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FORM-FINDING AND ANALYSIS OF BENDING-ACTIVE SYSTEMS USING DYNAMIC RELAXATION

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Summary. A common challenge for architects and engineers in the development of structurally efficient systems is the generation of good structural forms for a specific set of boundary conditions, a process known as form-finding. Dynamic relaxation is a well-established explicit numerical analysis method used for the form-finding and analysis of highly non-linear structures. With the incorporation of bending and clustered elements, the method can be extended for the analysis of complex curved and bending-active structural systems. Bending-active structures employ elastic deformation to generate complex curved shapes. With low computational cost, dynamic relaxation has large potential as a design and analysis technique of novel large span structural systems such as spline stressed membranes and small scale robotics, bio-mechanics and architectural applications made of novel materials such as electro- active polymers (EAP).

1 INTRODUCTION

Large elastic deformation phenomena are well known to engineers. At a (sub)microscopic level, these phenomena are attractive for their potential as concept generators for micro-lens surfaces and gels [1], [2] nano-tubes [3] and elastic shells [4]. However, in larger scales such as in structural engineering large elastic deformations are considered as failures and are designed against. There are only few examples in the built environment where implementing elastic deformation as a form-generating strategy has been explored [5-10].

Current design theory holds that excessive elastic deformations are undesirable in structures. Our research challenges that philosophy; we believe that large elastic deformations can be successfully modeled, analyzed and interpreted as a form-finding strategy for bending-active systems. By integrating elastic deformations in the form-finding process, novel lightweight spatial structures constructed from flexible yet strong engineering materials can be explored.

This paper focuses on the analysis of bending-active systems using dynamic relaxation. Dynamic relaxation is an established explicit numerical form-finding and analysis method [11]. In Section 2, the method and the element formulations for bending and clustered elements are presented. Section 3 presents the numerical modeling of a dielectric-elastomer minimum-energy structure and a stressed spline membrane. Finally, conclusions are presented in Section 4.

2 THEORY AND ELEMENT FORMULATION

2.1 Dynamic Relaxation Basic Scheme

Dynamic relaxation (DR) traces the motion of each node of a structure for small time increments, Δt, until, due to artificial damping, the structure reaches a static equilibrium [12]. In form-finding, the process may be started from an arbitrary geometry, with the motion initiated by a stress or force application. For analyses, the process starts with a valid geometry and the motion is caused by a sudden load application. The DR description summarized below is for structures with axial links and assumes a ‘‘kinetic’’ damping of the structural system to obtain a static equilibrium state. When kinetic damping is employed, the motion of the structure is traced and when a local peak in the total kinetic energy of the system is detected, all velocity components are set to zero. The process is then restarted from the current geometry and repeated until the energy of all modes of vibration has been dissipated and static equilibrium is achieved.

Dynamic relaxation is based on Newton's second law that governs the motion of any node i in direction x at time t:

$$R_{ix}^{t} = M_i \dot{v}_{ix}^{t} \tag{1}$$

where R_{ix}^{t} is the residual force (difference between external and internal forces) at node i in direction x at time t, M_i is the lumped mass at node i which is set to optimize the convergence and ensure the stability of the numerical process. $\dot{v}_{ix}^{t}$ is the acceleration at node i in direction x at time t.

Expressing the acceleration term in Equation (1) in a finite difference form and rearranging the equation gives the recurrence equation for updating the velocity components:

$$v_{ix}^{t+\Delta t} = \frac{\Delta t}{M_i} R_{ix}^{t} + v_{ix}^{t-\Delta t/2} \tag{2}$$

Hence, the updated geometry projected to time $t+\Delta t/2$ is given by:

$$x_i^{t+\Delta t} = x_i^{t} + \Delta t v_{ix}^{t+\Delta t/2} \tag{3}$$

Equations (2) and (3) apply for all unconstrained nodes of the mesh in each coordinate direction. Moreover, the equations are nodally decoupled: the updated velocity at a node depends only on the previous velocity and residual force of the same node. Nodes are not directly influenced by updates at other nodes.

The updated geometry is then employed to determine the new link forces and together with the applied load components P_{ix} to define the updated residual forces R_{ix}:

$$R_{ix}^{t+\Delta t} = P_{ix} + \sum \left(\frac{F}{L}\right)^{t+\Delta t} \left(x_j - x_i\right)^{t+\Delta t} \tag{4}$$

where $F^{t+\Delta t}$ is the force in member m connecting node i to an adjacent node j at time $t+\Delta t$, $L_m^{t+\Delta t}$ is the length of member m at time $t+\Delta t$. The procedure is thus time stepped using Equations (2) – (4), until a kinetic energy peak is detected. Velocity components are then reset to zero (with a small adjustment made to the geometry to correct to the true kinetic energy time peak), and the process is repeated until adequate convergence (equilibrium) is achieved.

2.2 Spline Bending Formulation

'Splines' originally denoted continuous flexible wooden or rubber strips used by draughtsmen to draw smooth curves for ship lines or railway curves. In this paper, the term refers to tubular structural elements that are bent from an initially straight state. It was shown that the torsional stiffness need not enter the analysis of a bent spline [7]. Therefore, the bending action in the spline is idealized as a series of bending moments between the nodes of the finite elements that compose the spline. The basic idea behind the bending formulation is that the bending moments across the elements result from changes in the curvature engendering shear forces at their nodes. Shear forces are then taken into account in the residual forces in the DR scheme.

Adriaenssens and Barnes [13] proposed a spline type formulation that deals with moments and shear forces in deformed tubular members. The formulation adopts a finite difference modeling of a continuous beam. Figure 1a represents consecutive nodes along an initially straight tubular element, and Figure 1b two adjacent deformed segments, a and b, viewed normal to the plane of nodes ijk. The two elements are assumed to lie on a circular arc of radius R. The spacing of nodes along the traverse must be sufficiently close but the segment lengths need not be equal. The radius of curvature R through i, j and k and the bending moment M in the arc can be defined as:

$$R = \frac{l_c}{2 sin\alpha} \tag{5}$$

$$M = \frac{EI}{R} \tag{6}$$

where EI is assumed to be constant along the beam, E is modulus of elasticity and I second moment of area. The free body shear forces S_a, S_b of elements a and b complying with moment M at j are thus given by:

$$S_a = \frac{2EIsina}{l_a l_c} \tag{7}$$

$$S_b = \frac{2EIsina}{l_b l_c} \tag{8}$$

where l_a, l_b, l_c are the distances between nodes ij, jk and ik, respectively. The three non-collinear nodes i, j and k define a reference plane ijk. The shear forces S_a and S_b are applied at nodes i, j and j, k, respectively and act normal to the links ij and jk respectively and in the ijk plane.

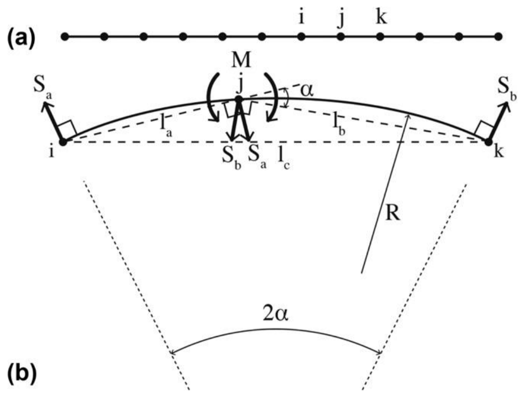

Figure 1: (a) Consecutive nodes along an initially straight tubular beam traverse; (b) Two adjacent deformed segments, *a* and *b*, viewed normal to the plane *ijk*.

The calculations and transformations required in the DR scheme are thus rather simple. Nodes along the spline element are considered sequentially in sets of three, each lying in different planes when modeling a spatially curved tubular element bent from an initially straight condition. The formulation is useful for modeling grid shells with continuous tubular members, and also for membranes in which flexible battens are employed to give shape control such as in sails.

2.2 Clustered Formulation

Clustered elements describe sliding or continuous tensile elements and were introduced by Moored and Bart-Smith [14]. Clustered elements can group two or more links. Figure 2 (right) illustrates a clustered four-node system, where the clustered element replaces two tensile elements (*links 2* and *3*). The clustered element can be seen as a cable running over a small frictionless pulley on *node 3*. Therefore, *links 2* and *3* in the clustered element carry the same tensile force. Additionally, *node 3* in the clustered structure has fewer kinematic constraints compared with the same node in the un-clustered system (see Figure 1 left) as it can move around its current position.

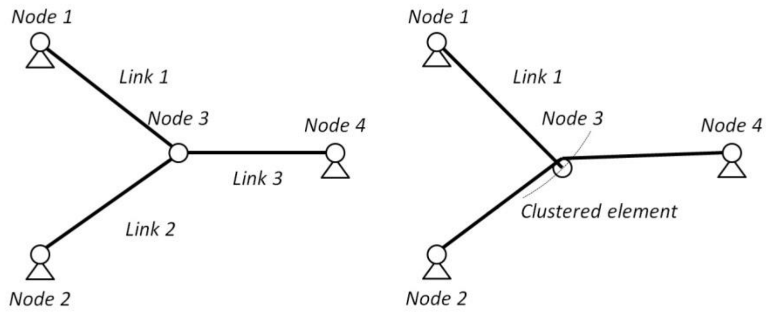

Figure 2: Illustration of an un-clustered four-node system (left) and a clustered configuration (right).

A clustered element for dynamic relaxation was proposed by Bel Hadj Ali et al. [15]. Similar to [14], a clustering matrix S is used to link the clustered structure with its corresponding un-clustered configuration. The clustering matrix $S \in \mathbb{R}^{\bar{e} \times e}$ is defined as follows:

$$S_{ij} = \begin{cases} 1, & if\ the\ link\ e_j\ is\ part\ of\ the\ clustered\ element\ e_i \\ 0, & if\ not \end{cases}$$

where $\bar{e}$ is the number of elements in the clustered structure and e is the number of elements in the traditional (un-clustered) structure. The equilibrium of the clustered system is thus linked with the equilibrium of the un-clustered system. Element characteristics such as the elastic modulus, the cross-section area, the fabrication length and pre-stress are also linked to the un-clustered system using the clustering matrix S:

$$\bar{p} = Sp \tag{9}$$

where $\bar{p}$ corresponds to a characteristic of the clustered system and p is the same characteristic for the un-clustered system. The internal force in the m^{th} element of the clustered structure at a time t is given by:

$$\bar{f}_m^t = \frac{\bar{E}_m \bar{A}_m}{\bar{l}_{0,m}} \left(\bar{l}_m^t - \bar{l}_{0,m}^t \right) + \bar{f}_m^0 \tag{10}$$

where $\bar{E}_m$, $\bar{A}_m$ and $\bar{f}_m^0$ are the elastic modulus, the cross-section area and initial pre-stress of the clustered member m. $\bar{l}_{0,m}$ and $\bar{l}_m^t$ are the fabrication length and the current length of clustered member m. The internal forces of the un-clustered structure can be related to the clustered-element internal forces through:

$$f_m^t = S^T \bar{f}_m^t \tag{11}$$

where $\bar{f}_m^t$ is the internal force in the m^{th} element of the un-clustered structure at a time t. Linking the equilibrium of the clustered system with the equilibrium of its corresponding un-clustered configuration allows dynamic relaxation to correctly model sliding or continuous tensile elements while maintaining its computational advantages.

3 FORM-FINDING AND ANALYSIS EXAMPLES

3.1 Dielectric-Elastomer Minimum-Energy Structures

Dynamic relaxation provides an inexpensive alternative for the simulation of dielectric-elastomer minimum-energy structures (DEMES). DEMES are electro-active bending-active structures composed of a prestressed dielectric elastomer membrane adhered to a thin flexible frame [16]. The strain energy of the prestressed membrane is transferred to the initially straight frame deforming the structure until equilibrium. The shape of the structure is controlled by prestress and reflects a minimum energy state in the structure [17]. DEMES have been proposed for shape-shifting applications in various disciplines such as robotics [16], bioengineering [17] and architecture [18]. However, predicting the behavior of DEMES remains a challenging task requiring complex analytical or numerical models.

In this paper, we analyze a DEMES with a rounded triangular shape (Figure 3) using dynamic relaxation and compare the form-found shape with the shape obtained with a physical model. The frame of the model has a length of 52mm and a width of 4mm. A similar structure was studied by O'Brien et al. [19]. The input for dynamic relaxation (nodal coordinates and connectivity) is a planar mesh of links connected with nodes based on the scheme of Figure 3. Nodes at the base of the system are pinned. Clustered elements with an axial stiffness of 0.08N/mm are used to model the membrane while bending elements with a bending stiffness of 7.8mm^2 are employed for the frame.

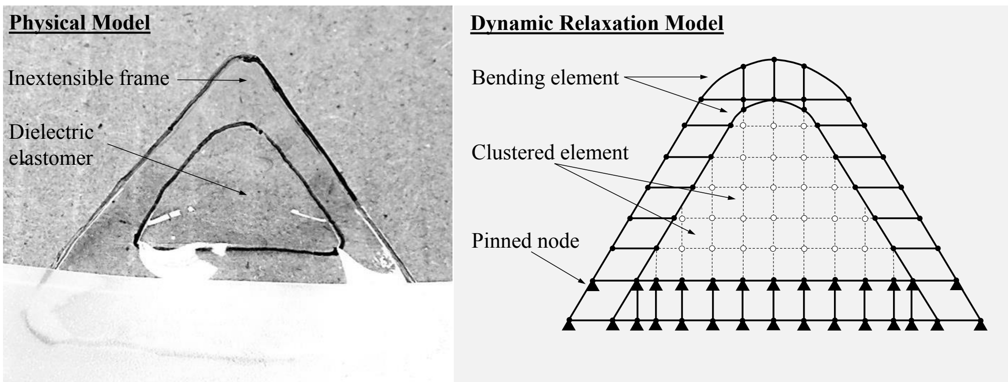

Figure 3: Illustration of the elements in the numerical model in relation with the physical model.

DEMES equilibrium shapes are controlled by the prestress in the membrane. Therefore, clustered elements are given initial lengths providing the desired prestress. To initiate the simulation in the numerical model, the top of the mesh is given a small initial deformation. The prestress in the membrane induces the bending of the frame until an equilibrium shape is obtained. Figure 4 shows the equilibrium shape obtained with the dynamic relaxation DEMES model in relation with the shape of the physical model. The shape resulting from the application of prestress in the clustered elements of a flat structure is similar to the shape of the DEMES physical model.

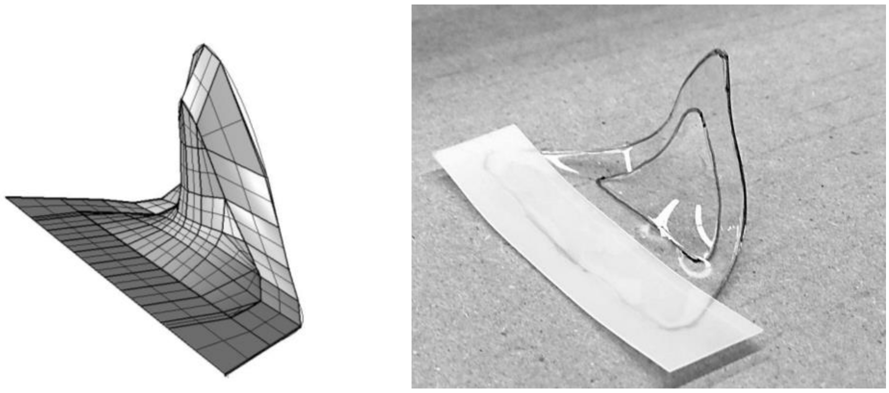

Figure 4: DEMES equilibrium shape obtained with dynamic relaxation in relation the physical model.

The equilibrium shape of the numerical model and the physical model correspond to different prestress states. The membrane in the physical model is prestressed at 200% of its initial length, while in the numerical model clustered elements have a prestress of 150%. This discrepancy is most likely due to uncertainties in the numerical model [20] as well as due to the hysteretic DEMES behavior [21].

3.2 Stressed Spline Membranes

Stressed spline membrane structures are bending-active structures that combine spline elements with a prestressed membrane. Splines need to be sufficiently flexible to be curved into the required shape and to enough strength to resist the forces arising from bending and the loading combinations. Therefore, materials such as Fibre Reinforced Plastics (FRP) that have low Young's modulus and high strength are favored for spline applications over traditional structural materials such as steel and timber.

In this paper, we analyze focus on a branched spline system in which the splines themselves provide bracing. The structure is based on three arcs of a circle joined to each other at one end at a central height of 3.5m using a branching splice joint while their other end lies on the circumference of a 8m radius circle, 120degrees away from each other (Figure 5). Between the arc ends on the circumference, boundary arches with a central height of 2.3m are inclined at 65degrees from the vertical plane and fixed. The geometry provides a double curvature in the prestressed membrane and therefore increased stiffness.

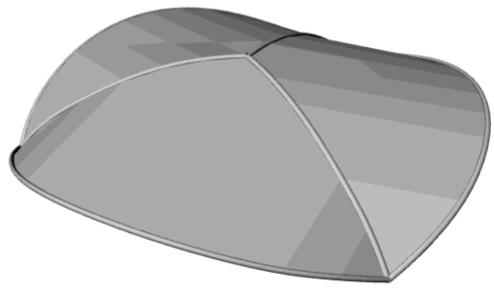

Figure 5: Initial shape of the prestressed spline membrane structure.

The apex joint is required to be stiff and flat to be a branched splice. Therefore, it is modeled using splice and virtual elements (Figure 6). Adjacent splines are connected through splices that run from the penultimate node of one spline via the central node to the penultimate node of the next spline. Virtual members are also added to the apex of the structure to keep it horizontal during the form-finding and the load analysis. The additional members link the penultimate nodes of the structural splines triangulating the apex into a rigid joint. In practice, the branched splice might be a stiff casting onto which the splines are slotted.

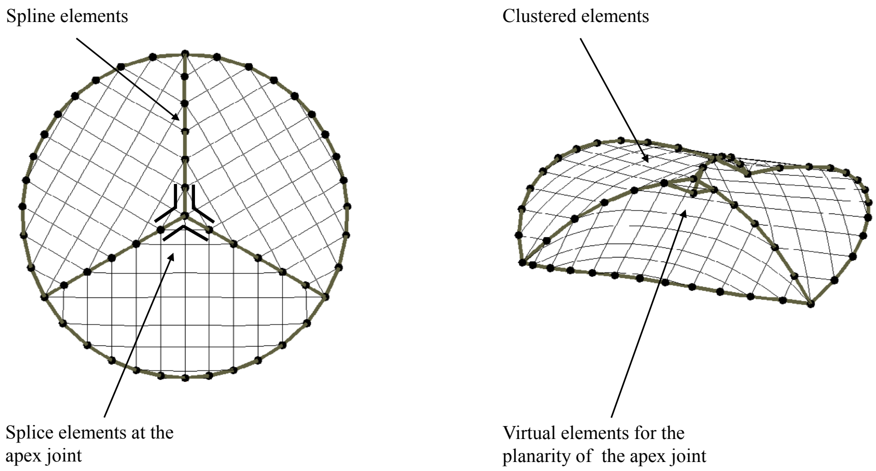

Figure 6: Elements for the apex joint.

Splines are composed of tubular Glass Fibre Reinforced Plastic (GFRP) elements with a diameter of 120mm and a wall thickness of 5mm. GFRP tubes have a Young's modulus of 40000MPa and admissible stresses of $100N/mm^2$ for compression and $700N/mm^2$ for tension as well as bending. The membrane is made out of PVC. It has a warp and weft stiffness of 1MN/m and an admissible strength of 12kN/m. Moreover, a prestress of 1kN/m is applied in the membrane in both directions. The design load cases considered include self-weight along with symmetric and asymmetric snow and wind loading. Four loading cases were analyzed: 1.

asymmetric wind loading and self-weight, 2. asymmetric snow loading and self-weight, 3. asymmetric snow and wind loading as well as self-weight, 4. uniform snow loading and self-weight. Stresses under these load cases remain below element strength (Table 2). Furthermore, although splines tend to straighten out under loading, deformations remain within acceptable levels.

Table 1: Engineering materials and their properties

Loading condition: *$p=0.20kN/m^2$*	Max. stress in the spline [N/mm^2]	Max. force in the membrane [kN]	Deflection at the center [mm]
1. Asymmetric wind loading and self-weight: *p and -4p*	376.20	8.53	81
2. Asymmetric snow loading and self-weight: *2.5p and p*	394.30	6.79	277
3. Asymmetric snow and wind loading as well as self-weight: *2.5p and -4p*	381.90	8.67	146
4. Uniform snow loading and self-weight: *2.5p and 2.5p*	443.8	9.51	444

4 CONCLUSION

This paper focuses on the form-finding and analysis of bending-active structure. With the incorporation of bending and clustered elements, dynamic relaxation is extended for the analysis of complex curved and bending-active structural systems. The formulation provides a fast and valuable tool to investigate structural behavior in the radial direction of in-plane bending. Two bending-active structures were investigated using spline and clustered elements: a basic DEMES and a stressed spline membrane. When compared with actual physical models, it was found that dynamic relaxation correctly predicts the equilibrium shapes of DEMES. In the stressed spline membrane, the pre-stress in the membrane acts as a continuous restraint for the splines allowing them to be very slender and therefore bent to a tighter radius. The examples show that the presented formulation has great potential for the modeling of bending-active elements that undergo large elastic deformations and opens the door to the development of a whole new realm of novel structural curved systems.

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