

## FORMFINDING AND STATICAL ANALYSIS OF CABLE NETS WITH FLEXIBLE COVERS

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#### ABSTRACT

Nowadays cable nets are built as roofs, enclosures for animals in zoos, facades, safety barriers in buildings, etc. Some of the advantages are high resistance against damage and high transparency.

In the first part of this paper the problems of the cable net calculation are described. The key problem is the generation of the assembly plan. This procedure is according to the cutting pattern generation process. In case of cable nets we have to create as big as possible fields. The reason for this purpose is the boundary patterning, this means: the patterns are restricted to the region along the boundary cables and equidistant rectangular meshes remain in the inner part of the fields.

The calculation of the formfinding and the statical analysis of equidistant rectangular nets will be pointed out step by step. We also show the specific characteristic of small sized 60°-degree cable nets, produced e. g. by the German company Carl Stahl. This kind of cable net is characterized by a high flexibility and the adaptability to free forms. By the so called S-twist in the cable pieces between the clamps we have to consider special statical properties. The usage of physical nonlinear material laws and the node stiffness caused by the clamps will be explained.

As opposed to the calculation of cable nets we have to consider additional material properties as shear- and crimp- stiffness for membranes and foils. The cutting pattern generation can be performed here as usual with surface seams.

The disadvantage of cable nets is that an additional cover is needed if the usage requires a weather protection. In the last chapter we will show the calculation of the composite material cable net – membrane cover.

## 1 INTRODUCTION

Because of the simple fabrication and assembly cable nets have equidistant meshes, in other words, they have constant node distances. Due to the fact that the calculation of the figure of equilibrium with force densities results always in irregular node distances, standard membrane calculation steps has to be used in a modified way. The flattening and remeshing of a pre-stressed 3d geometry (calculated by formfinding) is the first task in cable net calculation.

## 2 FLATTENING AND REMESHING USING KNOWN MATERIAL LAWS

After the formfinding calculation 3d shapes exist as triangle or polyline surface representations. The forms can be flattened by different theories (also if the formfinding process was calculated by using quadrangular meshes, as is usual for steel meshes). A new developed software module (EASY boundary mapping) maps the complete 3d surface under consideration of the following boundary conditions:

1. Equal area under consideration of minimal distortions energy. The area of the mapped surface in 2d is equal to the surface area in 3d.
2. Equal boundary distance under consideration of minimal distortions energy. The circumference of the mapped surface is equal to the circumference of the 3d surface.

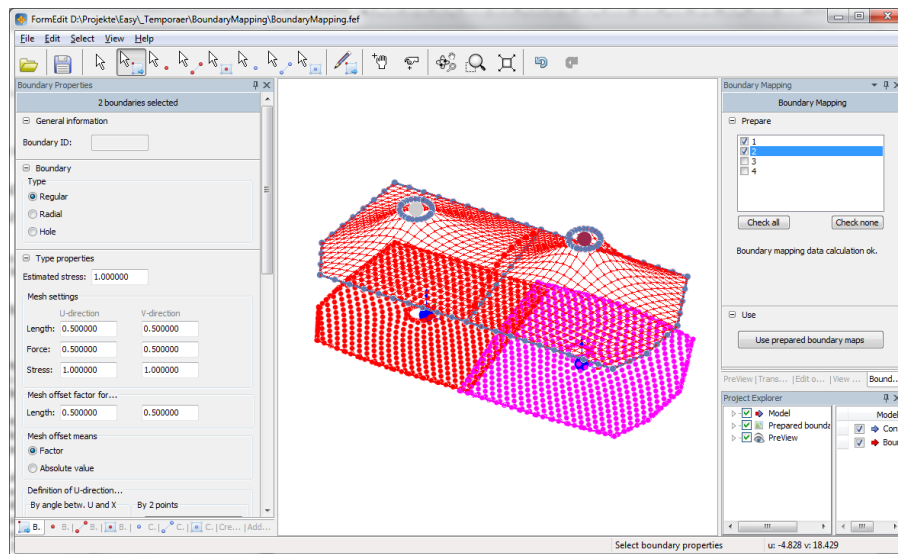


Figure 1: Equal area 3d mapping. The 3d surface was mapped to the red and violet area

For both procedures additional conditions can be introduced (alignment, specific angles, etc.). After the mapping process the boundary lines are extracted and prepared for the remeshing process automatically. The remeshing is performed under consideration of the given meshing parameters (mesh size, mesh angle, mesh direction, etc.)

The cable net characteristic of the steel mesh can be modelled during the remeshing process and additional adjustment elements for the simulation of complete meshes are introduced. With the help of the remeshed net a new formfinding and statical analysis under prestress and external loads can be performed. Initially a mesh model without additional node rigidity is used (mesh model 1).

### 3 SHAPE DETERMINATION AND STATICAL ANALYSIS – MESH MODEL 1

Based on the flat model generation (see 2) and linear material laws the cable net can now be hooked in the control point frame and the stresses can be visualized. The following calculation steps have to be performed:

1. Fast model generation (see 2).
2. Linear formfinding in the control frame (for the calculation of initial values of the nonlinear calculation).
3. Definition of force density controlled adjustment elements.
4. Assignment of linear material parameter.
5. Generation of the figure of equilibrium under prestress (nonlinear process).
6. Stress and deformation calculation under prestress and external loads (nonlinear processes).

For the static calculation a characteristic line, determined by the KIT (Karlsruhe Institute of Technology - Fachgebiet Bautechnologie), of the X-TEND steel mesh from CARL Stahl (diameter 1.5 mm, mesh size 60 mm) was used.

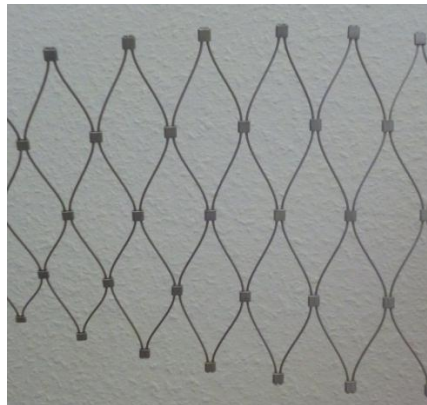


Figure 2: Stainless steel cable mesh X-TEND from Carl Stahl

The so called S-twist between the nodes was modelled by the nonlinear characteristic curve of the cable. Therefore nonlinear material properties of a defined cable net were introduced and calculated under different loads. In the case of nonlinear material properties the process is equivalent to the points 1-6 (see 3) except point 4 is replaced by “assignment of point-wise defined nonlinear characteristic curves”.

#### 4 SHAPE DETERMINATION AND STATICAL ANALYSIS – MESH MODEL 2

In mesh model 1 the cable nodes were mathematically assumed as fully hinged. In a next step (mesh model 2) we extended the model by introducing the stiffness of the connection. For the simulation of the node stiffness we examined 2 approaches:

1. Bending stiff beam elements.
2. Springs between the cable net meshes.

We tested and compared both possibilities. Due to the fact that both options lead to almost identical results and the easy usage we implemented Option 2 in our software.

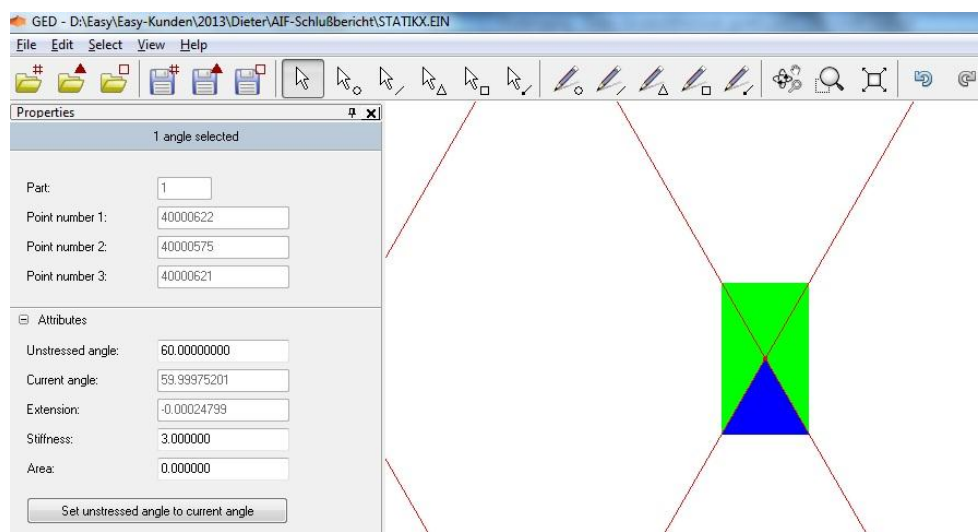


Figure 3: Visualisation of spring with opening angle and stiffness

The functional model was extended by springs. The node resistance value can be set individually between 2 cable directions. By doing this a realistic calculation of cable nets is available to users. This is based on the prerequisite that spring stiffness values and nonlinear characteristic curves for the cables with S-twist exist.

## 5 ADDITIONAL MATERIAL PROPERTIES FOR THE STATICAL CALCULATION OF MEMBRANES AND FOILS

A more precise description of the material behaviour is possible by introducing the shear-stiffness and also the so-called crimp-stiffness, which steers the interaction between warp ( $u$ ) and weft ( $v$ ). The extended model allows the statical calculation of membrane and foil structures with transverse extension and shear.

In formula (1) warp- ( $u$ ) and weft direction ( $v$ ) are dependent or correlated and shear is active.

$$\boldsymbol{\sigma} = \begin{bmatrix} \sigma_u \\ \sigma_v \\ \tau \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} & 0 \\ & m_{22} & 0 \\ sym. & & m_{33} \end{bmatrix} \begin{bmatrix} \varepsilon_u \\ \varepsilon_v \\ \Delta\alpha \end{bmatrix} = \mathbf{M} \cdot \boldsymbol{\varepsilon} \quad (1)$$

- $\sigma_u$ : stress in warp direction
- $\sigma_v$ : stress in weft direction
- $\tau$ : shear stress
- $m_{12}$ : crimp module
- $m_{11}$ : modulus of elasticity for warp
- $m_{22}$ : modulus of elasticity for weft
- $m_{33}$ : shear stiffness
- $\varepsilon_u$ : strain in warp
- $\varepsilon_v$ : strain in weft
- $\Delta\alpha$ : shear deformation

## 6 MEMBRANE PREPARATION FOR THE INTEGRATED STATICAL CALCULATION

In order to be able to calculate a cable net in combination with a membrane the calculation model has to connect both of them. We considered 2 possibilities:

1. The connection is made topologically, means with identical points on the membrane and the cable net.
2. A much more general solution was also implemented: The membrane is connected with the cable net on arbitrary points. Thereby the “connected” membrane point will be enslaved with 3 adjacent cable net points. The enslavement is achieved by retaining the natural coordinates during deformation.

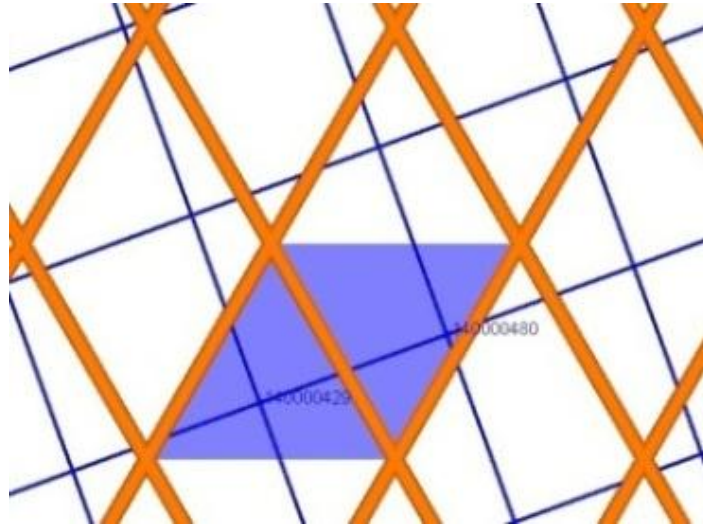


Figure 4: Membrane point 14000429 enslaved on 3 cable net points.

The single working steps for the connection of the cable net and membrane will be listed below:

1. 3d form generation of the cable net.
2. Cutting pattern generation of the membrane related to the cable net geometry.
3. Membrane points triangle enslavement to cable net.
4. Net generation of the cutting patterns.
5. Cable net - membrane model generation by using identical boundary points and triangle enslaved seam points.

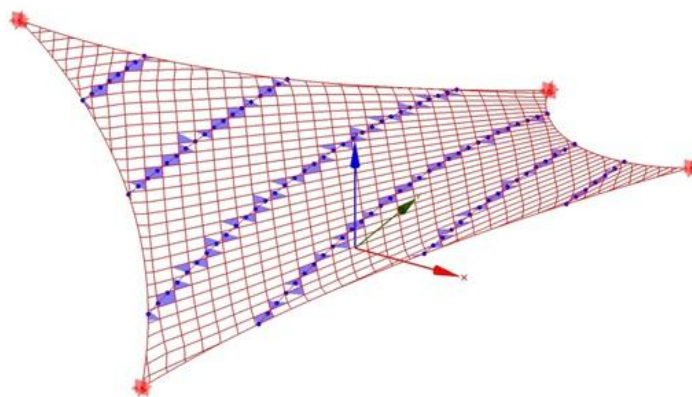


Figure 5: Triangle enslaved seam points

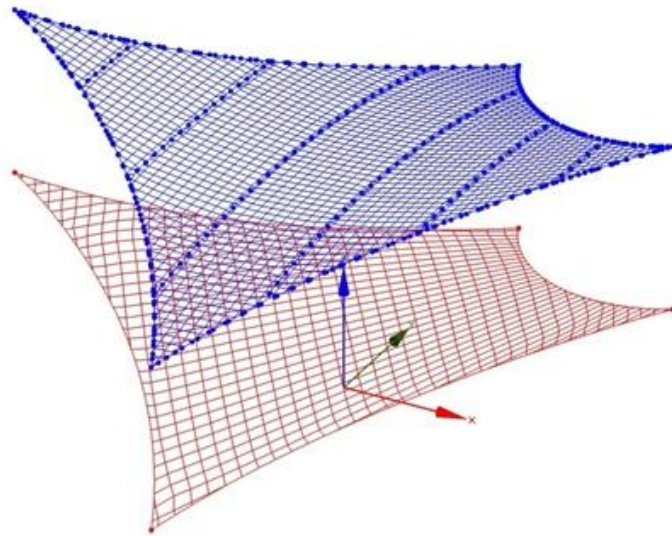


Figure 6: Complete system (shown with offset) – connected topologically. Cable net (red), membrane (blue)

## 7 MODEL DEVELOPMENT FOR THE INTEGRATED STATIC CALCULATION

We included the triangle enslaved points in the static programs whereby the calculation of connected models became possible. The connected models are describing the cable net and the membrane in a realistic way. By that we mean that mechanical characteristics of the membrane and cable net can be set in a correct way. Different membrane pre-stress conditions referring to a given cable net can be simulated. If the cable net changes, the process (chapter 6) has to be repeated (to generate new membrane cutting patterns). Particularly the application of external loads (wind, snow, etc.) on the models (chapter 6) and the contact problem of the triangle enslaved points were performed successfully.

The solution strategy for the contact problem can be described as follows: The “flexible” membrane point is connected to the “stiff” steel point case-by-case. That means: If a steel point is compressed by a membrane point contact exists, if a steel point resists tension forces, contact breaks down and the membrane point releases from the steel point. Thus the membrane point is free and no longer triangle enslaved. Compression and tension means in this context: force components are acting into or away from the triangle plane.



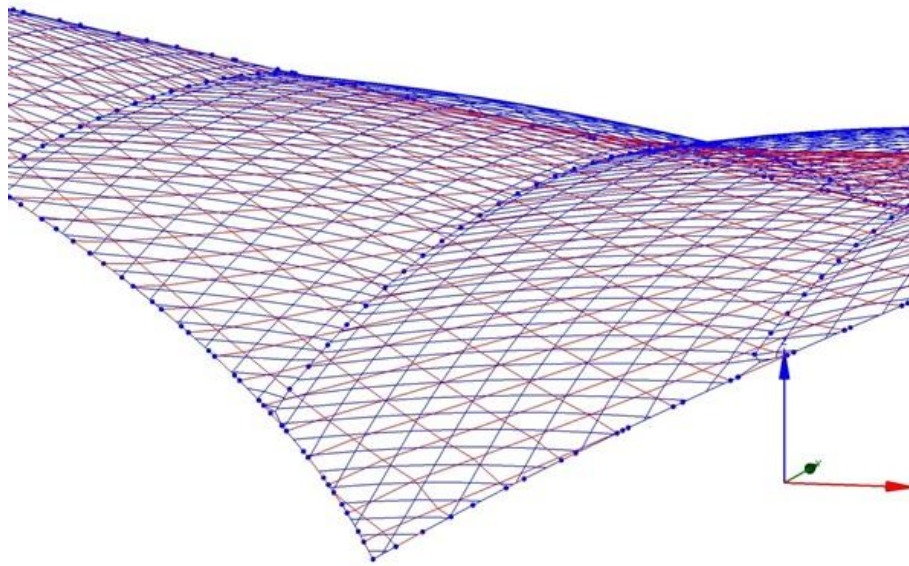


Figure 7: Wind load on a linked system

## 8 CONCLUSIONS

The results are based on a research project which was sponsored by the German Federal Ministry of Economics and Technology (BMWi). The Karlsruhe Institute of Technology (KIT) was our partner in this project. The project had some influence in the software package EASY from technet GmbH and the new modules have been proven successfully in practice.

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