

CONCEPTION AND DESIGN OF MEMBRANE STRUCTURES CONSIDERING THEIR NON-LINEAR BEHAVIOR

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Summary. The lack of unified verification approaches and standards like the Eurocodes for various materials is a limiting factor to further propagation of architectural membranes. This paper will discuss the possibilities and challenges of integrating the design and verification of membrane structures into the Eurocodes' philosophy. Therefore an overview of existing guidelines will be given, followed by a discussion of the underlying principles of the Eurocodes. Especially the non-linear behavior of architectural membranes distinguishes them from other structures. Therefore the focus of this contribution is to discuss the implications of this non-linearity on verification approaches. Theoretical considerations as well as in-depth examples help to clarify the necessary basis. Finally the consequences of non-linearity on the verification of the primary structure and hybrid structures are presented.

1 INTRODUCTION AND MOTIVATION

Architectural membranes provide minimal use of material combined with an attractive and impressive language of shapes. These shapes are – in contrast to most shapes in civil and structural engineering – directly mechanically motivated: based on the chosen prestress level and the boundary conditions, form finding determines the shape of equilibrium that allows the membrane to act in pure tension. The algorithms and approaches for successful computation of membrane and cable net structures exist [1-5] and are widely used. In contrast, the lack of consistent standards for verification and unified codes still is a limiting factor for further realization and success of architectural membranes.

In the following sections, steps of the conception and design of tensile structures under special consideration of their non-linear behavior shall be discussed. In section 2, a short overview of verification codes that are nowadays applied to membrane structures will be given. Since the Eurocodes generally provide the central framework of today's verification procedure in Europe, section 3 will discuss the inscription of architectural membranes' design in the existing codes, mainly characterized by their non-linear behavior. The problem of verification in the non-linear context increases, when different structural members are mixed. This is the case for primary structures for membranes in general and especially for hybrid structures, where the supporting system undergoes large displacements. The problems arising through this combination shall be addressed in section 4. Finally, concluding remarks will

provide a summary and give an outlook on future research activities towards a unified, consistently based verification standard in alignment with the Eurocodes' approach.

2 VERIFICATION STANDARDS FOR MEMBRANE STRUCTURES

In contrast to most other materials used in the building and construction industry, currently there is no unified code for the verification of architectural membranes. Some codes and design guides exist on national level, like for example the ASCE 55-10 [6] (USA), the ITBTP design guide [7] (France) or the German practice, combining the DIN 4134 [8] and the dissertation of J. Minte [9]. Most of these codes and guidelines are based on a stress factor approach that compares the results of an analysis with characteristic (i.e. unfactored, representative actions) loads to a permissible strength.

As an example one may take the approach from the ITBTP guide [7],

$$T_C \leq T_D = \frac{k_q \cdot k_e}{\gamma_t} \cdot T_{rm} = \frac{T_{rm}}{\gamma_{stress}}, \quad (1)$$

where the design strength T_D is derived from the (characteristic) tensile strength T_{rm} , reduced by the factors k_q and k_e , as well as the so-called safety coefficient γ_t , taking into account the environmental degradation. The design strength T_D represents the permissible strength that is ultimately assessed against the calculated tensile force T_C under the respective load combination, assuming characteristic values for the actions. The quality factor k_q shall adjust the member capacity to the execution quality; the scaling factor k_e reflects the increased risk of critical defect with increasing surface area. For the sake of comparison, the individual factors – k_q , k_e and γ_t – may be summarized in one stress reduction coefficient γ_{stress} (often termed “stress factor”), as demonstrated in equation 1.

Though the various codes and guidelines show differences in their respective prescribed load combinations and the way in which the stress reductions are applied, they can basically be compared to the procedure described in equation 1, summarizing the respective factors and coefficients to the overall stress factor γ_{stress} . As stated in different publications [10,11] the mentioned guidelines agree on comparable “levels of uncertainty”, reflected in the different stress factors. These reduction approaches are schematically represented in table 1:

Table 1: Stress factors γ_{stress} in selected verification approaches.

Standard	Factors	Incorporated influences	γ_{stress}
ASCE 55-10 [6]	L_t, β	life cycle factor, strength reduction based on different load combinations	4.0 – 7.8
ITBTP Design Guide [7]	k_q, k_e, γ_t	execution quality, scale factor, environmental degradation	(4.0) 5.0 – 7.0
German practice, based on DIN 4134 [8] and J. Minte [9]	A_{res} (γ_f, γ_M, A_i)	loading uncertainties, material safety, test scaling, time influence, environmental degradation, temperature	2.9 – 6.4

In summary one may conclude that permissible stresses are obtained by reducing the

characteristic strength of the textile by a reduction factor γ_{stress} in the order of 4.0 to 7.0 (the extreme values of 2.9 and 7.8 from table 1 are rather rare cases).

3 ARCHITECTURAL MEMBRANES AND THE EUROCODE REGULATIONS

In Europe, the design of structures generally is codified in the so-called Structural Eurocodes (EC). These have been introduced for the most commonly used materials like steel (EC 3), concrete (EC 2) or wood (EC 5). As mentioned above, such a unified standard does not exist for membrane structures up to now. Based on first attempts towards a unified design and verification approach like the TensiNet Design Guide [10], CEN250 Working Group 5 has initiated the development of a new Eurocode. This code shall specifically be applicable for membrane and tensile structures and provide guidance for their very special design and simulation demands. In the next sections, a short overview over the Structural Eurocodes' underlying principles will be given; in the following, the challenge of incorporating the non-linear behavior of tensile structures will be discussed.

3.1 Eurocode regulations

Like all codes, the Eurocodes tempt to provide the necessary verification and assessment procedures to guarantee “safety” of the structures in scope. The underlying principle as described in the EC 0: “Eurocode – Basis of structural design” [12] is based on reliability theory. The main idea behind the semi-probabilistic approach is to define a probability of failure P_f that represents an acceptable level of safety (cf. Fig. 1, right). This probability of failure can be linked to a reliability index β . To give an order of magnitude, for “usual” buildings (reliability class RC2) an annual failure probability in the order of 10^{-6} is deemed acceptable. All further verifications of certain limit states – leading to the term “limit state design” (LSD) – as prescribed in the different Eurocodes are designed in such a way, that they guarantee this level of probability of failure.

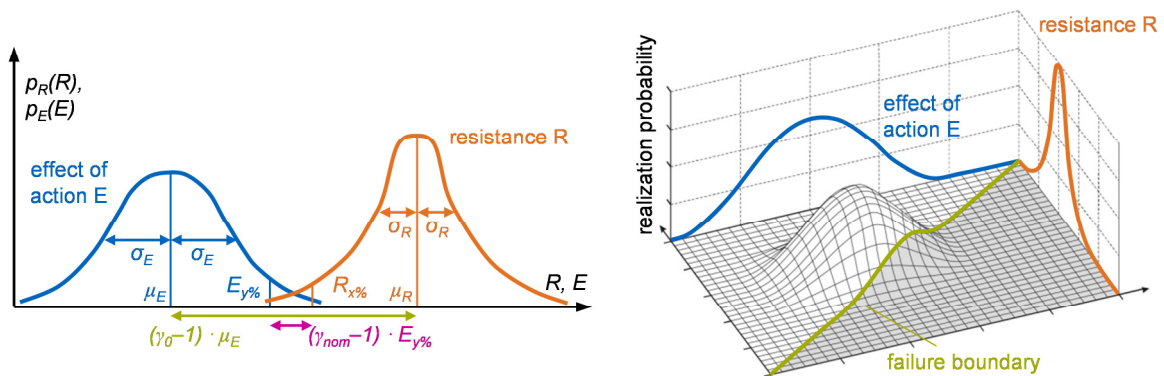


Figure 1: Sketch of the global safety factor concept and definition of the descriptive parameters: the mean value μ_E and μ_R , the standard deviation σ_E and σ_R , the realization probabilities $p_R(R)$ and $p_E(E)$ and the defined factors of safety, γ_{nom} and γ_0 (left); Failure probability P_f (volume under the dark grey area) as a function of the variations of effects of actions E and resistance R , failure boundary (green) separating the failure domain (right).

In the context of LSD, two major limit states can be distinguished, the *Serviceability Limit State* (SLS) and the *Ultimate Limit State* (ULS). While the first group (SLS) is focused on

functioning, comfort and appearance, the second group (ULS) concerns the safety of people and of the structure. Both verifications are based on the definition of relevant design situations and load cases [12].

In earlier design approaches, a global safety factor concept has been used instead of the reliability based approach of the Eurocodes (cf. Fig. 1, right): Assuming both the loads (and with them the effects of action E like e.g. the resulting stress in a member) and the resistance R being subject to statistical variations, the definition of mean values (e.g. the mean stress μ_E) and fractile values (e.g. the 95% fractile of the stress, $E_{95\%}$) were used for verification. As described in Figure 1, this concept led to two values for the quantification of safety, the central factor of safety γ_0 , and the nominal factor of safety γ_{nom} .

The concept of distributed probabilities for loads and resistances is still at the basis of Eurocodes: Fractile values are used to define the “characteristic” values for the actions and resistances, F_k and R_k , respectively. These characteristic values are directly used to verify for the *Serviceability Limit State*.

The verification for the *Ultimate Limit State* is based on a comparison of a design value of an effect of action, E_d , and a design value of the corresponding resistance, R_d . This basic verification concept can be written as

$$E_d = E_d \{G \oplus Q\} \leq R_d = \frac{R_k}{\gamma_M}, \quad (1)$$

where R_d is defined through the characteristic resistance R_k , divided by a partial factor γ_M that reflects the uncertainties in the definition of the material properties (the better material properties can be predicted, the smaller γ_M can be assumed, e.g. $\gamma_{M,steel} \approx 1.0$ to 1.1). The design value of the effect of action, E_d , is the outcome of a load combination of permanent and variable actions G and Q , respectively. These actions are collected in load combinations (sign “ \oplus ”) that shall reflect different relevant scenarios the structure may be faced with during its projected lifetime. In addition, partial factors γ_F are applied to the respective loads in order to account for the uncertainties in the load values; combination factors ψ represent the probability of occurrence in the respective load combinations (e.g. a combination of dead load of the structure, wind and traffic load). In the specific Eurocodes (depending on the materials used in their construction) detailed instructions for the assessment of structures are given, as well as specific values for the various partial factors.

Besides the lack of a material-specific Eurocode for architectural membranes, another important problem can be identified in advance: Due to the non-linear behavior of tensile structures, the influence of single actions on the effects of actions cannot directly be identified, thus opposing the concept of applying factored loads in order to account for a certain level of uncertainty. Hence for non-linear structures more detailed considerations are necessary, since only few indications are given in the Eurocode 0.

3.2 Non-linear behavior of tensile structures

As it is widely known, architectural membranes and other prestressed, tensile structures

draw their load-bearing capacities out of their shape – generally double curved – and their ability to undergo large deformations that allow providing considerable geometric stiffness. In order to reliably simulate these large deformations, the need for a geometrically non-linear analysis is obvious. This need for non-linear analysis has important consequences on possible verification approaches.

In the context of the present contribution, consequences on load combinations shall be mentioned; consequences for the determination and proceeding of “design loads” as presented in the previous section will be discussed.

As mentioned above, a core ingredient for the Eurocodes’ philosophy is the concept of load combinations. At a first glance, it is important to note that without linear behavior of the structure, the widely used superposition approach is not applicable any longer. While this may at first be considered a minor inconvenience, the complete consequences are much more important: a major simplification made in the Eurocodes is to state that for the determination of the effects of actions (e.g. deflections or resulting stresses in the structure) – in most cases – a factoring of the action is equivalent to factoring the effects of this action, expressed in formula (6.2) of the Eurocode 0 [12] as

$$E_d = \gamma_{Sd} \cdot E\{\gamma_{f,i} \cdot F_{rep,i}; a_d\} \xrightarrow{\text{linear structural behavior}} E_d = E\{\gamma_{Sd} \times \gamma_{f,i} \cdot F_{rep,i}; a_d\}, \quad (2)$$

where E_d is the effect of action due to the action F_{rep} (i is the summation index for different loading actions) applied to the design geometry a_d . F_{rep} is the representative value of the action that is multiplied by the partial factor γ_f for possible unfavorable deviations of the representative value. At the left, the resulting effect of actions E is factored by a partial factor γ_{Sd} for the uncertainties in modeling, at the right the underlying actions are factored by γ_{Sd} directly. This concept of factoring the loads allows calculating with factored actions in order to obtain the design values of the effects of actions, which seems quite attractive for the verification of structures.

For the analysis of tensile structures with their large deformations, this concept has some major deficiencies: Since the stress state is strongly connected to the shape of the structure, a factoring of the load would also lead to an “unrealistic” deformation (cf. also section 4).

The Eurocode 0 addresses this problem of non-linearity by indicating a distinction between two different types of non-linear behavior in paragraph 6.3.2(4) “Design values of the effects of actions”, represented graphically in Figure 2: A distinction is made between structures where the effect of action, E , increases more than the representative value of the action, F_{rep} , (category a) respectively less (category b). The behavior characterized by category a) often is termed “over-linear” while category b) describes “under-linear” behavior.

Note: for alignment with the commonly used terms in the following no distinction will be made between the characteristic value F_k and the representative value of the action, F_{rep} . Only one single action F will be used.

The simplified representation in Figure 2 shows the difference between the two types of behavior. As mentioned above, for the case of a linear behavior of the structure, the two cases coincide, thus equation (2) becomes valid and the simplification can be applied.

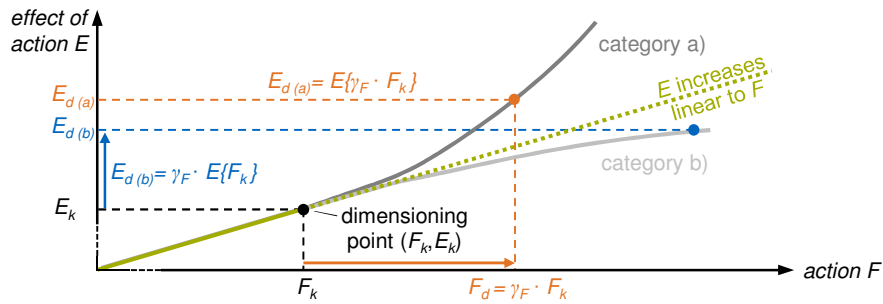


Figure 2: Simplified graphical representation of the two types of non-linear behavior of structures as proposed in the Eurocode 0. Design values of the effects of action, E_d , starting from a dimensioning point (F_k, E_k) .

In case of a non-linear structural behavior, it is important to correctly classify the type of structures to one of the above categories. This can be problematic, since the direct output of a non-linear simulation based on non-factorized characteristic actions F_k is only the dimensioning point (F_k, E_k) , not a complete graph as shown in Figure 2. In a more abstract sense, this classification of the non-linear behavior represents the determination of the inclination of the F-E-graph. Two related approaches are briefly discussed in the following.

Since the inclination of the graph is not needed in an analytical, continuous sense, but in a reasonable surrounding of the dimensioning point, it would be sufficient to obtain one more point in addition to the dimensioning point. This determination of an additional point in order to approximate the graph's evolution can be compared to classical sensitivity analysis.

Another approach is based on the fact that often non-linear simulations use path-following methods like load control [13]. With these methods, equilibrium configurations on the path are available, that allow approaching the graph's inclination.

3.3 In-depth example of a prototype structure of a reduced hypar

The presented approaches apply to structures with non-linear behavior in general. For the case of architectural membranes the Eurocode 0 gives an indication considering their behavior: “Except for rope, cable and membrane structures, most structures or structural elements are in category a)” [12], and in consequence cable and membrane structures are in category b). In order to underline this assumption and demonstrate some effects of non-linearity, a reduced model of a classical hypar (cf. Fig. 3), will be discussed as a prototype structure. The simplifications taken from the hypar membrane to the model of two prestressed truss members (single degree of freedom (DOF) system) allow keeping the derivations intelligible.

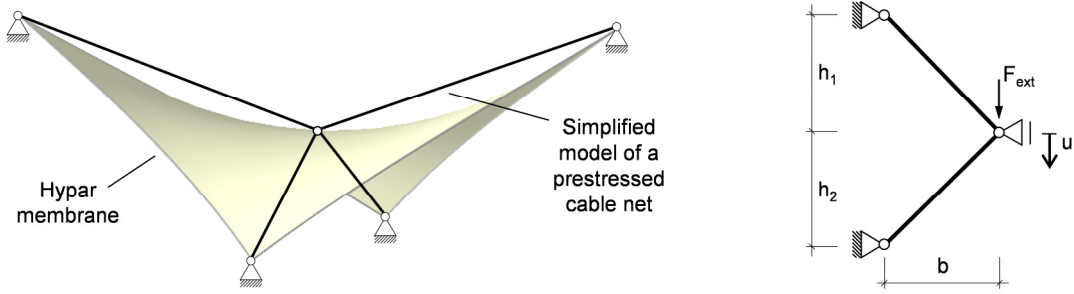


Figure 3: Reduction of a spatial prestressed hyper membrane to a simplified model of a prestressed cable net (left); further simplification as a plane structure consisting of two prestressed truss members (right).

In order to analyze the non-linear behavior of this structure, its residual force equation is derived, based on the principle of virtual work w.r.t. to the displacement variables \mathbf{u} :

$$-\delta W = -(\delta W_{int} + \delta W_{ext}) = -\left(\frac{\partial W_{int}}{\partial \mathbf{u}} + \frac{\partial W_{ext}}{\partial \mathbf{u}}\right) \cdot \delta \mathbf{u} = (\mathbf{R}_{int} - \mathbf{R}_{ext}) \cdot \delta \mathbf{u} = 0 \quad (3)$$

Here, the residual force vector $\mathbf{R} = \mathbf{R}_{int} + \mathbf{R}_{ext}$ reduces to a scalar for the 1-DOF-system. In case of conservative loading, the external residual force \mathbf{R}_{ext} is equal to the load F_{ext} . The internal virtual work of a single member i can be written as

$$-\delta W_{int,i} = \int_{V_0} [(S_{11,i} + S_{0,i}) \cdot \delta \epsilon_{GL,i}] dV \stackrel{\text{simplification}}{=} A_i \cdot L_i \cdot [(S_{11,i} + S_{0,i}) \cdot \delta \epsilon_{GL,i}], \quad (4)$$

where L_i and A_i are the length and cross section of the member i , respectively. In the members we assume constant strains and stresses along the elements. The stresses from elastic deformation (S_{11}) and prestress (S_0), measured as 2nd Piola-Kirchhoff stresses (PK2), are energy conjugate to the Green-Lagrange strains ϵ_{GL} . For truss members, the strains ϵ_{GL} can be expressed as a function of the reference length L and the current length ℓ :

$$\epsilon_{GL,i} = \frac{1}{2} \frac{\ell_i^2 - L_i^2}{L_i^2}, \text{ and consequently: } \epsilon_{GL,1} = \frac{1}{2} \frac{u^2 + 2h_1 u}{L_1^2} \text{ and } \epsilon_{GL,2} = \frac{1}{2} \frac{u^2 - 2h_2 u}{L_2^2} \quad (5)$$

Rewriting the virtual strains $\delta \epsilon_{GL,i}$ leads to

$$\delta \epsilon_{GL,i} = \frac{\partial \epsilon_{GL,i}}{\partial u} \cdot \delta u, \text{ and thus } \delta \epsilon_{GL,1} = \frac{u + h_1}{L_1^2} \cdot \delta u \text{ and } \delta \epsilon_{GL,2} = \frac{u - h_2}{L_2^2} \cdot \delta u. \quad (6)$$

When introducing the simplifying assumptions of equal height $h_i=h$, initial length $L_i=L$, cross section $A_i=A$, and prestress $S_{0,i}=S_0$, the expression of the internal residual forces $\mathbf{R}_{int}=\sum \mathbf{R}_{int,i}$ can be written as:

$$\begin{aligned} \mathbf{R}_{int} &= \frac{A}{L^3} \left((u+h) \left[\frac{1}{2} E \cdot (u^2 + 2hu) + S_0 \cdot L^2 \right] + (u-h) \left[\frac{1}{2} E \cdot (u^2 - 2hu) + S_0 \cdot L^2 \right] \right) = \\ &= \dots = \frac{EA}{L^3} \cdot (u^3 + 2h^2 u) + 2 \frac{u}{L} S_0 \cdot A \end{aligned} \quad (7)$$

Additionally assuming linear elastic material, the elastic stresses S_{11} have been replaced by $S_{11}=E \cdot \varepsilon_{GL}$, introducing Young's modulus E .

For the evaluation of internal forces as effects of actions, the internal forces N_1 and N_2 of the members can be written as

$$N_i = \frac{\ell_i}{L_i} \cdot A_i \cdot S_{PK2,i} = \frac{\ell_i}{L_i} \cdot A_i \cdot (E \varepsilon_{GL,i} + S_{0,i}) \stackrel{\text{here}}{=} \frac{\ell_i}{L_i} \cdot \left[\frac{EA}{2} \cdot \frac{\ell_i^2 - L^2}{L^2} + S_0 \cdot A \right], \quad (8)$$

which can also be formulated for the individual members as a function of the displacements (the current length ℓ_i is a function of u). In this example, the factor ℓ_i/L represents the transition from the reference configuration (and with it the reference orientation) and the current configuration, oriented in the member's current direction. In all presented developments, a deformation of the section A is neglected (equivalent to Poisson's ratio $\nu=0$).

With these formulations at hand, three selected parameters are analyzed w.r.t. to their non-linear evolution regarding their possible verification according to the approach of Eurocode 0: (i) the displacement u and (ii) the normal forces N_1 and N_2 . For these selected parameters, a classification according to the Eurocode's proposition discussed in section 3.2 will be made.

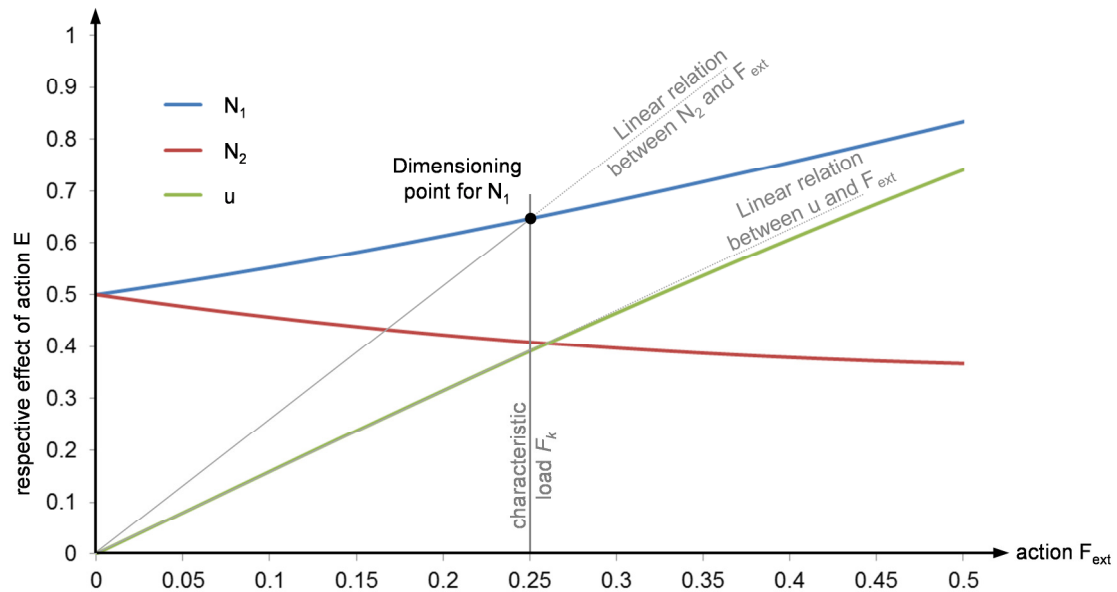


Figure 4: Representation of the selected effects of actions resulting from the action F_{ext} ; the distinction of the Eurocode's categories of non-linearity can be made by comparison with a fictive linear relation between E and F (grey straight lines). For the normal force N_2 , the factoring of the load must not be applied, as N_2 is reduced by increasing F_{ext} (in the surrounding of F_k). For the displacement u , a SLS verification is applied (no factoring).

The first examined parameter u serves as example for a verification of the *Serviceability Limit State*. Obviously the question of increasing the load magnitude by a factor is rather artificial in this case, since the SLS has to be verified with characteristic values: only “as-realistic-as-possible” predictions of the deformations to be expected are of value at the design stage. Nonetheless it is considered an effect of action E and thus is plotted in the graph in

Figure 4. For the effect of action u it can be observed, that it increases less than the action F_{ext} itself, thus – theoretically – classifying the structure in category b) (cf. Fig. 2 and Fig. 4).

The question of correctly applying the load factor to the respective value – the action F_k or the effect of action E_k – is more important for the *Ultimate Limit State* verifications, examined here for the member forces N_1 and N_2 . From the plot in Figure 4 one can observe that for the upper member, the normal force N_1 increases less than the action F_{ext} . Hence the ULS verification of N_1 would be placed in category b), which is in accordance with current design practice for membrane structures: the stresses are calculated based on characteristic loads, these stresses are then assessed against admissible stresses. For the second member, the question of assessing in the loaded state – and in consequence the question of factoring the load F_{ext} or its effect – is irrelevant as the unloaded state ($F_{\text{ext}}=0$) represents the most demanding situation for the member. This phenomenon can obviously be explained via the prestress that's reduced as the member is compressed by the increasing load.

One can conclude that current design's practice – application of the factors on the effects of action rather than on the action itself – complies with the basic instructions for non-linear structures of EC 0. Nevertheless an important problem arises from non-linearity: Though in the design guides and codes factored load combinations are prescribed, their effect is very delicate to be judged. If one would apply these factored loads in the non-linear calculation, not only effects of action with a magnitude differing from the considerations above may result, but these values are also doubtful as they are based on “exaggerated” displacements, which usually are considered large for tensile structures. This will make it very difficult to judge about the true value, as one cannot be sure whether these values are conservative.

4 TENSILE STRUCTURES AND THE PRIMARY STRUCTURE

The problem mentioned above is even more accentuated, when it comes to the interaction of different types of structures. For membrane structures, this is the case, as they all rely on some kind of primary structure, supporting the textile membrane. This primary structure, often made of steelwork, has to be verified on its own, applying its specific code (for the case of steelwork this would be the EC 3). These codes prescribe the use of adapted load combinations with individual partial factors for the different actions. Here the problem becomes obvious: When for the membrane the load factor is applied on the stresses based on characteristic loads, the necessary individual load factors cannot be applied anymore.

4.1 Interaction of textile membrane and the primary structure

When looking at the load transfer from textile membrane to the underlying structure two aspects may be observed that are closely related yet of individual importance. As the membrane transfers the surface loads to the primary structure through tensile forces, this transfer includes both the magnitude of the force as also its orientation (cf. Fig. 5). The question of the force orientation may seem of little importance, but when considering the large deflections that may occur, it might be of interest for the dimensioning of the primary structure. Thus, the question is rather twofold: which load from the membrane to apply in which orientation on the primary structure? As stated above, the approach usually taken for

the membrane – i.e. simulating the membrane with characteristic loads and applying the load factor on the effects of actions – leads to reasonable values for the membrane design. This approach should be continued for the primary structure. In addition to the load, the effects of the deformed geometry and with it the altered orientation of the interaction forces between membrane and primary structures have to be examined.

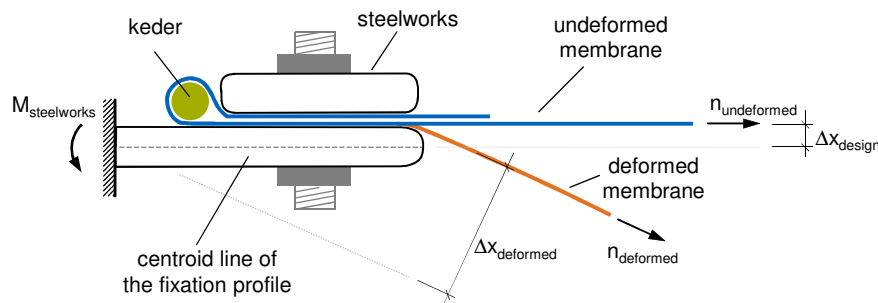


Figure 5: Transfer of the load from the membrane to the underlying primary structure. The factoring of loads may lead to the desired conservative effects of actions, but they also influence the geometry of the load transfer. Taking into account the deformed geometry, the design moment $M_{\text{steelworks}}$ to take into account is increased.

In Figure 5 a schematic representation of the load transfer from membrane to primary structure is presented. In order to determine the design value $M_{\text{steelworks}}$ between the fixation profile and the general steelworks, the tension from the membrane has to be multiplied with its respective lever arm. It is obvious that even when assuming the same tensile force n , the moment is also dependent on the lever arm Δx . While even for the design geometry an eccentricity Δx_{design} must be taken into account, this Δx may increase during deformation.

4.2 Hybrid structures

In most cases, the primary structure is considered stiff compared to the membrane. In consequence it is often treated as a fixed support; the loads from these imaginary supports are then verified for the primary structure in a separate assessment. This approach may be justified, when the structure can be considered very stiff.

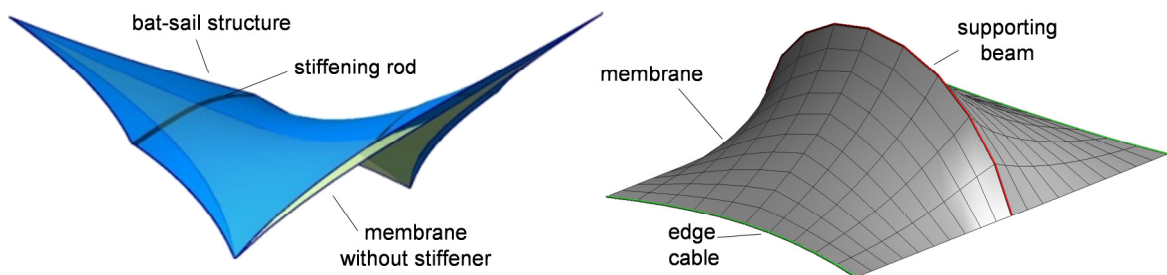


Figure 6: Examples for hybrid structures, uniting form found, stress defined membranes with non-form found elastic members like beams. These bending active elements are crucial for structures like the bat-sail (left) and the prototype structure (right). These structures and adapted computation approaches are discussed in [14-16].

If the supporting structure is too weak to be considered a fixed support, it has to be

included in the non-linear simulation of the membrane. These structures, where both the form found membrane and the elastic supporting structure have to be taken into account in one integrated computation approach are called “hybrid structures” [14-16].

Due to the large displacements the supporting structure may be subject to, the problematic of correctly computing and applying the forces as well as the geometry is even more accentuated for hybrid structures.

5 CONCLUDING REMARKS

This paper has presented the possibilities and challenges of integrating architectural membranes with their non-linear behavior into the reliability based design approach that represents the underlying principle of the Structural Eurocodes. The verification approaches currently in use and different codes and design guides that are applied have been presented, especially focusing on their verification approach. As basically all guidelines are based on a permissible strength (or allowable stress) design, the stress factors γ_{stress} have been compared.

A general overview over the Eurocodes’ design philosophy, the definition of “safety” by a failure probability P_f , has been presented and compared to the global safety factor concept. The limit state design approach and the respective effects of actions are of major importance in this context. The possibilities of integrating non-linear structures in the Eurocodes’ design concept have been discussed. In order to characterize and demonstrate the consequences of this non-linear character of tensile structures, a small scale example of prestressed cables has been discussed in detail, especially focusing on possible verification approaches.

An outlook on the interaction of the membrane with the primary structure has been given, taking into account the consequences of the non-linear behavior of membrane structures. Especially for comparatively weak primary structures – as it’s the case for hybrid structures, where the elastic supporting elements create the need for an integrated simulation approach – this interaction is of importance for possible verification approaches. Examples for these hybrid structures have been given.

To conclude we may state that – though important progress has been made concerning the unification of levels of “safety” and stress factors [10] – the non-linear behavior of architectural membranes presents a major challenge on the way towards a consistent verification methodology. These challenges may be subdivided in three major categories: (i) Still the material properties of textile membranes are far from being consistently derived and widely accessible: it is still very difficult for designers, to obtain reliable data on stiffness, creep or even material strength. This is accompanied by the need for consistent material models on the simulation side. (ii) As discussed in this paper, the determination of loads and load combination as well as the respective partial factors are still subject to current research and code development. The complex curved geometries of membrane structures make it very difficult to estimate some load cases like snow, their light weight in addition makes them prone to wind excitation. This aspect is part of the research efforts invested in Fluid-Structure-Interaction and Computational Wind Engineering [15]. (iii) Last but not least the non-linear character of architectural membranes as well as the design tasks (form finding, cutting pattern generation,...) make the simulation of membrane structures a challenging task for methodological research and software development. Even for well-defined examples,

available software environments provide very different results [17]. In summary it can be stated that the development in the field of tensile structures is far from being finished, important research is needed to solve the mentioned problems and face the arising challenges.

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