VI International Conference on Textile Composites and Inflatable Structures
STRUCTURAL MEMBRANES 2013
K.-U.Bletzinger, B. Kröplin and E. Oñate (Eds)

Restraining actively-bent structures by membranes

H. ALPERMANN*, C. GENGNAGEL*

* Chair of Structural Design and Technology, School of Architecture
University of the Arts Berlin
Hardenbergstrasse 33, 10623 Berlin, Germany
e-mail: alpermann@udk-berlin.de, web page: http://www.arch.udk-berlin.de/gengnagel

Key words: active-bending, membrane restraining, non-linear, FEM, hybrid structure

Summary. Membranes can be used to restrain actively-bent structures. The paper explains the structural behaviour behind using membranes to couple actively-bent elements and analyses the influence of different parameters.

1 INTRODUCTION

In actively-bent elements the curved end-geometry is achieved by means of elastic bending. The shape depends on the boundary conditions (e.g. supports, length, cross-section and material properties of the bent element) and normally results in non-constant curvature and stress distributions. By changing the boundary conditions, different curvatures can be achieved. Based on this, initially identical elements can be transformed to achieve different curvatures. As the curvature is inversely proportional to the bending stress of the element, different curvatures lead to different residual stress distributions. In order to minimise the residual stress, the cross-section height of the bent elements needs to be small. However, small cross-sections lead to relatively small stiffness.

One option to create stiff structures with actively-bent elements is to restrain the actively-bent elements using a membrane. The membrane restrains the bent elements to one another. Membrane-restrained structures can be used as standalone systems or as components within arch-, column- or girder-structures.

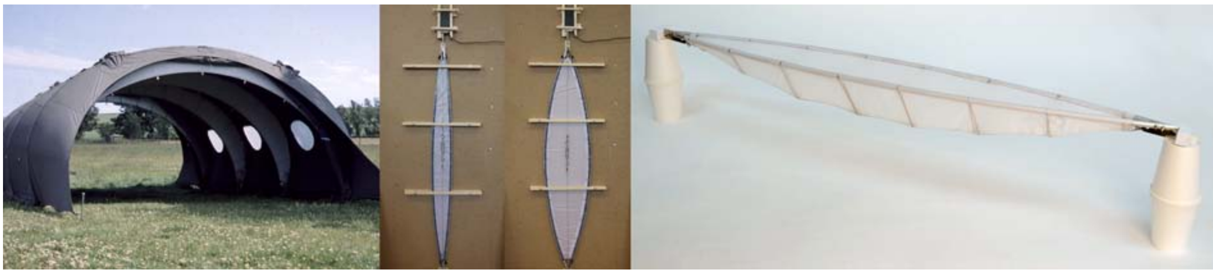

Figure 1: membrane restrained structures [2], [3], [5]

2 COUPLING OF STRUCTURAL ELEMENTS WITH MEMBRANES

The basic idea of coupling structural elements by means of a membrane is based on a force transfer between the elements. The membrane-coupling results in a multi component cross-section. The membrane is directly involved in the load transfer to the supports and thus is an indispensable part of the structure.

The goal of coupling the structural elements is to transfer shear forces between the elements. The shear stiffness of textile membranes is very low and not sufficient to ensure a shear-resistant structural behaviour. Therefore the shear forces are transferred in another way. The transfer of shear forces can be described by strut and tie forces in a truss model. In a truss as in figure 2, left, the shear forces are transferred by diagonal ties and vertical struts between the upper- and bottom-chord. As membranes can only carry tension forces, the diagonal tension forces can be transferred within a membrane between upper- and bottom-chord. The vertical compression forces cannot be transferred within the membrane but by additional struts. The struts can be integrated into the membrane as sail battens for example.

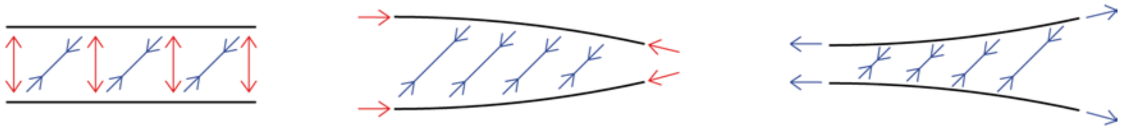

Figure 2: parts of truss-models for coupled elements with straight chords (left) or curved chords (middle, right)

If the chords of the truss are curved and both ends supported in vertical and horizontal direction, they will transfer the loads as an arch- or cable structure. Elastically bent beams are generally supported horizontally, since the horizontal supports are needed to maintain the bent shape. Arch and cable structures are much stiffer to take vertical loads compared to beam structures. So the vertical compression component of the shear force in the truss model can be transferred by the arch- or cable elements without the need of additional struts. The diagonal tension forces of the truss model lead to compression forces in the arches and tension forces in the cable.

The ability for an arch or cable structure to carry vertical loads depends on its curvature. Figure 3 shows four arches with the same span but different arch rise and for comparison a linear beam of equal span is also shown. The arches are elastically bent elements and therefore exhibit curvatures consistent with the bending shape. Within this bending shape the curvature is not constant, but zero at the supports and largest at the centre of the span. The beams of all systems have the same rectangular hollow cross-section measuring 50 mm in width and 25 mm in height with a thickness of 1.5 mm. The E-Modulus of the beams is 20,000 N/mm². In load case 1 the load is constantly distributed along the entire span, in load case 2 only the left half of the span is loaded. The maximum deformation occurs in point A for load case 1 and point B for load case 2. The maximum deformation in point A does not occur in the middle of the arches, as the curvature in the middle is higher than near to the ends.

The load-deflection curves are based on the maximum vertical deformation and they show a logarithmic gradient. The gradient of the load-displacement curve correlates to the stiffness of the arch with respect to the maximum vertical deformation. At the load where the curve approaches a horizontal line the structure will snap through. The snap through of the structure causes high deformations, until the structure finds a new equilibrium geometry as a cable structure. This structural behaviour is not shown in the load-deflection curves. The load-deflection curve of the beam has in both load cases an exponential gradient. Unlike the actively bent beam arches, the beam does not snap through since its deformed geometry results in immediate tension force activation from both load cases. As deflections in the beam increase, the transfer of bending forces will decrease while the transfer of tension forces will increase.

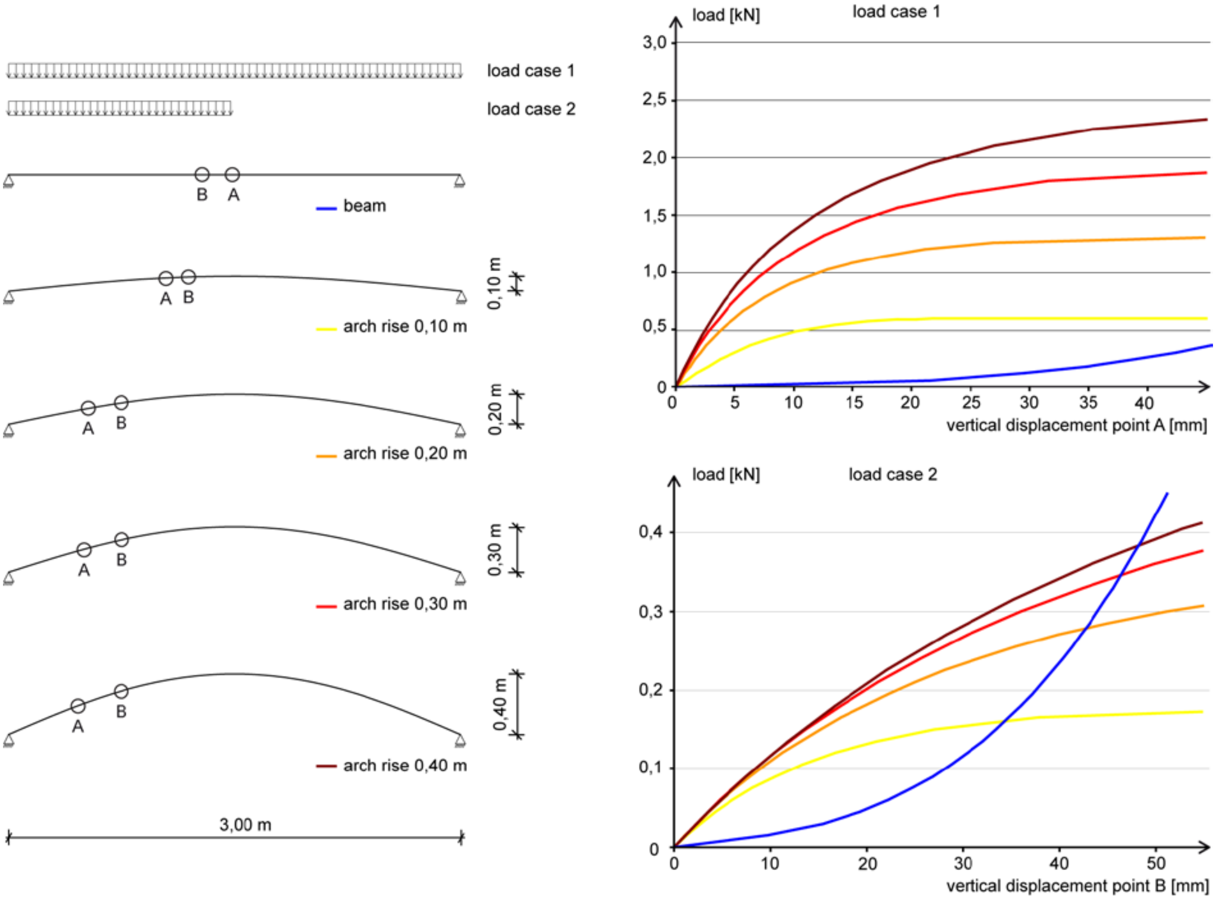

Figure 3: structural behaviour of a beam and different arch structures

The stiffness of the arches increases logarithmically with respect to the arch rise for both load cases. The stiffness of the beam is significantly lower than the stiffness of the arches. This means that a minimum curvature is required to enable membrane coupling. The coupling effect will be more efficient if the curvature is higher.

The principle membrane forces in the membrane occur only in one direction and they are orientated diagonally like in the truss-model (figure 2). If the membrane is not stressed by an additional load (e.g. pre-stress), the uniaxial membrane forces will lead to wrinkles in the membrane. The efficiency of the coupling effect is not only depending on the stiffness of the arch, but also on the E-modulus of the membrane. Therefore even if the level of principle membrane forces in the membrane is low, choosing membranes with high stiffness is advised.

3 COUPLING OF TWO ELASTICALLY BENT BEAMS

The coupling-effect will be exemplified by means of numeric analysis. The following structure consists of two elastically bent beams, which are connected to each other at their ends. At these ends axial and shear forces can be transferred between the two beams. The ends of the beams are connected by a cable to introduce and maintain the pretension in the beams. The profiles selected for the beams are rectangular hollow profiles measuring 50 mm in width and 25 mm in height with a thickness of 1.5 mm. The E-Modulus of the beams is 20,000 N/mm². The two beams behave as upper and lower chords connected by a membrane. The membrane is fixed to the beams in tangential and radial direction and not pre-stressed. The membrane is modelled with two-dimensional, quadrangular elements, which can only sustain tension forces. The specific orthotropic material properties of the membrane are considered in the FE-elements according to Münsch-Reinhardt. The E-modulus of the membrane was chosen as 600 kN/m for both fibre directions and the shear modulus was taken as 40 kN/m. The membrane orientation is turned by 45° to the x-axis. The structure is analysed as two-dimensional system and as such buckling out-of-plane is neglected.

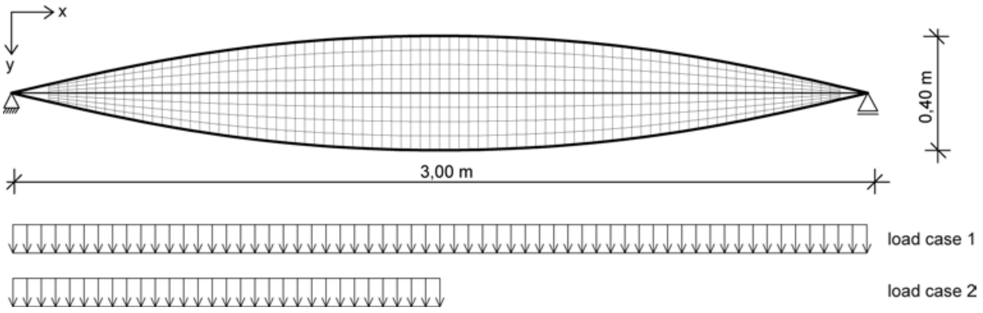

Figure 4: FE-model membrane restrained system

Two different load cases were simulated. The first load case consists of a continuous vertical line load along the bottom chord, the second load case consists of a continuous vertical line load on the left half of the bottom chord. The maximum displacement is in point A for load case 1 and in point B for load case 2. The load-deflection curves show for both load cases a linear gradient for the maximum vertical displacement. The deflections are higher in load case 2 than in load case 1. To determine the influence of the coupling effect, a second system was analysed. Here the membrane is replaced by vertical cables (cable restrained system). The cross-section and E-Modulus of the cables is with respect to the stiffness equal to the E-Modulus of the membrane in warp- or weft-direction. In load case 1 the deflections of the cable restrained system are lower than the deflections of the membrane-restrained system. As the membrane is turned by 45°, the stiffness of the membrane in vertical direction is lower than the stiffness of the cables. Therefore more loads are transferred by the bottom chord and less by the upper chord. This leads to higher deflections of the bottom chord.

In load case 2 the deflections of the cable restrained system are much higher than the deflections of the membrane restrained system. The cables ensure that both chords transfer loads to the support, but since the forces cannot be transferred in diagonal direction a coupling effect is not possible. This shows that the coupling effect improves the structural behaviour significantly.

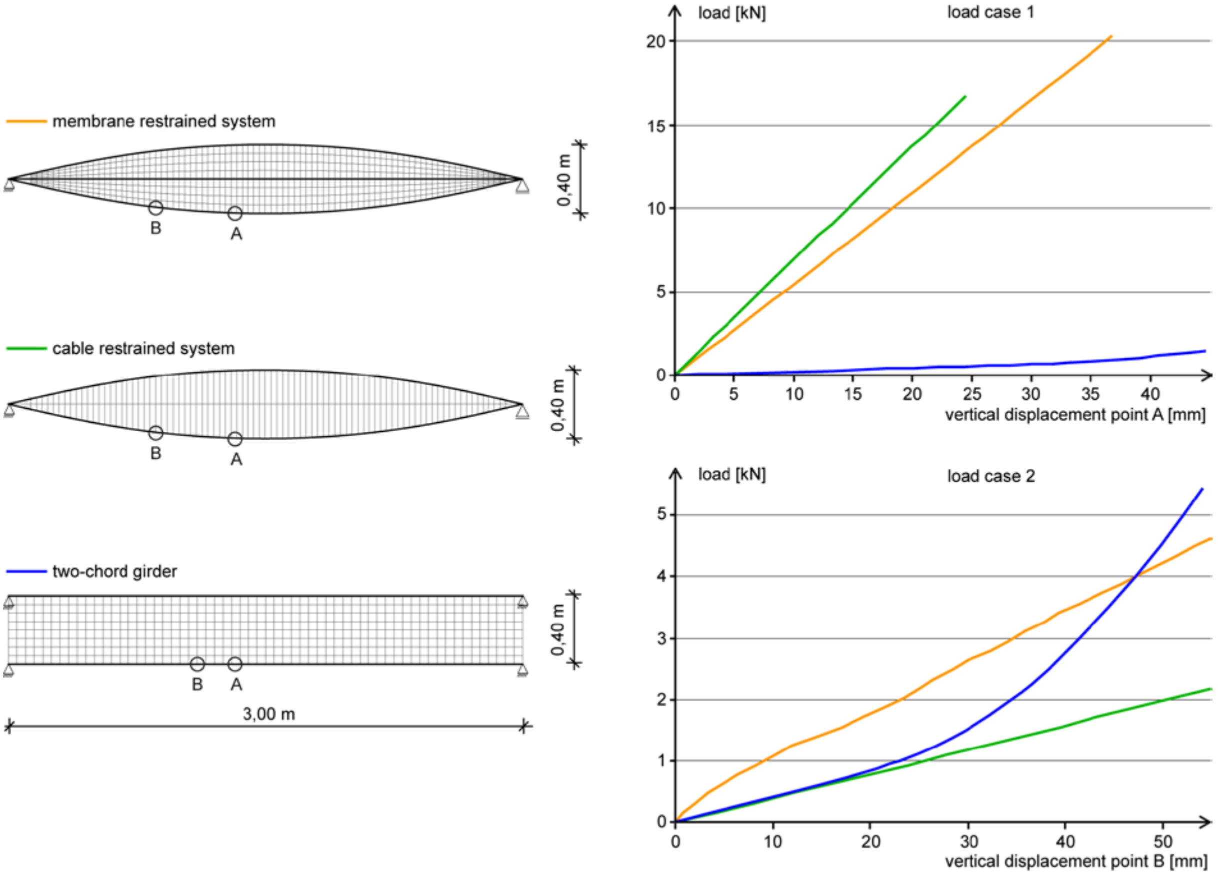

Figure 5: structural behaviour of membrane restrained system, cable restrained system and two-chord girder

The membrane and the cable restrained systems were also compared with a third system: a two chord girder (figure 5). Both chords are straight and supported in vertical and horizontal directions at both ends. If a shear force transfer by the membrane is assumed, the two pinned supports at each end will perform like a single clamped support for the girder. Additionally there is a direct load transfer in the membrane: The membrane forces are orientated diagonally and run from the bottom chord to the upper supports. Due to these two effects near the supports, one might expect the two-chord girder to be stiffer than the other systems. However, the deformations of the two-chord girder are much higher in load case 1. In load case 2 the deformations of the two-chord girder and the cable-restrained system are nearly identically for deformations up to 20 mm. The load-deflection curves of the two-chord girder show an exponential gradient because the system's stiffness increases when the chords act as cables by transferring tension forces. In the two-chord girder the membrane is not able to couple the chords like in the truss-model, as the stiffness of the chords for vertical loads is not high enough. For the coupling effect it is always necessary to ensure both a membrane for a diagonal load transfer as well as beam curvature for sufficient stiffness.

The principle membrane forces in load case 1 are orientated horizontally and vertically in the middle of the system and diagonally at the ends. The deformations of the chords are higher near the ends than in the middle, as the curvature is smaller near the ends. Near the ends the membrane couples the chords. The membrane is stressed in warp- and weft direction everywhere.

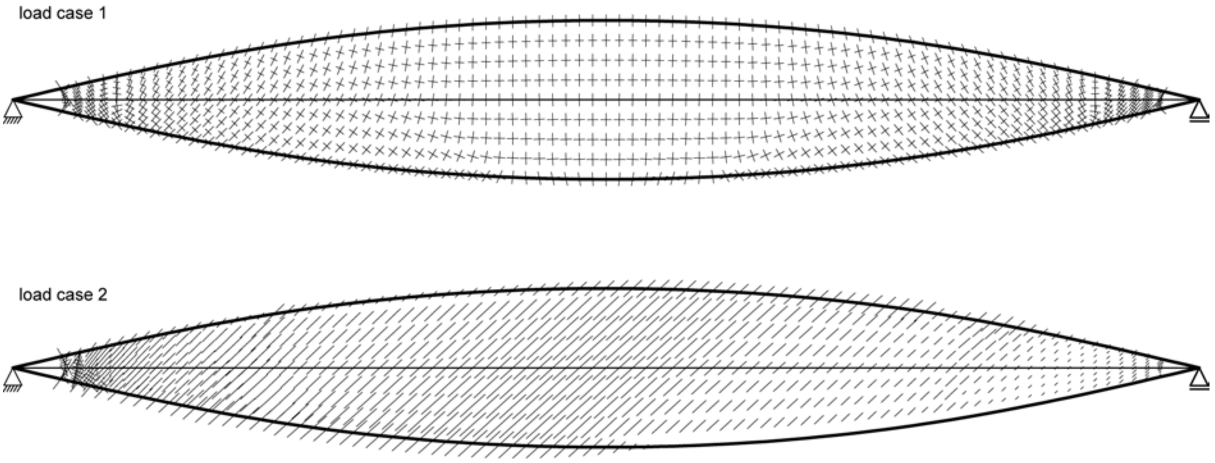

Figure 6: principle membrane forces

In load case 2 the principle membrane forces are orientated diagonally only. This correlates to the truss model (figure 2). The whole membrane is only stressed uniaxially, so wrinkles parallel to the membrane forces may occur.

The membrane restrained system was analysed for different heights (figure 7). The load deflection-curves show a linear gradient for all heights. In both load cases the deformations decrease with increasing height of the system. This occurs due to the increasing stiffness of the arch and cable structures, when the arch rise and the cable sag increases (chapter 2). The influence of the height to coupling effect can hardly be determined by these results.

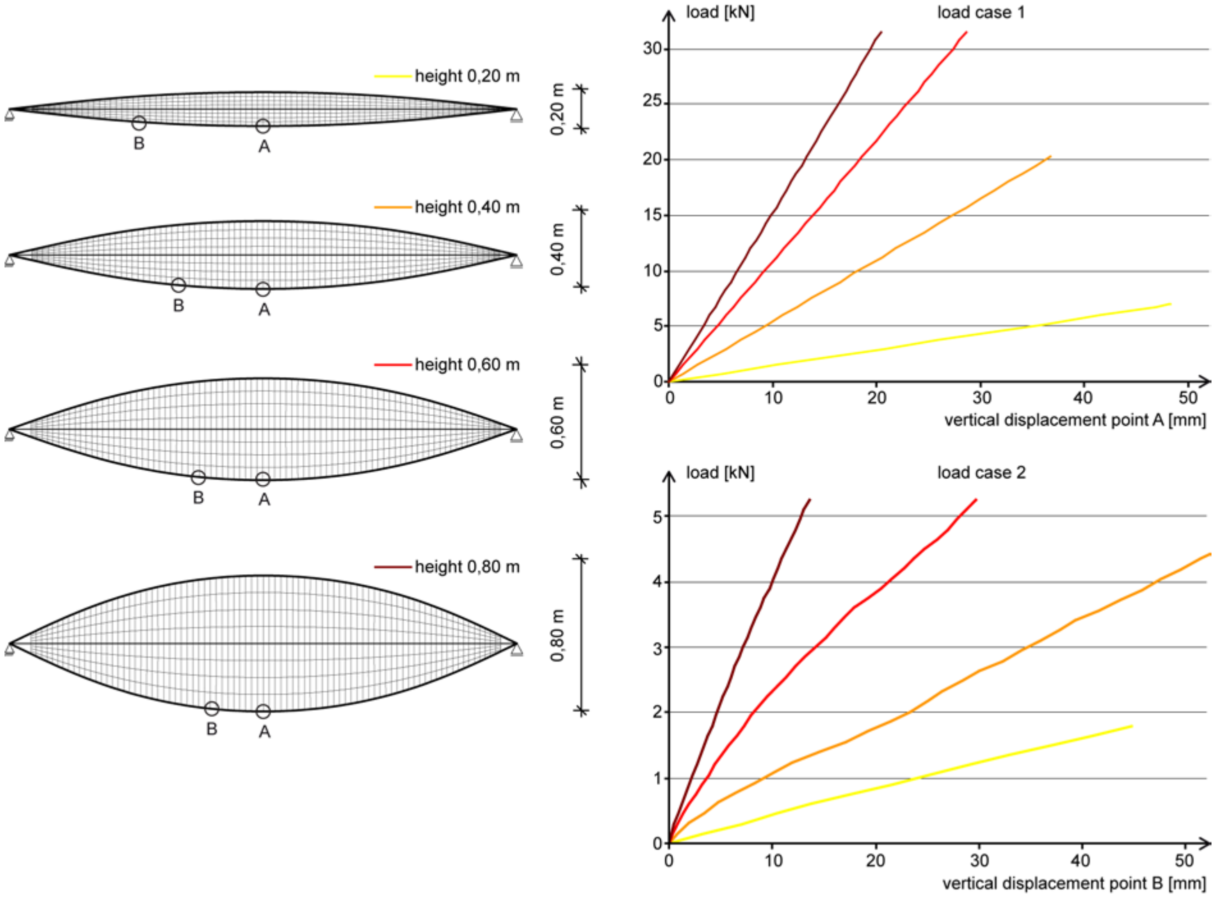

Figure 7: structural behaviour of membrane restrained systems with different height

The influence of the coupling-effect can be determined by calculating the quotient of the deformations of the membrane restrained system by the deformations of the cable restrained system. In both systems the stiffness increases with the height of the systems. Whereas in the cable restrained system the stiffness between the chords in vertical direction is higher, the membrane offers diagonal force paths for a coupling effect.

In load case 1 the deformations of the membrane restrained system and the cable restrained system are quite similar. This means that the coupling effect is negligible for the structural behaviour. As the loads are almost affine to the geometry of the chords, they will be transferred to the supports predominantly by the chords as arch and cable structures.

In load case 2 the deformations of the membrane restrained system are significantly smaller than for the cable restrained system. For the systems with a height of 0,20 m, the deformations of the membrane restrained system lie between 70 and 80% of those for the cable restrained system. As the height increases, the difference in deformations between the two systems also increases. The deformations of the membrane restrained system with a height of 0,80 m represent only about 10% of the deformations of the cable restrained system with the same height.

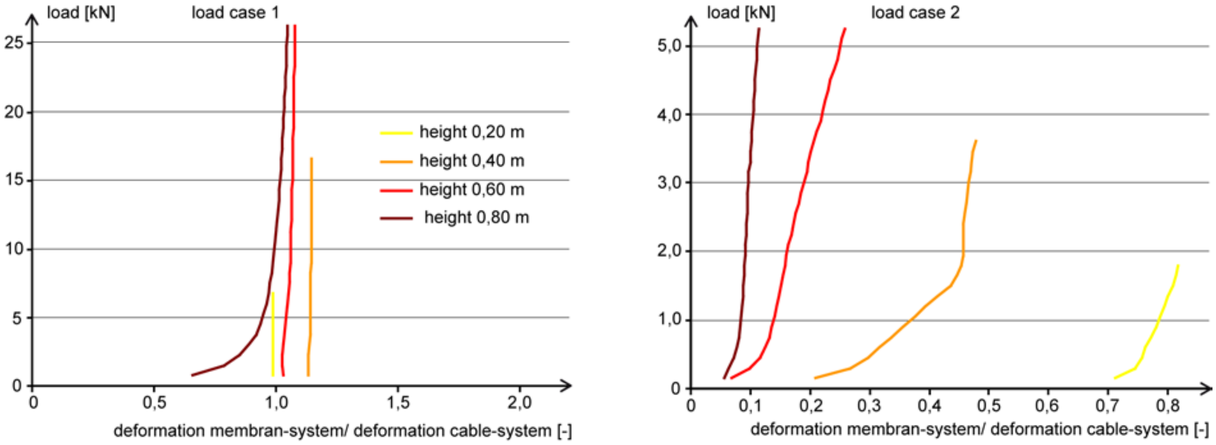

Figure 8: comparison of membrane- and cable restrained system

The coupling effect increases with the curvature of the chords. Therefore the curvature plays a significant role in the structural behaviour of these membrane restrained systems.

4 CONCLUSION

The concept of membrane coupling is based on a shear force transfer between two edge-elements. Since the shear-stiffness of textile membrane is negligible, the shear force transfer is achieved by the formation of tension ties in the membrane and not by the shear stiffness of the membrane itself. The edge elements must be curved in order to provide sufficient stiffness to take vertical forces. When using a truss model analogy, the formation of tension ties in the membrane act as the diagonal tension members of a truss while the curved edge beams act as the vertical compression struts of a truss. Membrane coupling does not provide a completely shear resistant connection, but instead provides force paths which offer a limited shear load transfer between the chords. The shear transferring connection in combination with the upper and lower chords form a multi-part cross section which increases the stiffness and the load-bearing capacity significantly compared with an uncoupled system.

REFERENCES

[1] J. Lienhard, H. Alpermann, C. Gengnagel, J. Knippers: *Active Bending, a Review on structures where bending is used as a self formation process*, Conference Proceedings IASS- APCS Symposium From Spatial Structures to Space Structures, 2012 Seoul.

[2] H. Alpermann and C. Gengnagel, *Hybrid membrane structures*, IASS Symposium 2010 – Spatial Structures Temporary and Permanent, 2189-2196, (2010).

[3] H. Alpermann and C. Gengnagel, *Membrane restrained columns*, Structural Membranes, Proceedings of the V International Conference on Textile Composites and Inflatable Structures, Barcelona, 2011.

[4] H. Alpermann, C. Gengnagel: *Shaping actively-bent elements by restraining systems*, Conference Proceedings IASS- APCS Symposium : From Spatial Structures to Space Structures, 2012 Seoul

[5] H. Alpermann and C. Gengnagel, Membranversteifter Träger, Stahlbau 05/2012, Ernst & Sohn / Wiley-VCH Verlag Berlin, Seite 373 - 378, 2012