

# EXPERIENCE ON THE IMPLEMENTATION OF A NONLINEAR MATERIAL MODEL FOR MEMBRANE FABRICS IN A FINITE ELEMENT PROGRAM

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**Key words:** Membrane Analysis, fabrics, warp fill behavior.

**Summary.** This paper describes the implementation of a nonlinear material law and presents examples to nonlinear warp fill behavior and wrinkling.

## 1 INTRODUCTION

Since 1996 SOFiSTiK provides finite element membrane analysis with a simple linear elastic orthotropic material model. The only material nonlinearity was that it could not carry compression and so simulates wrinkles.

Based on a paper in Tensinews [1] we implemented a real nonlinear material model. The model can be calibrated to experimental results and can handle the yarn parallel warp and fill relationship.

In this paper first experiences with this model in complex finite element calculations will be presented. Also the effect of additional shear stiffness and the problem of restressing after wrinkle occurrence will be discussed.

## 2 IMPLEMENTATION

The nonlinear behavior in [1] is expressed as a stress-strain relation. This means that for a given stress  $\sigma_w$  and  $\sigma_f$  ( $w$ =warp direction,  $f$ =fill direction) it gives a corresponding nonlinear strain  $\epsilon_w$  and  $\epsilon_f$ . The values of the stress-strain matrix depend on the ratio of  $\sigma_w$  to  $\sigma_f$  using the factors  $\gamma_w$  and  $\gamma_f$ :

$$\gamma_w = \frac{\sigma_w}{\sqrt{\sigma_w^2 + \sigma_f^2}} \quad \gamma_f = \frac{\sigma_f}{\sqrt{\sigma_w^2 + \sigma_f^2}} \quad (1)$$

$$\begin{bmatrix} \epsilon_w \\ \epsilon_f \end{bmatrix} = \begin{bmatrix} \frac{1}{E_w(\gamma_w)} & \frac{-\nu_{wf}}{E_w(\gamma_w)} \\ \frac{-\nu_{wf}}{E_w(\gamma_w)} & \frac{1}{E_f(\gamma_f)} \end{bmatrix} \begin{bmatrix} \sigma_w \\ \sigma_f \end{bmatrix} \quad (2)$$

$$\begin{aligned}
 E_w(\gamma_w) &= \Delta E_w \left( \gamma_w - \frac{1}{\sqrt{2}} \right) + E^{1:1}_w \\
 E_f(\gamma_f) &= \Delta E_f \left( \gamma_f - \frac{1}{\sqrt{2}} \right) + E^{1:1}_f
 \end{aligned}
 \tag{3}$$

The finite element program implementation uses a quick internal iteration for the inverse problem, analyzing the stress for a given strain of the actual finite element iteration.

In [1] some typical material parameters are shown, e.g. for Mehler Technologies Valmex FR700(I):

$$\begin{aligned}
 E(1:1)_w &= 653,2 \text{ kN/m} & E(1:1)_f &= 444.5 \text{ kN/m} \\
 \Delta E_w &= 521.2 \text{ kN/m} & \Delta E_f &= 403.7 \text{ kN/m} & \nu_{wf} &= 0.327
 \end{aligned}$$

As we use isoparametric finite shell elements, also a shear modulus  $G$  is included and represents the stiffness against yarn shear distortion. If no further information is available we recommend a value of 1-3 % of the E-modulus for textile products.

### 3 NONLINEARITY

The major nonlinear effect occurs in case of decreasing the stress in one direction. The following animation illustrates the interaction of warp and fill yarns in this case:

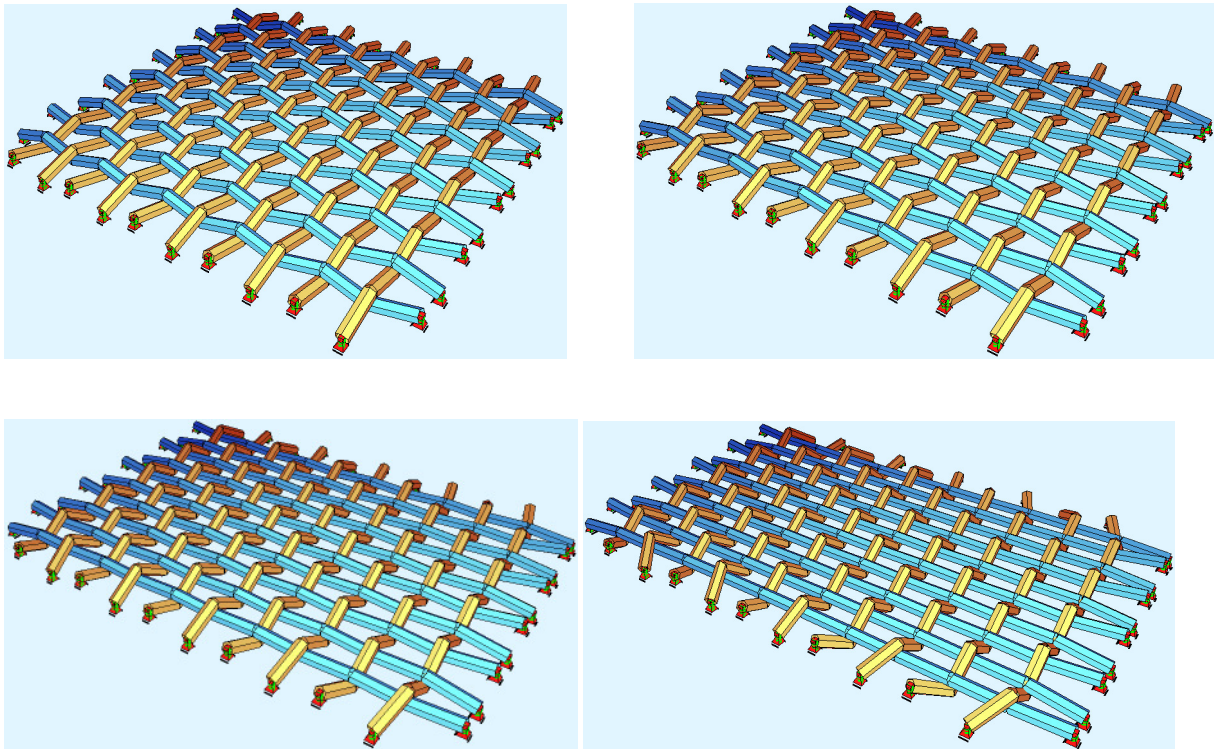


Figure 1: animation for increasing warp (blue) and decreasing fill stress (yellow)

Starting with an isotropic prestress the blue yarns (warp direction) are further stressed, the yellow yarns (fill direction) are released. As the blue yarns get straight, the yellow yarns must go up and down much more due to geometric nonlinear effects and shorten the material in the fill direction. The deformation in fill direction (yellow) is now disproportionately higher than calculated with simple poisson ration  $\mu$ . The same example analyzed with the nonlinear material law for quad elements also show this nonlinear effect (quad compressed in fill direction – plotted in red):

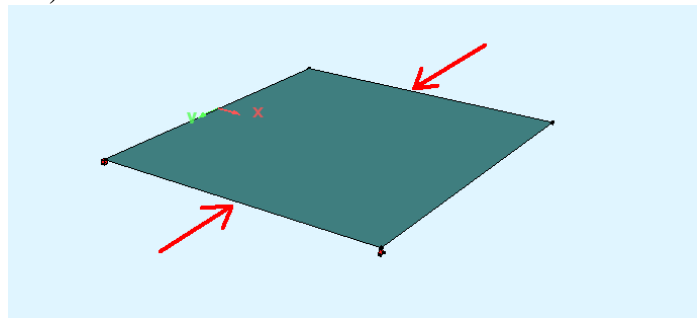


Figure 2: one single finite quad element under fill compression

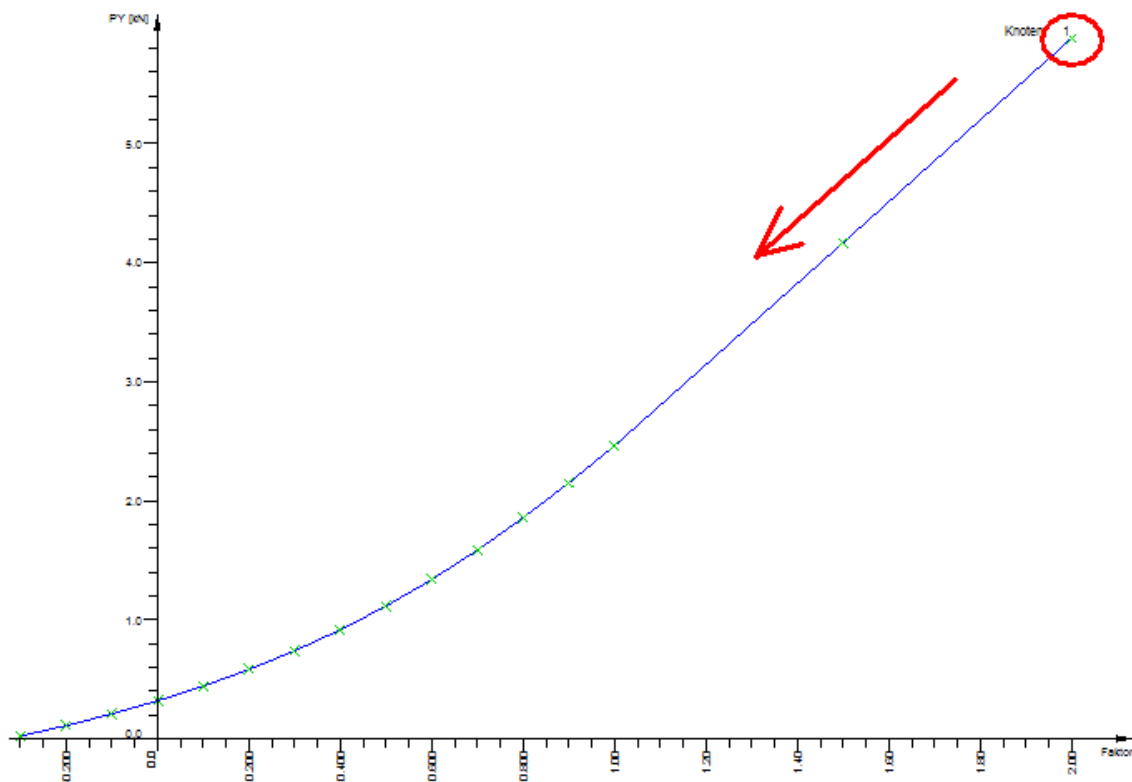


Figure 3: fill stress – fill compression strain (starting point right (red), compression direction to the left)

With decreasing fill stress (vertical PY) the strain in fill direction changes disproportionately high. On other words: decreasing the fill deformation (fill compression) the stress in the fill direction will not decrease as fast as in a linear analysis.

## 4 WRINKLES

The main problem is the wrinkling. A further compression in fill direction only creates wrinkles but will not change the stress in warp direction (that occurred at the start of wrinkling)! This means that the material behavior changes completely because no more poisson ratio effect exists: further  $\epsilon_{f-f}$  compression has no effect on the now uniaxial stress state:

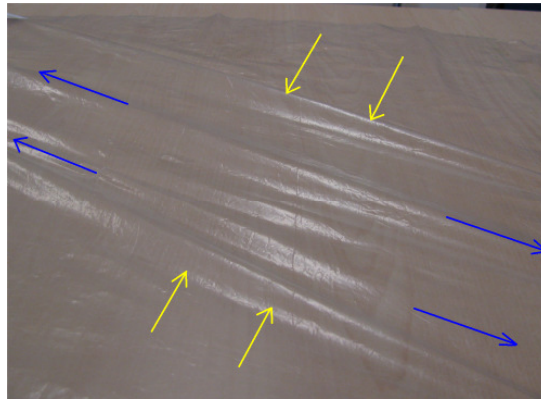


Figure 4: further wrinkle compression (yellow) may not change the force in the stress direction (blue)

On the other hand a fill-wrinkled membrane under following fill tension must act without stress change until the wrinkle deformation is restored. Please notice that a real quad finite element also has a little shear modulus  $G$  and may have main stresses  $\sigma_I$  and  $\sigma_{II}$  that differ from the warp and fill direction. So the following wrinkle procedure also works for isotropic materials (PVC). The program is based on the real total deformation and strain and works as follows:

- compute the linear main stress  $\sigma_I$  and  $\sigma_{II}$  from given total strain (linear law)
- if both stresses are positive -> standard case -> compute nonlinear warp-fill stresses
- else:
  - calculate main tensile direction  $\beta$
  - calculate strain in this direction  $\epsilon_{I-I}$  and transverse  $\epsilon_{II-II}$
  - assume wrinkle transverse to main tensile direction = assume  $\sigma_{II-II}=0$
  - set material law  $E$  for this stress assumption and rotate to  $\beta$
  - iterate wrinkle starting strain  $\epsilon_{II-II-0}$  to achieve  $\sigma_{II-II}=0$  with this material law  $E$
  - if  $\epsilon_{II-II} < \epsilon_{II-II-0}$  a real wrinkle occurred with wrinkle (damage) strain  
damage = stressfree wrinkle strain =  $\epsilon_{II-II} - \epsilon_{II-II-0}$
  - if created  $\sigma_I$  is negative -> wrinkled in both directions
  - transform stresses  $\sigma_I$  and  $\sigma_{II-II}$  back to local quad coordinate system
  - in the general case with shear modulus  $G$  not 0.0 this gives  $\sigma_x$ ,  $\sigma_y$  and  $\sigma_{xy}$

## 5. EXAMPLES

A prestressed roof with plus-minus curvature is loaded with wind from below. For linear elastic material behavior the stress in warp direction (short span) decreases rapidly and on a downside air pressure of  $0.32 \text{ kN/m}^2$  first wrinkles occur (warp stress gets zero). Using the same material with the new nonlinear material law the wrinkles occur a little bit later on a downside air pressure of  $0.37 \text{ kN/m}^2$ :

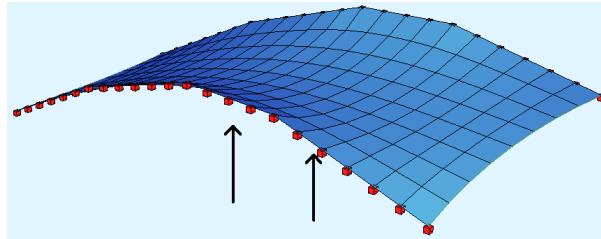


Figure 5: roof with wind loading from below

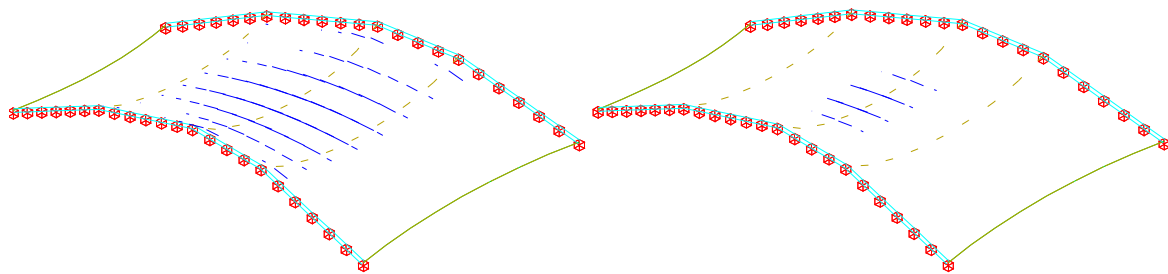


Figure 6: left: wrinkles with linear material, right: with new nonlinear warp-fill law (both  $0.40 \text{ kN/m}^2$  windload)

Another example is a membrane structure with a highpoint where wrinkles occur later using the new implemented nonlinear material law. For high upside wind the tangential prestress falls out nearly completely and the membrane carries the load by only radial warp stress:

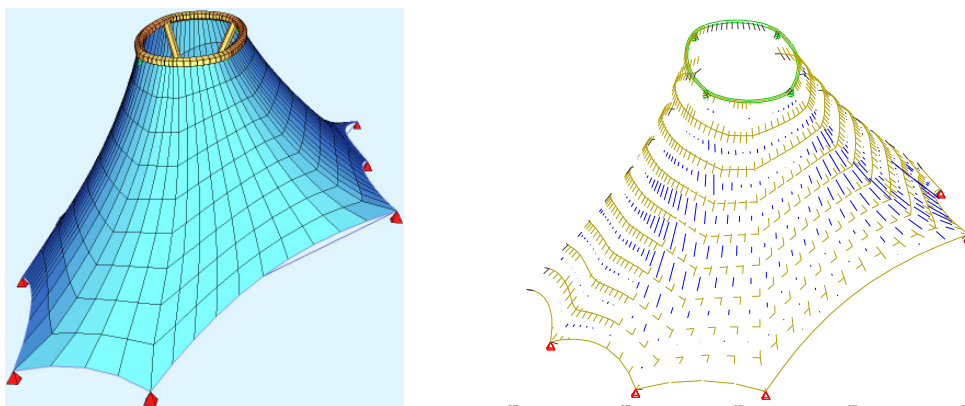


Figure 7: right : wrinkles for upside wind

The air supported cushion form [2] shows an extreme curved wrinkle zone on the lee side:

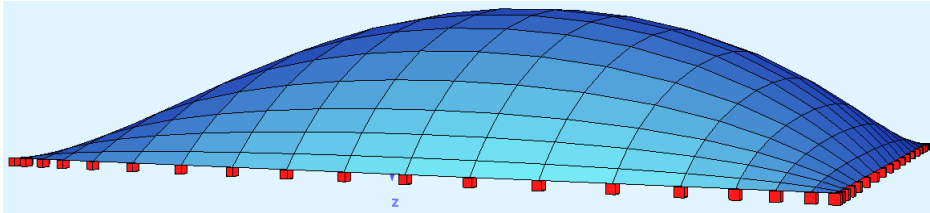


Figure 8: cushion : formfinding with inner air pressure

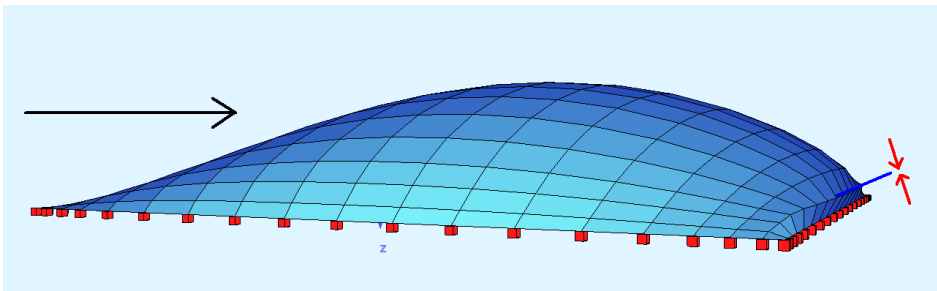


Figure 9: wind on cushion with extreme curved wrinkle zone (wrinkle compression red)

## 6 CONCLUSIONS

The paper reports of the implementation of a nonlinear membrane material law. Specifically for high wrinkle effects, special algorithms were tested to fulfill the requirement of constant stress in case of increasing wrinkles.

## REFERENCES

- [1] Cédric Galliot + Rolf Luchsinger, *A simple non-linear material model for PVC-coated polyester fabrics*, Tensinews Newsletter No. 18 April 2010
- [2] Jürgen Bellmann, *Air Volume Elements for Distribution of Pressure in Air Cushion Membranes*, International Conference on Textile Composites and Inflatable Structures, STRUCTURAL MEMBRANES 2011, Barcelona 2011