VI International Conference on Textile Composites and Inflatable Structures
STRUCTURAL MEMBRANES 2013
K.-U.Bletzinger, B. Kröplin and E. Oñate (Eds)

STRESS ANALYSIS OF INFLATED POLYHEDRA FOR THE 32-PANEL SOCCER BALL

ANDRÁS LENGYEL[*] AND KRISZTIÁN HINCZ[†]

[*]Department of Structural Mechanics
Budapest University of Technology and Economics
Műegyetem rkp. 3, H-1521 Budapest, Hungary
e-mail: lengyel@eik.bme.hu, web page: https://www.me.bme.hu/en

[†] Department of Structural Mechanics
Budapest University of Technology and Economics
Műegyetem rkp. 3, H-1521 Budapest, Hungary
e-mail: hincz@eik.bme.hu, web page: https://www.me.bme.hu/en

Key words: Soccer ball, polyhedron, inflation of membrane, dynamic relaxation

Abstract. This paper presents a stress analysis of a membrane modelling the 32-panel soccer ball. The most popular soccer ball type and three variations are considered. The discretized mesh of the stress-free polyhedron-shaped membrane is subjected to internal pressure and the sphericity and the stress distribution of the models are compared.

1 INTRODUCTION

In spite of that soccer has been one of the most influential sports games for decades, scientific analysis on the geometry of the soccer ball has received significantly less attention. Since its origin in the 19th century the history of modern soccer witnessed a vast variety of balls shapes. They mostly consisted of flat panels of leather sewn together and inflated by means of a bladder. The 32-panel icosahedral ball type has become the most widely used model since the 1970's and is still one of the models sold in the largest numbers worldwide. Though materials used for fabrication turned from leather to synthetics, its composition remained an assembly of twelve pentagonal and twenty hexagonal panels representing one of the Archimedean solids, the truncated icosahedron. It served as the official ball for several world championships.

A recent approach in soccer ball design introduced curved panels thermally bonded in order to create a more precise approximation of the sphere (e.g. Teamgeist or Jubilani), however, the 32-panel ball has remained the most popular ball.

Early attempts to create a round ball of flat pieces based on heuristic constructions lacking probably any scientific foundations. As the game gained increasing importance ball shapes based on regular and semiregular polyhedra were created requiring geometrical background for the analysis. There have been a number of approaches for the definiton of the roundness of spatial solids from different fields of science. Among them the most widely used term considered in mathematics is the so-called isoperimetric quotient (IQ), introduced by Polya [1] in its present form:

$$IQ = 36\pi \frac{V^2}{A^3} \tag{1}$$

where V and A denote the volume and the surface area of the body, respectively. The scaling factor 36π is introduced so that IQ yields 1 for the unit sphere. A central problem in mathematics of spatial bodies is the isoperimetric problem, which states: among a set of bodies of given surface area, which one encloses the largest volume (largest IQ), i.e. which one approximates the sphere the most closely?

In a previous study the authors of this paper addressed the issue of roundness of the 32-panel icosahedral soccer ball [2], using a set of geometrical properties to quantify the roundness of a spatial solid introduced previously by the first author [3]. This paper considers four different types of 32-panel soccer ball geometry, all commercially available: the truncated icosahedron, which is an Archimedean solid, the Geo 0.84 model by Nike, the equal panel area ball by Puma, and the Hyperball® by Prof. P. Huybers [4]. Our aim is to simulate the inflation of a polyhedron-shaped membrane representing the soccer ball. Discretized geometric models of all types are created and subjected to internal pressure. The key parameters of the analysis are the density of the finite mesh, the applied pressure, and the material stiffness of the membrane. The roundness of the inflated shapes are evaluated and compared. The next three sections outline the model for the computations, the most important results, and the major conclusions.

2 MODELLING

2.1 Geometrical model

The initial (undeformed) shape of the ball is defined by the truncated icosahedron or its variations. The *first* model is the Archimedean truncation of the icosahedron representing the most popular ball configuration. The Archimedean truncation yields regular pentagons and hexagons, i.e. all edges are of the same length. The *second* model is Geo 0.84 by Nike where the ratio of edges between hexagons and edges of the pentagon is approximately 0.839. This configuration is obtained by a nonregular truncation of the icosahedron resulting in larger pentagons. It is claimed that such truncation results in the optimal stress distribution in the inflated structure. The *third* model (Puma) is a truncation of the icosahedron in a way that it yields pentagons and hexagons of equal surface area. The *fourth* model is not purely a truncation of the icosahedron but a more sophisticated form even though it is made of 32 panels. The geometry of the Hyperball is derived from the isodistant truncation of the icosahedron (i.e. when all panels are at

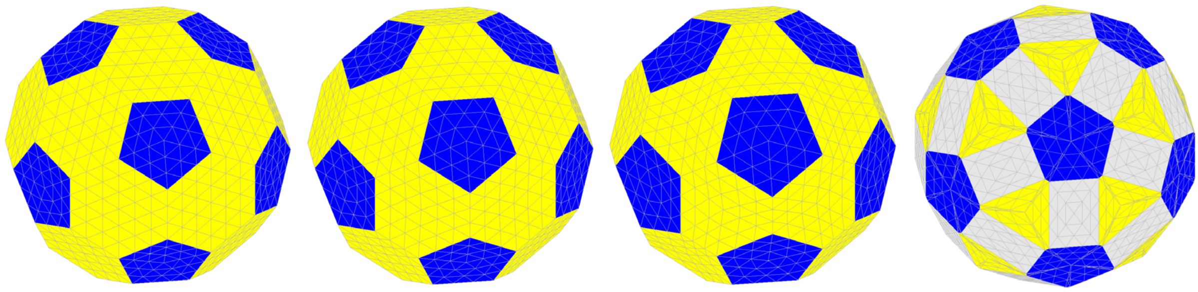

Figure 1: Triangular mesh models for four ball types: (left to right) the truncated icosahedron (Archimedean), Nike Geo 0.84, Puma equal face area ball, Hyperball.

the same distance from the centre of the inscribed sphere). This polyhedron is modified by further truncation of the common edges of hexagons forming rectangular faces. Both truncations are defined in an isodistant way meant to produce the roundest shape. The key feature of this model is the reduction of the large number of faces by dividing the rectangles to triangles and trapezoids, which then can be joined with the neighbouring pentagons and hexagons. (A detailed description is given in [4].) Sketches of the four models are given in Figure 1.

In order to have a realistic approximation of the sphere, all polyhedron faces are divided into a number of triangles. The triangulation has been performed in a way to preserve the icosahedral symmetry of the surface. All pentagons, hexagons, and rectangles are first divided into five, six, and four equal triangles, respectively by drawing radial lines to the vertices from the centroids of the faces, which are then further divided into smaller triangles by placing points along the edges at equal distances. In the case of the fourth model, the ratio of the sides of some of the triangles were inconveniently large, therefore an alternative triangulation has been applied in parts of the domain, also preserving symmetry.

2.2 Structural model

The surface of the polyhedron is regarded as a thin membrane of isotropic material characterized by its in-plane stiffness and Poisson's ratio. The triangular elements may develop stresses and strains in the plane determined by the position of their vertices and free rotational displacements may occur at the edges. Internal pressure inside the closed membrane is represented by a distributed load of equal intensity on all elements.

We apply the dinamic relaxation method (DRM) [5] to determine the equilibrium shape of the structure. It is an iterative technique widely used for tensile structures. Fictitious lumped masses are assigned to the vertices of the mesh. In each time step of the iteration, the position, the velocity, and the acceleration of each vertex is computed, and the equations of motions are numerically solved. The final deformed shape is obtained when the vertices are balanced and have zero velocity.

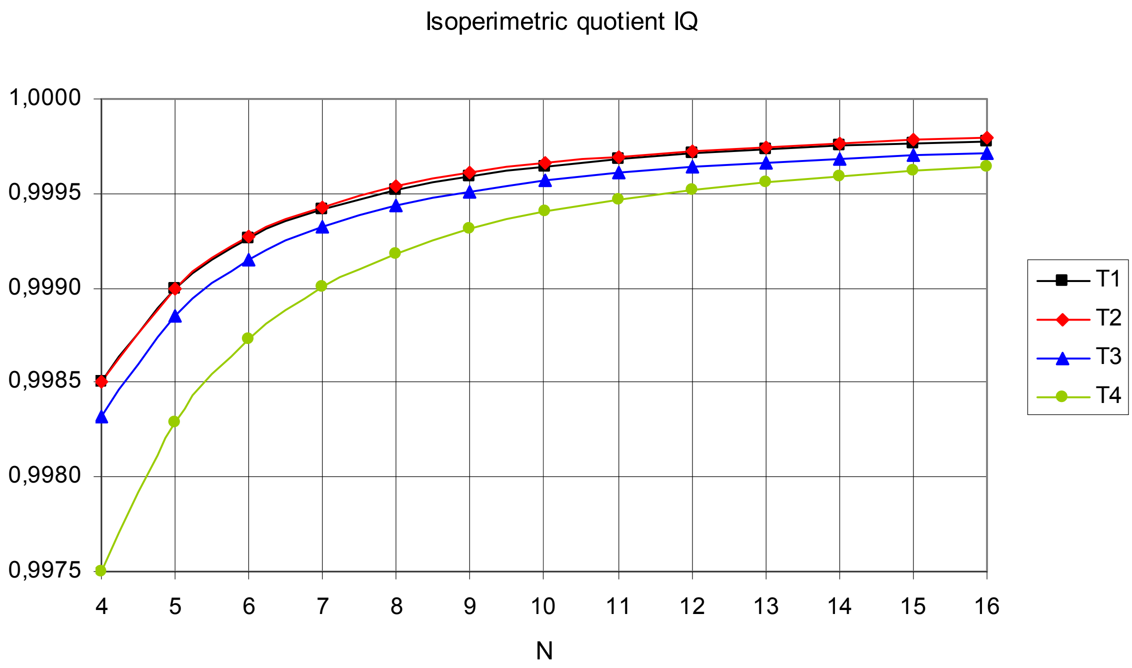

Figure 2: Isoperimeric quotient IQ for the four ball types (1: the truncated icosahedron (Archimedean), 2: Nike Geo 0.84, 3: Puma equal face area ball, 4: Hyperball) against mesh density parameter N.

3 RESULTS

All initial (uninflated) models are scaled to have volume equal to that of a 100 mm radius sphere. Once DRM yields the final equilibrium positions of all nodes in the mesh, geometrical and mechanical evaluation of the deformed shape is performed. The isoperimetric quotient IQ is computed via the surface area A and volume V. Values of IQ show convergence with respect to the density of the mesh. The density is characterized by the number N of the sections along the edges of any face of the polyhedra. (E.g. $N = 3$ is applied for the generation of shapes in Figure 1.) It is found in a series of test computations that IQ does not vary significantly for $N > 12$ (see Figure 2) therefore density $N = 12$ has been applied for further analysis.

The best isoperimetric quotient has been obtained for the standard model and for Geo 0.84, followed by the equal face area model, and the Hyperball.

The stiffness of the membrane and the applied pressure have been set to obtain realistic inflated shapes. FIFA Inspected quality for soccer balls requires balls to have circumference 68 to 70 cm among various specifications. We set normal stiffness $S = 40\,\mathrm{kN/m}$, Poisson's ratio $\nu = 0.2$, and internal pressure $p = 1.0\,\mathrm{atm}$ for the analysis.

Stresses for all four models have been computed, see Figures 3 to 6. (In this paper the term 'stresses' refer to specific forces in sections of the membrane in units of force/length as customary for tensile structures and not in units of force/area as in continuum mechanics.)

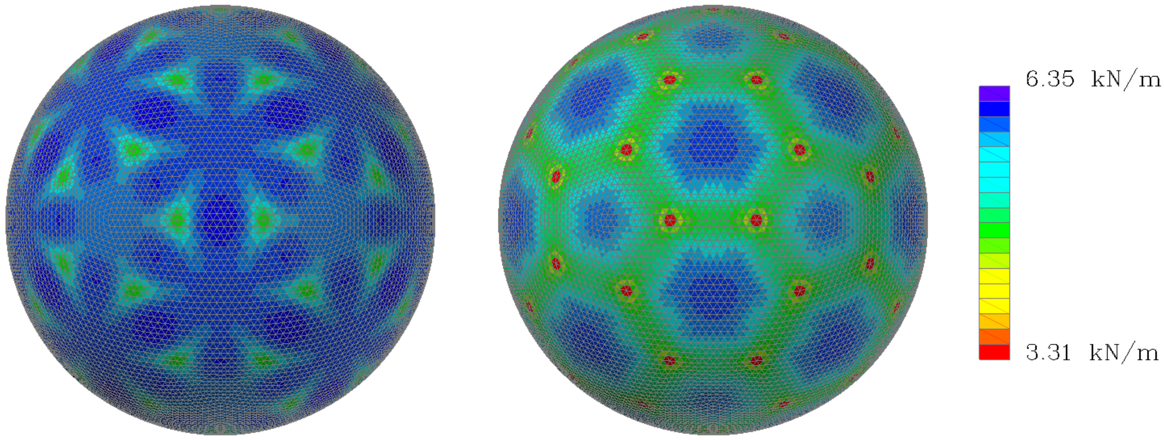

Figure 3: Distribution of maximum (left) and minimum (right) principal stresses in the membrane for model 1. Blue and red colours denote high and low stress levels, respectively.

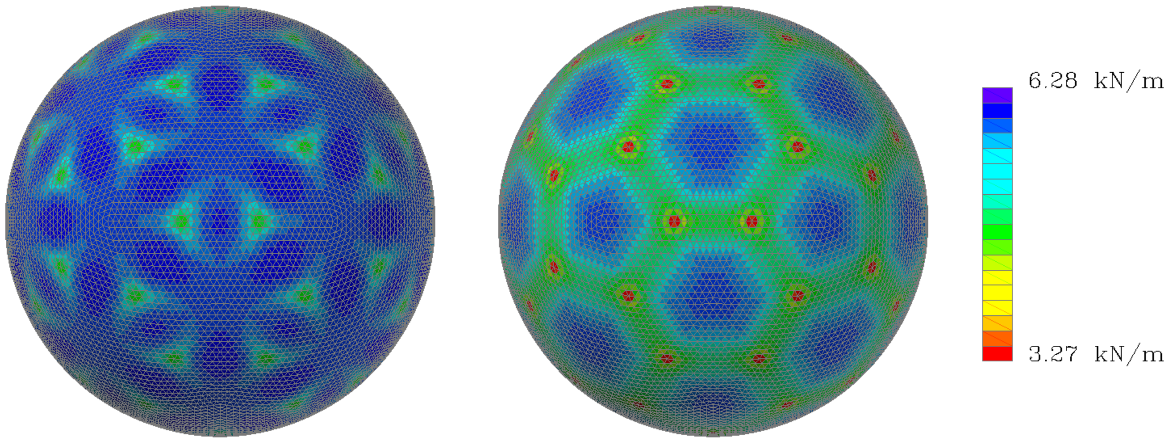

Figure 4: Distribution of maximum (left) and minimum (right) principal stresses in the membrane for model 2. Blue and red colours denote high and low stress levels, respectively.

Figure 3 shows the distribution of principal stresses for model 1. The maximum principal stresses (left) are large in the middle of the hexagonal panels and slightly smaller in the pentagonal faces. Strips of large stresses connect the centres of neighbouring faces indicating the directions of characteristic stretching in the membrane. Local low stress regions are formed around the vertices of the panels. Minimum principal stresses (right) range over a wider interval. Peaks are measured in the middle of the hexagonal panels whereas peaks in the pentagonal faces are slightly smaller. Stresses along the edges are approximately uniform with medium intensity, and the smallest values are again obtained around the vertices. It indicates that these regions are moderately affected by inflation.

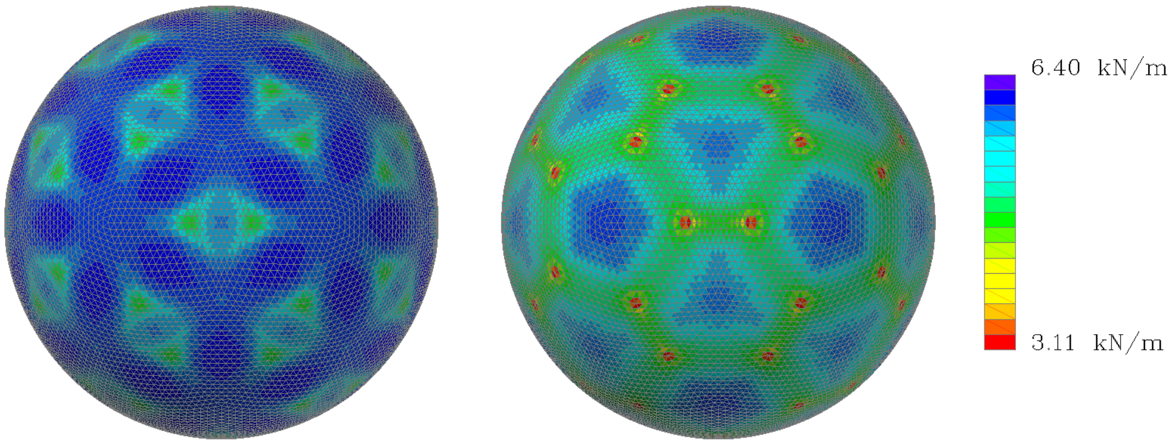

Figure 5: Distribution of maximum (left) and minimum (right) principal stresses in the membrane for model 3. Blue and red colours denote high and low stress levels, respectively.

Stresses of models 2 (see Figure 4) and 3 (see Figure 5) are characterisically similar to those of model 1. The pentagonal faces remain regular but the symmetry of the hexagonal faces reduces to D_3. With larger pentagons, the ratio of the long and the short edges of hexagons increases. Distribution of stresses corresponds to the symmetry of the shape of the structure. It is found that peaks of the maximum principal stresses occur in the strips perpendicular to long edges while stresses at the centres of panels and in strips near short edges decrease. Distribution of minimum principal stresses varies moderately. As the size of the pentagonal faces increase relative to that of the hexagonal ones, the peaks in the middle of all faces increase or decrease with the surface area, accordingly, compare models 1 to 3.

The layout of model 4 (see Figure 6) is characteristically different from the other three regarding the number of faces and the ratio of edges. Three sides of each hexagons are significantly smaller than the rest resulting in a significantly different stress distribution. Regions around the vertices are joined in pairs, and the hexagonal patterns become practically triangular. Largest values of the maximum principal stress occur in the hexagonal and the rectangular faces forming a contiguous region of nearly uniform stresses surrounding isolated pentagonal parts of smaller stress. It indicates that the loadbearing is primarily provided by the frame of hexagons and rectangles. (In the actual fabrication rectangles are divided and joined with the neighbouring faces.) The minimum principal stresses are more evenly distributed than in the previous cases. Smallest values again occur around the vertices.

4 CONCLUSIONS

Maximum and minimum stresses as well as isoperimetric quotients for all inflated models are summarized in Table 1. Ranking of the models with respect to IQ in decreasing

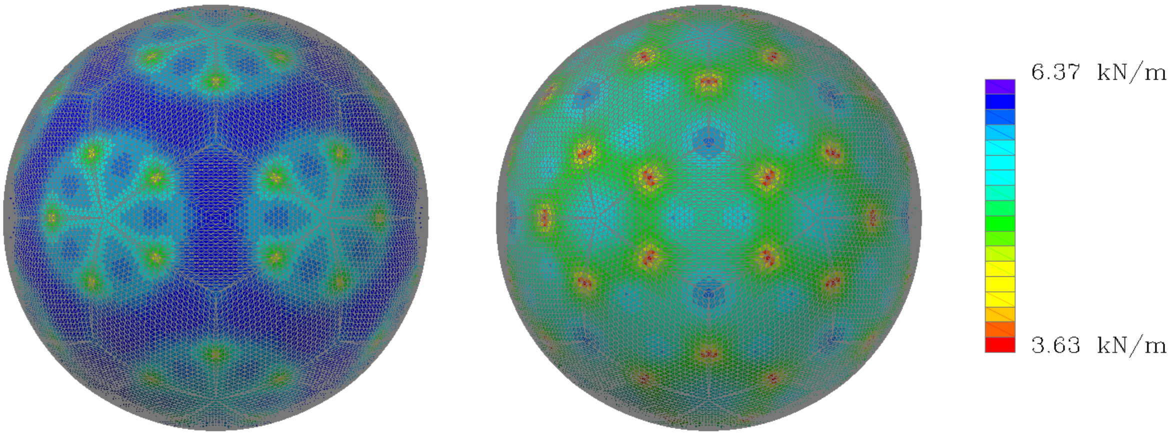

Figure 6: Distribution of maximum (left) and minimum (right) principal stresses in the membrane for model 4. Blue and red colours denote high and low stress levels, respectively.

order is Nike Geo 0.84, the standard truncated icosahedron, the Puma equal face area ball, and the Hyperball. It is found that the largest σ_1 stresses are practically in inverse correspondence with roundness: $\sigma_{1,max}$ decreases with IQ increasing except that Hyperball has rank 3. It suggests that lower peak stresses are expected with better roundness. Also, with increasing IQ, the smallest σ_1 values in the membrane increase closing the gap between maximum and minimum, i.e. more evenly distributed stress field is expected. Principal stresses σ_2 vary more, though it can also be concluded that among the truncated icosahedral models the gap between the maximum and minimum decreases with IQ increasing. It is interesting to note that Hyperball performs significantly better in this respect, which can be attributed to the double truncation to smoothen the peaks of the polyhedron. Overall, Geo 0.84 and the standard ball produce the best roundness values and are the most efficient in reducing the largest stresses and their deviation. The special geometric configuration of the Hyperball proves capable of involving vertex regions into the loadbearing.

ACKNOWLEDGEMENTS

The research reported here was partially supported by OTKA grant no. K81146 awarded by the Hungarian Scientific Research Fund. This paper was also supported by the János Bolyai Research Scholarship of the Hungarian Academy of Sciences.

REFERENCES

[1] Pólya, G. *Mathematics and Plausible Reasoning Vol. I. Induction and Analogy in Mathematics*, Princeton University Press, 1954.

[2] Lengyel, A. and Hincz K. Optimization of the roundness of the soccer ball. In:

Table 1: Comparison of four ball models with respect to the largest and the smallest values of principal stresses σ_1 and σ_2, and the isoperimetric quotient IQ.

	Model	Principal stresses (kN/m)				IQ
		$\sigma_{1,max}$	$\sigma_{1,min}$	$\sigma_{2,max}$	$\sigma_{2,min}$	
1	Standard TI	6.350	4.769	6.082	3.310	0.999710726
2	Nike Geo 0.84	6.279	4.819	6.032	3.274	0.999725590
3	Puma equal face area	6.399	4.666	6.107	3.109	0.999639643
4	Hyperball	6.366	4.419	5.943	3.627	0.999519980

Domingo, A, Lázaro, C, (eds) *Evolution and Trends in Design, Analysis and Construction of Shell and Spatial Structures: Proceedings of the IASS Symposium 2009, Valencia.* Valencia, Spain, 2009.09.28-2009.10.02. pp. 1100-1110.

[3] Geiger, A. and Lengyel, A. On the roundness of polyhedra for soccer ball design. *Int J Space Struct* **23**:(3) pp. 133-142. (2008)

[4] Huybers, P. *A new kind of football with improved roundness*, International Patent Application PCT/NL2008/002421, dated 17 March 2008.

[5] Day, A.S. An introduction to dynamic relaxation. *The Engineer* (1965) **219**: 218-221.