90 degree angle to the horizontal direction with a water level, equal to the respective height. Compulsory displacement is applied on the control node in the normal direction of the surface, with 0.01 m of incrementation for each iteration step. The whole structure consists of triangular soap film elements and the initial tension of soap film value is 20 kN/m.

Table-10 shows the relation between volume, V and tension of soap film σt for selected incremental step of compulsory displacement. According to the graph, it is clear that the minimal value of tension for soap element (marked as **D**) exists, thus the result which surpassed the minimal value could also be traced (marked as **E** and **F**).

In this analysis, when the surface volume reaches approximately 1.9 m^3, divergence occurs and there was no result for the following phase. This is due to the 'extra large' deformation for the soap film element and to solve this, it is proposed to increase the value of mesh to obtain better result.

Table 1 : Analysis of soap film subjected to water pressure

Label	Compulsory displacement value, *dcp* (m)	Volume, V (m^3)	Tension of soap film, σt (kN/m)
A	-0.4159	0.0883	25.2982
B	-0.4127	0.1948	11.8968
C	-0.3728	0.6776	5.0219
D	-0.3313	1.0531	4.6543
E	-0.2971	1.4345	4.8185
F	-0.2772	1.9052	5.2943

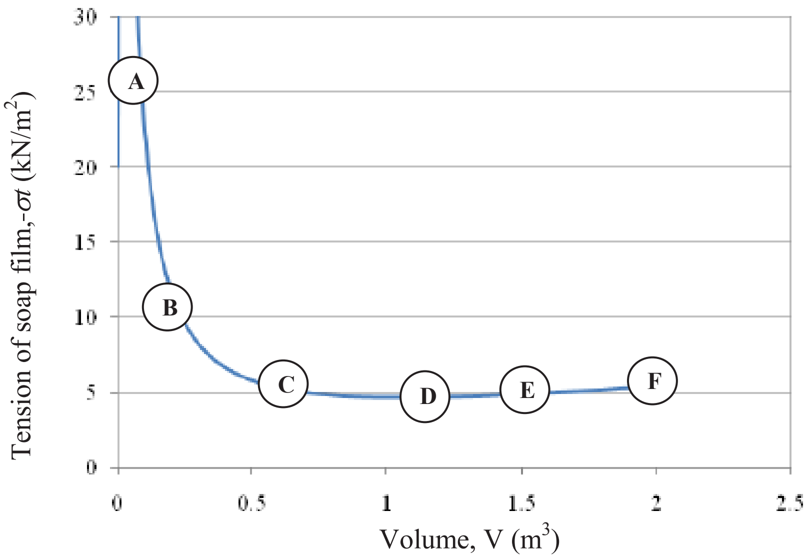

Figure-4 : The relation of tension of soap film, σt (kN/m) and volume, V (m^3)

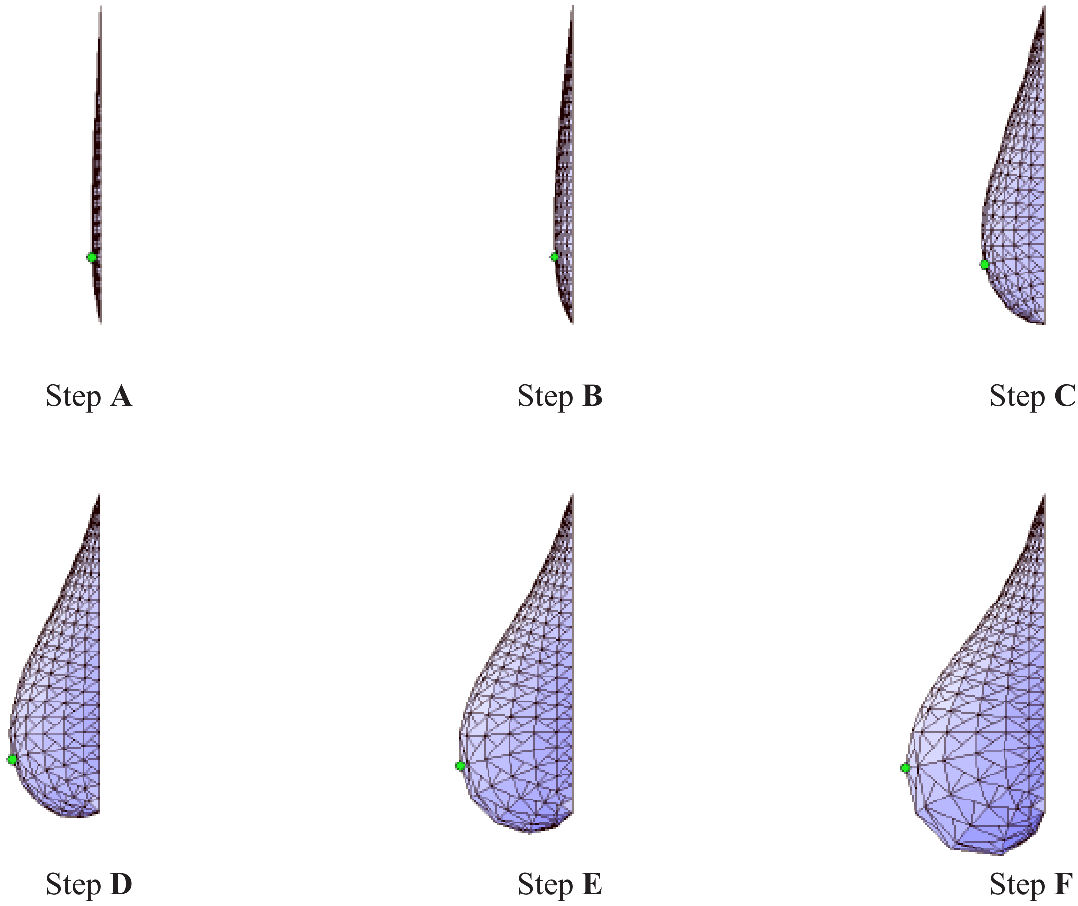

Figure-5 : Deformation diagram for soap film structure (Case 1)

CASE 2 : STATIC WATER PRESSURE SUBJECTED ON SOAP FILM

As shown in Figure-6, a hexagonal shape soap film is fixed along the border, subjected with a water loading. The water pressure is equivalent to 12 m of water level, pumped from beneath the structure. Using averaging surface tension by simultaneous control, a control point is located in the middle of the hexagonal structure, applied in the upward direction with the increment value of 0.1 m for each iteration step. Figure-7 shows the relation between tension of soap film, σt (kN/m) and surface volume, V (m^3), and Figure-8 shows the deformation diagram of the structure as water pressure is applied throughout the analysis.

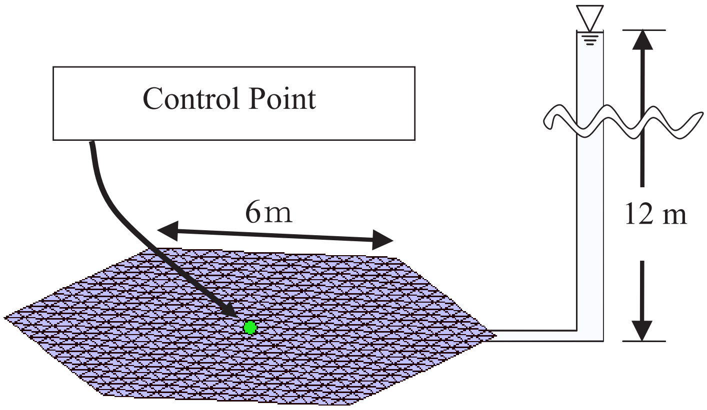

Figure-6 : Analysis example for Case 2

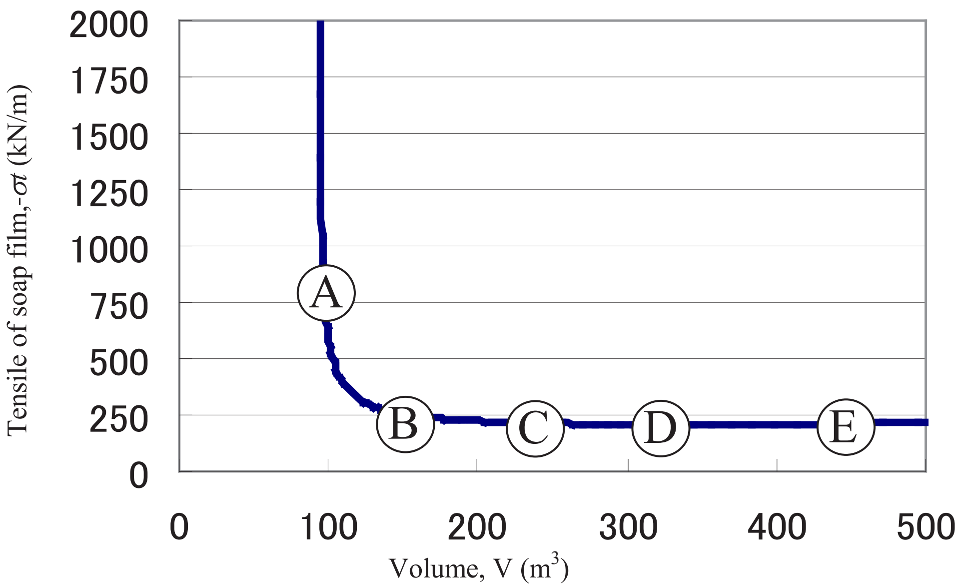

Figure-7 : The relation of tension of soap film, σt and volume, V

Similar to the previous analysis, the minimal value of tension for soap film also exists (marked as **D**) and so as the solution that surpasses it. As the structure is continuously deforming and the surface volume increases, it is clear that the deformed hexagonal shape structure seems to turn into a 'droplet' shape. However, in this analysis, the water pressure that works on the nodes which are located on the surface could not be assumed as zero. This is due to the approximation of polyhedron structure where all nodes are scattered in a three dimensional area, it is not possible to assume the external force as zero which assumes that all nodes are equal in a plane area. Thus, a real 'droplet' could not be achieved exactly as seen in nature.

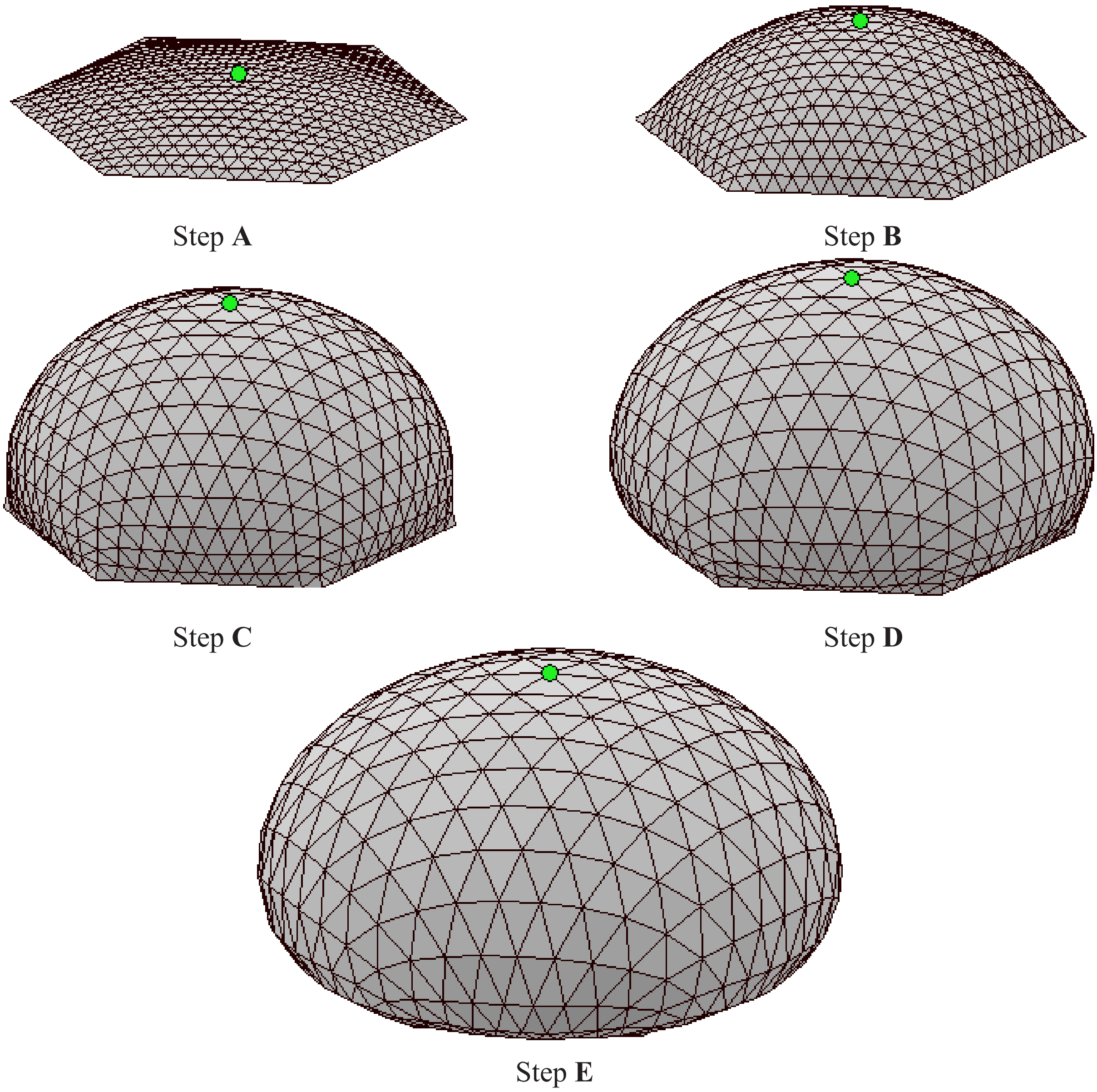

Figure-8 : Deformation diagram for soap film structure (Case 2)

In conclusion, by applying this method, an isotonic tension on an equilibrium surface could be obtained even though the water pressure is applied from beneath the structure.

4 CONCLUSION

In this paper, the idea of 'the simultaneous control averaging tension' is proposed based on the increment method by the simultaneous control.

The proposed method is the improvement of 'the simultaneous control averaging internal pressure' proposed by the authors, and in this method, the mean value of converted tension for all nodes are calculated, the 'converted average of tension' is renewed and converged in each iteration step.

Therefore, in the proposed method, the tension of soap film, obtained from given value of loading condition, is able to be applied as the initial tension for the surface. Compared to 'the simultaneous control averaging internal pressure' which calculates nodal force as an unknown quantity, it is more rational analysis flow if it is applied in the design calculation.

In addition, for the proposed method, the adaptability of location dependency loading was improved gradually. With this method, cases such as isolating fresh water from sea water by using embedded membrane structure tank or a gas container made out by a giant membrane structure in outer space and etc. is the applications in the future. Finally, new morphology creation and expansion are expected to be a reality.

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