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Advanced cutting pattern generation – Consideration of structural requirements in the optimization process

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Summary. This paper presents extensions to optimized cutting pattern generation through inverse engineering regarding structural requirements. The optimized cutting pattern generation through inverse engineering is a general approach for the cutting pattern generation which is based on the description of the underlying mechanical problem. The three dimensional surface, which is defined through the form finding process, represents the final structure after manufacturing. For this surface the coordinates in three dimensional space Ω3D and the finally desired prestress state $\boldsymbol{\sigma}_{prestress}$ are known. The aim is to find a surface in a two dimensional space $\boldsymbol{\Omega}_{2D}$ which minimizes the difference between the elastic stresses $\boldsymbol{\sigma}_{el,2D\rightarrow 3D}$ arising through the manufacturing process and the final prestress $\boldsymbol{\sigma}_{prestress}$. Thus the cutting pattern generation leads to an optimization problem, were the positions of the nodes in the two dimensional space $\boldsymbol{\Omega}_{2D}$ are the design variables. In this paper various improvements to the method will be shown. The influence of the seam lines to the stress distribution in the membrane is investigated. Additionally, the control of equal edge length for associated patterns is an example for important enhancement.

1 INTRODUCTION

Membrane Structures are lightweight structures, which combine optimal stress state of the material with an impressive language of shapes [1]. The shape of membrane structures is defined by the equilibrium of surface stress and cable edge forces in tension. The process of find the shape in equilibrium is known as form finding. In the past a various number of methods are developed to solve the inverse problem of form finding [2] – [11]. Recent researches in the field of Fluid-Structure-Interaction accomplish the design process with the strong possibility of simulating the coupled problem of wind and membrane structure [12]. Throughout the whole design process of membrane structures the variation of prestress constitutes the main shaping parameter. Details of cutting pattern and compensation are affected by residual stresses from developing curved surfaces into the plane and anisotropic material properties. With the knowledge of this, numerical methods for the design and analysis of membrane structures should be able to deal with all sources of stress state in a proper way. In the next sections, methods for a proper cutting pattern generation of membrane structures, which is able to treat the prestress and residual states of stress in a correct continuum mechanical way.

2 CUTTING PATTERN GENERATION

It is well known that a general doubly curved surface cannot be developed into a plane without compromises which results in additional residual stresses when the structure will be erected. In addition, the elastic deformation due to pre-stress has to be compensated. Typically, a two stage procedure is applied consisting of (i) forced flattening of the curved surface into a plane by pure geometrical considerations and (ii) compensation of both, the intended pre-stress and the additional elastic stresses of the flattening procedure [13]. Usually, an additional problem occurs if the flattening strains are determined using the curved, final surface as undeformed reference geometry, thus neglecting the correct erecting procedure. In highly curved regions the error might be remarkably large.

2.1 Optimized Cutting Pattern Generation

An alternative approach is suggested which uses ideas from the inverse engineering [14], [15]. The idea is to correctly simulate the erection procedure from a plane cutting pattern as undeformed reference geometry to the final deformed surface as defined by the design stresses. Consequently, the definition of strains is non-conventional, i.e. inverse, as the coordinates of the undeformed cutting pattern are introduced as unknowns. The proper cutting pattern layout is found by an optimization technique by minimizing the deviation of the stresses due to pre-stress and elastic deformation from the defined stress distribution of form finding. The advantages of this procedure are that (i) the true erection process is modelled, (ii) automatically, all sources of stress deviation are correctly resolved, e.g. residual stresses from development, (iii) all mechanical and geometrical reasons of compensation are considered, and (iv) again, the procedure is consistently embedded into non-linear continuum mechanics allowing for a formulation as close as possible to reality. Figure 1 roughly visualizes the optimization procedure. The challenge in this way of doing cutting pattern generation is to handle the numerical solution strategies for the optimization problem. In [15] two different methods of solving the optimization problem are reviewed. Both of them are well known methods for solving unconstrained optimization problems. In the following, both methods are briefly discussed.

Method I: Least-squares optimization

$$\min_{\mathbf{X}} \rightarrow \Pi = \frac{1}{2} t \int_{\Omega_{3D}} \left(\boldsymbol{\sigma}_{el,2D\rightarrow 3D} - \boldsymbol{\sigma}_{ps} \right) : \left(\boldsymbol{\sigma}_{el,2D\rightarrow 3D} - \boldsymbol{\sigma}_{ps} \right) da_{3D} \tag{1}$$

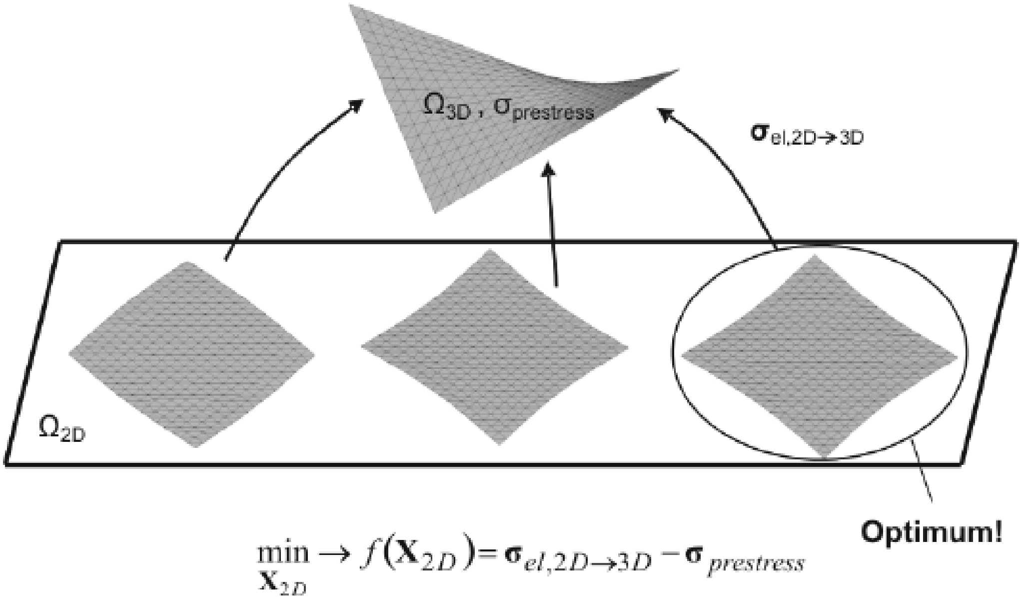

Fig. 1: Optimization problem for cutting pattern generation

A necessary condition for a minimum in the least squares is a stationary point in the functional Π w.r.t. a variation in the reference geometry. This results in the following variational form of the optimization problem:

$$\delta w^{I} = t \int_{\Omega_{3D}} \left(\boldsymbol{\sigma}_{el,2D\rightarrow 3D} - \boldsymbol{\sigma}_{ps}\right) : \frac{\partial \boldsymbol{\sigma}_{el,2D\rightarrow 3D}}{\partial \mathbf{X}} \delta \mathbf{X} da_{3D} = 0 \tag{2}$$

To solve the nonlinear field equation (2) standard procedures like the finite element method in combination with the Newton-Raphson method are applied. It turned out in [15] that equation (2) shows a very small convergence radius due to the non-convex character of the optimization problem. This disadvantage could be avoided by method II.

Method II: Minimization of the “stress difference” energy

The idea of method II is to solve the optimization of the stress difference in a “mean way”. To do so a Galerkin approach is used. In this case the stress difference is integrated over the domain and a weighting function is applied to the stress difference. Again, a variation of the equation should lead to the optimum cutting pattern (see eq. (3)).

$$\delta w^{II} = t \int_{\Omega_{3}} \left(\boldsymbol{\sigma}_{el,2D\rightarrow 3D} - \boldsymbol{\sigma}_{ps}\right) : \delta \boldsymbol{\eta} da_{3D} = 0 \tag{3}$$

To avoid the non-convexity of the optimization the weighting function is chosen as the Euler-

Almansi strains since they are energetically conjugated to Cauchy stresses (see eq. (4)).

$$\delta w^{II} = t \int_{\Omega_3} \left(\boldsymbol{\sigma}_{el,2D\to3D} - \boldsymbol{\sigma}_{ps} \right) : \frac{\partial \mathbf{e}_{el,2D\to3D}}{\partial \mathbf{X}} \delta \mathbf{X} da_{3D} = 0 \tag{4}$$

With both methods the resulting cutting patterns are only slightly different. A detailed comparison and more details about the numerical issues of both methods could be found in [14] – [15].

2.2 Consideration of seam stiffness in the computation

After the computation of the cutting patterns, an important step is quantification of the residual stresses in the membrane. These are occurring due to the fact that a non-developable surface could not be developed into a flat surface without any compromises. This results in residual stresses after the simulation of the erection process. From a continuum mechanical point of view, this means that the cutting patterns are the reference configuration and the configuration from form finding is an intermediate stage which is not totally in equilibrium. Performing a geometrical nonlinear analysis to this configuration will result in a slight displacement which ends up in the residual stresses. Starting with this in mind, the exact description of the erection process is crucial. There are several of influences to the computation. One of them is the correct description of the geometry, respectively the structural members of the structure. From this point of view it is totally clear that the modelling of the seams is an important step to the “right” structural model. The influence of the seams is related to the fact that the membrane is doubled in these regions since overlapping material is needed for joining the adjacent strips together. To take this issue into account in the computation, cable elements are included which consider the stiffness from this doubled material. The question which arises is how large is the influence of this doubled material in the membrane? There is no unique answer this. Typical examples for a noticeable effect of the concentration of material are highpoints of membranes were a concentration of seam lines is located. Figure 2 shows the well known Chinese hat membrane were the seam lines are concentrated towards the upper ring. The diagram in figure 2 shows the change in stresses σ in the membrane and change in forces F in the cable elements by increasing the cross section area A of the seam line cable elements. It can be seen that the membrane stresses are decreasing while the forces in the cable elements increase for larger seam line cross section areas. What is very interesting in this example is the amount of stress decrease in the membrane. For a ratio of two for A to A_{max} the membrane stresses decrease to 65% of the initial stresses. This effect is not equal for each kind of membrane and each situation but this example shows that the effect should be investigated and included in each computation of a membrane structure.

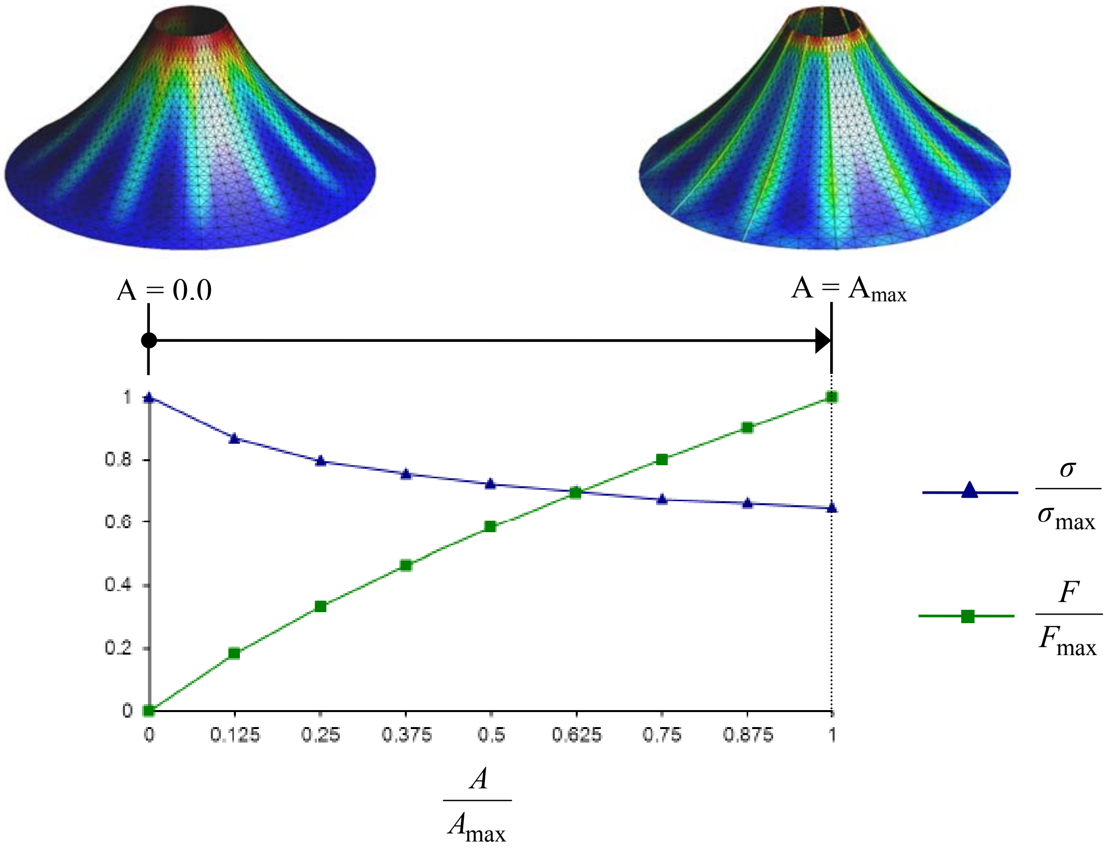

Fig.2: Influence of seam line stiffness

2.3 Control the same length of adjacent edges in cutting pattern generation

When building a membrane structure the process of erection is structured like (i) take the patterns of the membrane and join them together, (ii) erect the primer structure (e.g. steel frames) and (iii) mount the membrane into it. In the first step of joining the patterns it is obvious that the adjacent edges of the pattern need to fit together or at least must have the same length. When neglecting this constraint the resulting membrane will show wrinkles along the seam line due to the fact that the initial strains are included into the membrane by joining unequal edges together. From the numerical point of view the requirement of same edge length of adjacent edges shows up in an equality constraint in the optimization problem. In figure 3 the requirement of same edge length of adjacent edges is shown for the example of a four point tent consisting of 6 patterns with 5 seam lines.

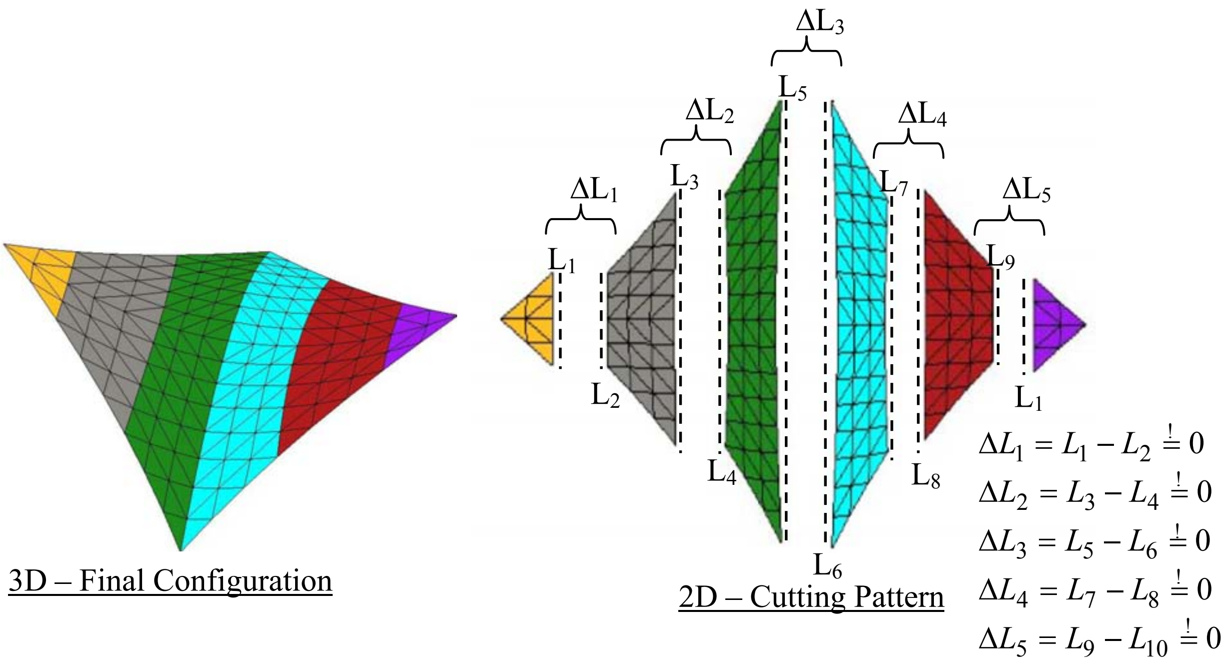

Fig. 3: Same length of adjacent patterns

From a mathematical point of view the description of the equality constraint is rather simple. The differences of the lengths should be 0 for all seams. That allows the formulation of an equality constraint for each seam which is shown in equation (5).

$$h_i(\mathbf{X}) = \Delta L_i = 0 \tag{5}$$

All differences of the lengths are added to get a single equation of the equality constraint. To avoid cancellations from positive and negative length differences the square of the values is chosen. With this equality constraint the optimization problem by using the Least-Squares approach is stated in equation (6).

$$\begin{aligned} &\min \rightarrow f(\mathbf{X}) = \frac{1}{2} t \int_{\Omega_{3D}} \left(\boldsymbol{\sigma}_{el,2D\rightarrow 3D} - \boldsymbol{\sigma}_{ps}\right) : \left(\boldsymbol{\sigma}_{el,2D\rightarrow 3D} - \boldsymbol{\sigma}_{ps}\right) da_{3D} \\ &\text{such that : } h_i(X) = \Delta L_i = 0 \end{aligned} \tag{6}$$

Knowing from section 2.1 that the objective function has a relatively small convergence radius the starting point is a crucial issue to the optimization problem. Additionally we know from 2.1 that the method II (Minimization of the “stress difference” energy) is a suitable related problem and provides a very good starting point for the optimization problem. In that sense a staggered approach is used to solve the optimization problem: First solve the unconstraint problem with method II. Use the resulting cutting pattern as the starting geometry for the constraint optimization problem which can be tackled by various optimization algorithms. The chosen solution strategy herein is the Lagrange method. The Lagrangian function for the unconstraint problem is given in equation (7).

$$L(\mathbf{X}, \mu_i) = f(\mathbf{X}) + \mu_i h_i(\mathbf{X}) \quad (7)$$

The corresponding Karush-Kuhn-Tucker conditions are given in equation (8).

$$\begin{aligned} \nabla f(\mathbf{X}) + \mu \nabla h_i(\mathbf{X}) &= 0 \\ h_i(\mathbf{X}) &= 0 \end{aligned} \quad (8)$$

For solving this nonlinear system of equations the Newton-Raphson method can be used.

3 CONCLUSIONS

Computational methods for the optimized cutting pattern generation have been presented. The governing equations for cutting pattern generation were derived by using the idea of inverse engineering from a full continuum mechanical approach. Improvements regarding structural requirements have been included into the optimization process. Concerning seam line stiffness and controlling length of adjacent patterns are tackled directly in optimization process of the cutting pattern

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