ANALYSIS OF CABLE STRUCTURES BY MEANS OF TRIGONOMETRIC SERIES

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Summary. *Cable structures are light-weight, expressive from an architectural point of view and allow covering large spans without intermediate supports. They are far superior to the conventional bearer systems of buildings and constructions. On the other hand, cables are very deformable and possess complex non-linear behavior under load. In spite of availability of special computer programs, designed for iteration analysis of cable systems, the problems of optimization require elaboration of analytical structural models. In the present paper, the behavior of flexible cables is analyzed by means of trigonometric series. This technique may be used for simulation of complex structures, comprising several tiers or flexible chords. It extends the scope of analytical approach on cables, influenced by substantially non-uniform loads. The present work also contributes to the analysis of flexible membrane systems, which are often simulated by a number of cables, arranged in mutual-perpendicular directions.*

1 INTRODUCTION

Cable structures are applied in buildings of various types and assignments. They substantially reduce assembly expenditures and amount of steel needed for the construction. In contrast to beams, girders and trusses, cables can span large areas without installation of intermediate supports.

Light-weight cable structures are intended to be covered with flexible high-strength polymer membranes or architectural fabrics, allowing the roof to be light-translucent. In addition, structural behavior of membranes is usually simulated by means of a number of flexible cables, arranged in mutual - perpendicular directions[1,2].

On the other hand, cables are very deformable and can sustain tensile forces only. They exhibit complex non-linear behavior under load. For a given set of parameters, such as sag and curvature of cables, values of preliminary stresses, cross section areas etc., structural analysis is performed by means of iteration techniques, with the help of specialized computer programs. These parameters are not usually known in advance, especially when the problem of optimization arises and the designer is to elaborate a cost-effective construction. This complex task may be solved by analytical investigation of the structure in order to get appropriate results directly or to substantially confine number of unknown structural parameters and their possible combinations. In addition, analytical approach is required to determine the influence of structural parameters on structural behavior. It allows checking numerical results, obtained by iteration techniques[2].

In the present paper, flexible cables are analyzed by means of trigonometric series[3,4]. Coefficients of load distribution are calculated in accordance to the formulations of Fourier's theory[5,6]. The cable shape function is represented by means of a multitude of coefficients. They depend on each other and may be expressed in terms of one coefficient. The technique for obtaining the coefficient is given. It is based on the differential equation of equilibrium of a flexible cable.

2 A FLEXIBLE CABLE UNDER LOAD

A flexible cable, influenced by non-uniform external load $q(x)$, is illustrated in figure 1. The present study concerns so-called shallow cables, with sag-to-span ratio (f/L) less than 1/8. In the assumption that the cable is perfectly flexible its shape is entirely determined by the load distribution. The horizontal support reactions *H*, called thrust, arise on both sides of the cable.

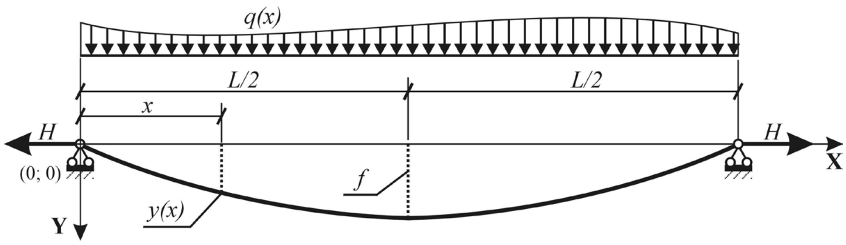

Figure 1: A cable under load

Cable shape, determined by a function $y(x)$, as well as external load $q(x)$ (figure 1) may be represented with the trigonometric sine-series as follows:

$$y(x) = S\left(x, \overrightarrow{kY}\right) \tag{1}$$

$$q(x) = S\left(x, \overrightarrow{kq}\right) \tag{2}$$

where $\overrightarrow{kY}$ and $\overrightarrow{kq}$ are the coefficients of the series[3,4]:

$$S\left(x, \vec{k}\right) = \sum_{m=1}^{nM} k_m \cdot sin\left(\frac{m \cdot \pi \cdot x}{L}\right) \tag{3}$$

where *m* is a number of a term of the series; *nM* is the quantity of considered series terms; *L* is the span of the cable.

The expression (1) satisfies the boundary conditions of the cable: $y(0) = y(L) = 0$ due to the numbers *m* are integer. In accordance to (2) load values also should be zero at $x = 0$ and

$x = L$. However, the more the terms of the series are considered the smaller the discrepancy to the given load distribution arises. It is additionally diminished by reduction of influence of the load near the supports of the cable.

Coefficients $\overrightarrow{kq}$ are calculated by means of the theory of Fourier series[5,6]:

$$kq_m = \frac{2}{L} \cdot \int_0^L \left(q(x) \cdot sin\left(\frac{m \cdot \pi \cdot x}{L} \right) \right) dx \tag{4}$$

The differential equation of the cable, influenced by an external load $q(x)$, is written as follows[3]:

$$\frac{d^2}{dx^2} y(x) = \frac{-q(x)}{H} \tag{5}$$

where H is the thrust. It depends on the axial force N in the cable and the angle γ between the tangent to it and X-axis:

$$H = N(x) \cdot \cos[\gamma(x)] \tag{6}$$

Variation of the force N along the span of a shallow cable is negligible. It may be considered approximately constant and equal to the thrust H:

$$N(x) \approx H \,. \tag{7}$$

Thus, the force N is obtained from the Hook's law:

$$N = N\left(\overrightarrow{kY}\right) = EA \cdot \frac{L_{c,1}\left(\overrightarrow{kY}\right) - L_{c,0}}{L_{c,0}} \tag{8}$$

where EA is the tensile stiffness of the cable; $L_{c,0}$, $L_{c,1}$ are total lengths of the cable in initial state and under external load q, respectively.

Substituting (1) and (2) into (5), considering (7), yields the set of nM equations:

$$m^2 \cdot \frac{kY_m}{kq_m} = \frac{1}{N\left(\overrightarrow{kY}\right)} \cdot \left(\frac{L}{\pi} \right)^2 \tag{9}$$

where $m = 1 \ldots nM$.

The right-hand side of every equation (9) is constant for a given vector of coefficients $\overrightarrow{kY}$. Consequently, the dependence for different indexes m and j may be written as follows:

$$m^2 \cdot \frac{kY_m}{kq_m} = j^2 \cdot \frac{kY_j}{kq_j} \tag{10}$$

It results in the following relationships between the coefficients $\vec{kY}$ of the series (1):

$$kY_m = \left(\frac{j}{m}\right)^2 \cdot kmj_m \cdot kY_j \tag{11}$$

where

$$kmj_m = kq_m / kq_j \tag{12}$$

The ratios kmj_m depend only on the type of the load, for example equal or inverse-symmetric one, and do not depend on particular load values.

The equation (11) means, that all coefficients $\vec{kY}$ of the series (1) depend on each other and may be expressed in terms of one coefficient kY_j. The index *j* must be selected according to the condition: $kq_j \neq 0$. In most cases it is advisable to apply $j = 1$, however for inverse-symmetric load it should be $j = 2$.

Substituting (11) into (9) yields the following equation in one unknown kY_j:

$$kY_j = \frac{kq_j}{N\left(kY_j\right)} \cdot \left(\frac{L}{j \cdot \pi}\right)^2 \tag{13}$$

In order to solve (13), cable length L_c, involved in (8), should be expressed in terms of the coefficient kY_j.

3 DERIVATION OF THE LENGTH OF THE CABLE

Cable length may be found from the simplified expression, valid for shallow cables only[7]:

$$L_c = \int_0^L \left[1 + \frac{1}{2} \cdot \left(\frac{d}{dx} y(x)\right)^2 - \frac{1}{8} \cdot \left(\frac{d}{dx} y(x)\right)^4\right] dx \tag{14}$$

Substituting (1) into (14), considering (11), yields the following expression:

$$L_c = L + k_2 \cdot \left(kY_j\right)^2 - k_4 \cdot \left(kY_j\right)^4 \tag{15}$$

where k_2 and k_4 are the coefficients:

$$k_2 = \frac{\pi^2}{4 \cdot L} \cdot j^4 \cdot \sum_{m=1}^{nM} \left(\xi_m\right)^2 \tag{16}$$

$$k_4 = \frac{\pi^4 \cdot j^8}{8 \cdot L^3} \cdot \sum_{i=1}^{5} k_{4,i} \tag{17}$$

where

$$\xi_m = kmj_m / m \tag{18}$$

and

$$k_{4,1} = \frac{3}{8} \cdot \sum_{m=1}^{nM} (\xi_m)^4 \tag{19, a}$$

$$k_{4,2} = 1.5 \cdot \sum_{m_1=1}^{nM-1} \sum_{m_2=m_1+1}^{nM} (\xi_{m_1} \cdot \xi_{m_2})^2 \tag{19, b}$$

$$k_{4,3} = \frac{1}{2} \cdot \sum_{m_1=1}^{nM} \sum_{m_2=1}^{nM-1} (\xi_{m_1})^2 \cdot \xi_{m_2} \cdot \xi_{m_3} \tag{19, c}$$

where $m_3 = 2 \cdot m_1 \pm m_2$, under the condition: $m_3 \in [(m_2 + 1); \quad nM]$;

$$k_{4,4} = \sum_{m_1=1}^{nM-2} \sum_{m_2=m_1+1}^{nM} \sum_{m_3=m_1+1}^{nM-1} \xi_{m_1} \cdot \xi_{m_2} \cdot \xi_{m_3} \cdot \xi_{m_4} \tag{19, d}$$

where $m_4 = \pm m_1 + m_2 \pm m_3$, under the condition: $m_4 \in [(m_3 + 1); \quad nM]$;

$$k_{4,5} = \sum_{m_1=1}^{nM-2} \sum_{m_2=m_1+1}^{nM-1} (\xi_{m_1})^2 \cdot \xi_{m_2} \cdot \xi_{m_3} \tag{19, e}$$

where $m_3 = 2 \cdot m_1 + m_2$, under the condition: $m_3 \in [(m_2 + 1); \quad nM]$.

Coefficients k_2 and k_4 depend on the type of the load, but not on its particular values.

The coefficient kY_j in (15) may be replaced with a cable ordinate $f = y(X)$ at a given point $X \in (0...L)$ along the span:

$$kY_j = \frac{f}{\Psi} \tag{20}$$

The coefficient Ψ is derived from (1) and (11):

$$\Psi = j^2 \cdot \sum_{m=1}^{nM} \mu_m \cdot sin\left(\frac{m \cdot \pi \cdot X}{L}\right) \tag{21}$$

where

$$\mu_m = kmj_m / m^2 \tag{22}$$

Consequently, the equilibrium shape of the cable is uniquely defined by the type of the external load and one point between supports.

Expression (21) may be written for three main points on the cable, situated in ¼, ½ and ¾ along length of the span:

$$\Psi = j^2 \cdot \sum_{n=1}^{nN} \sum_{t=1}^{3} sin\left(t \cdot \frac{\beta \cdot \pi}{4}\right) \cdot \left[\mu_{i_1(n,t)} - \left(2 - |\beta - 2|\right) \cdot \mu_{i_2(n,t)}\right] \tag{23}$$

where β = 1, 2 or 3 for $X = 1/4 \cdot L$, $1/2 \cdot L$ and $3/4 \cdot L$, respectively; *nN* is an integer number, which obeys the condition: $nN \le (nM + 1)/8$; i_1 and i_2 are the following indexes:

$$i_1(n,t) = 4 \cdot n - t + 4 \cdot (n-1) \cdot |\beta - 2| \tag{24, a}$$

$$i_2(n,t) = 4 \cdot n - t + 4 \cdot n \cdot |\beta - 2| \tag{24, b}$$

Substituting (20) into (15) results in the expression, from which the length of the cable may be found:

$$L_c(f) = \Psi_4 \cdot f^4 + \Psi_2 \cdot f^2 + L \tag{25}$$

where

$$\Psi_2 = k_2 / \Psi^2 \quad \text{and} \quad \Psi_4 = -k_4 / \Psi^4 . \tag{26}$$

4 DEFLECTION OF THE CABLE UNDER LOAD

Substituting (8) and (20) into (13), considering (25), yields a quintic equation in one unknown *f*:

$$f = \frac{\rho}{L_c(f) - L_{c,0}} \tag{27}$$

where

$$\rho = \left(\frac{L}{j \cdot \pi}\right)^2 \cdot \frac{L_{c,0}}{EA} \cdot \Psi \cdot kq_j \tag{28}$$

Cable ordinate f may be found from (27) by a diagram. On the other hand, graphical solution of the equation is a universal but inconvenient technique. In order to solve it analytically it is proposed to replace the ordinate f with cable deflection Δf , namely: $f = f_0 + \Delta f$. Terms of the equation, having degree higher than the second one, are omitted because the deflection is substantially smaller, than cable span and sag. Thus, the deflection is obtained as follows:

$$\Delta f = \frac{-A_1 + \sqrt{\left(A_1\right)^2 + 4 \cdot A_2 \cdot \left(\rho - A_0 \cdot f_0\right)}}{2 \cdot A_2} \tag{29}$$

where f_0 is the given ordinate of the cable in the initial state; A_0, A_1 and A_2 are the following coefficients:

$$A_0 = \Psi_4 \cdot (f_0)^4 + \Psi_2 \cdot (f_0)^2 + L - L_{c,0}, \quad A_1 = A_0 + 2 \cdot (f_0)^2 \cdot \left[2 \cdot \Psi_4 \cdot (f_0)^2 + \Psi_2\right], \quad A_2 = f_0 \cdot \left[10 \cdot \Psi_4 \cdot (f_0)^2 + 3 \cdot \Psi_2\right].$$

5 ANALYSIS OF TWO-CHORD PRETENSIONED CABLE TRUSS

Pretensioned cable truss[8], made of two flexible chords connected with ties (figure 2) is considered.

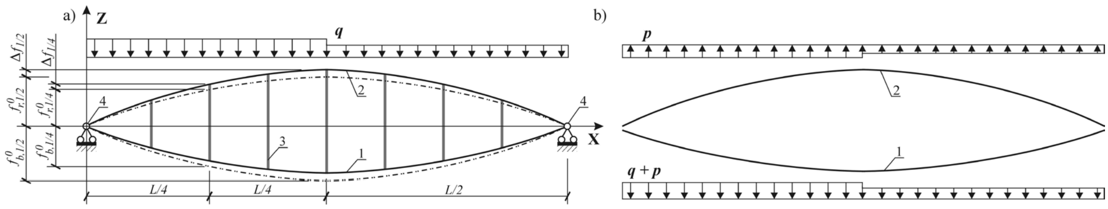

Figure 2: Two-chord cable truss. a – model of the structure; b – loads, acting on the chords; 1 – pre-stressed bearer chord; 2 – restraining chord; 3 – tie (spreader); 4 – fixed support

Loads, acting on the chords of the truss are the following: bearer chord - $Load_b = q(x) + p(x)$ and restraining chord - $Load_r = p(x)$, where *q(x)* is an external load assumed to influence from top to bottom; *p(x)* is, so-called, "link load", transmitting from a chord to the other one by means of ties; hereinafter index "*b*" refers to the bearer chord, and index "*r*" – to the restraining chord.

It is assumed that loads *p* and *q* may be split into uniformly distributed (index '*Eq*') and inverse-symmetric (index '*Inv*') parts (figure 3):

$$x = 0..\frac{L}{2} \qquad q(x) = q_{Eq} + q_{Inv} \qquad p(x) = p_{Eq} + p_{Inv} \tag{30, a}$$

$$x = \frac{L}{2}..L \qquad q(x) = q_{Eq} - q_{Inv} \qquad p(x) = p_{Eq} - p_{Inv} \tag{30, b}$$

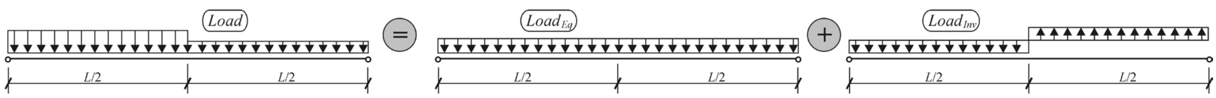

Figure 3: Decomposition of the load into uniformly distributed and inverse-symmetric parts

Loads, acting on the chords, may be represented by a similar way:

$$Load_{b,Eq} = q_{Eq} + p_{Eq} \quad \text{and} \quad Load_{b,Inv} = q_{Inv} + p_{Inv}, \tag{31, a}$$

$$Load_{r,Eq} = p_{Eq} \quad \text{and} \quad Load_{r,Inv} = p_{Inv}. \tag{31, b}$$

The ratios kmj_m, defined by (12), may be written for uniformly distributed and inverse-symmetric loads as follows:

$$eq_{\tau_i} = \frac{j}{\tau_i}, \qquad eq_{(m \neq \tau_i)} = 0 \tag{32, a}$$

$$inv_{2\cdot\tau_i} = \eta \cdot eq_{\tau_i}, \qquad inv_{(m \neq 2\cdot\tau_i)} = 0 \tag{32, b}$$

where $\tau_i = 2 \cdot i - 1$ is the number of non-zero ratio; i is an integer number 1, 2, etc.; η is the ratio of inverse-symmetric load to the uniformly distributed part:

$$\eta = Load_{Inv} / Load_{Eq} . \tag{33}$$

Expression (33), written for the bearer and restraining chords according to (31, a) and (31,b), results in the dependence between load coefficients η_b and η_r of the chords:

$$\eta_b = \frac{q_{Inv} + \eta_r \cdot p_{Eq}}{q_{Eq} + p_{Eq}} \tag{34}$$

Structural analysis of two-chord cable truss, influenced by non-uniformly distributed external loads, requires at least two points X_1 and X_2 to control deflections of upper and lower cables. According to (20), the relationship between ordinates $f_{1/4}$, $f_{1/2}$ and the coefficients $\Psi_{1/4}$ and $\Psi_{1/2}$ at points $X_1 = L/4$ and $X_2 = L/2$ is written as follows:

$$\frac{\Psi_{1/2}}{\Psi_{1/4}} = \frac{f_{1/2}}{f_{1/4}} = k_f \tag{35}$$

where $\Psi_{1/4}$ and $\Psi_{1/2}$ are obtained from (23), using $\beta = 1$ and $\beta = 2$, respectively.

The ordinates in the middle and in the quarter of the span ($f_{1/2}$, $f_{1/4}$) may be written for bearer and restraining chords as follows:

$$f_{b,1/2} = f^0{}_{b,1/2} - \Delta f_{1/2} \text{ and } f_{b,1/4} = f^0{}_{b,1/4} - \Delta f_{1/4}, \tag{36, a}$$

$$f_{r,1/2} = f^0{}_{r,1/2} + \Delta f_{1/2} \text{ and } f_{r,1/4} = f^0{}_{r,1/4} + \Delta f_{1/4}, \tag{36, b}$$

where $f^0{}_{1/4}$ and $f^0{}_{1/2}$ are initial ordinates of the bearer or restraining chord, which are usually given in advance; $\Delta f_{1/4}$ and $\Delta f_{1/2}$ are deflections of the cable truss, which are to be found.

Having substituted (23), written for $\beta = 1$ and $\beta = 2$, into (35), and taking into account (32), the expression for the coefficient k_f is derived as follows:

$$k_f = \frac{4}{\eta - \Theta} \tag{37}$$

where Θ is the coefficient:

$$\Theta = 2 \cdot \sqrt{2} \cdot \frac{\sum_{n=1}^{nN} \left[\frac{1}{(8 \cdot n - 3)^3} + \frac{1}{(8 \cdot n - 1)^3} - \frac{1}{(8 \cdot n - 7)^3} - \frac{1}{(8 \cdot n - 5)^3} \right]}{\sum_{n=1}^{nN} \left[\frac{1}{(4 \cdot n - 3)^3} - \frac{1}{(4 \cdot n - 1)^3} \right]}$$

It is obvious, that the coefficient Θ is constant for $nN \to \infty$, namely $\Theta = -3.0$. Therefore, the expression (37) is written as follows:

$$k_f = \frac{4}{\eta + 3.0} \tag{37'}$$

Substituting (37') into (35) yields the following dependence between load ratio η and the ratio of ordinates of the cable:

$$\eta = 4 \cdot f_{1/4} / f_{1/2} - 3 \tag{38}$$

Coefficient kq_j, required for the equilibrium equation (27), may be written according to (31) as follows:

$$kq_{b,j} = \left(q_{Eq} + p_{Eq}\right) \cdot kEq_{1,j} + \left(q_{Inv} + p_{Inv}\right) \cdot kInv_{1,j} \tag{39, a}$$

$$kq_{r,j} = p_{Eq} \cdot kEq_{1,j} + p_{Inv} \cdot kInv_{1,j} \tag{39, b}$$

where $kEq_{1,j}$ and $kInv_{1,j}$ are the coefficients of unit loads expansion (uniformly distributed and inverse-symmetric, respectively), defined by (4).

Either $kEq_{1,j}$ or $kInv_{1,j}$ is equal to zero for the same *j*. Hereinafter index *j* is omitted (it is adopted $j = 1$) and expressions (39) is simplified as follows:

$$kq_b = \left(q_{Eq} + p_{Eq}\right) \cdot kEq_1 \tag{39', a}$$

$$kq_r = p_{Eq} \cdot kEq_1 \tag{39', b}$$

where $kEq_1 = 4/\pi$.

Thus, equation (27) may be written for the chords as follows:

$$\frac{f_{b,1/2}}{q_{Eq} + p_{Eq}} = \frac{\rho_b}{\Delta L^b} \tag{40, a}$$

$$\frac{f_{r,1/2}}{p_{Eq}} = \frac{\rho_r}{\Delta L^r} \tag{40, b}$$

where ρ_b and ρ_r are the coefficients, calculated for the chords according to (28), using the following values $kq_j = kEq_1$ and $\Psi = \Psi_{1/2} = 1 - 1/3^3 + 1/5^3 - 1/7^3 \ldots \approx 0.96894$; ΔL^b and ΔL^r are elongations of the chords:

$$\Delta L = L_c - L_{c,0} \tag{41}$$

where L_c is the length of the chord under load; $L_{c,0} = L^0{}_{c,0} - \Delta L_p$ is the initial length of the chord, where $L^0{}_{c,0}$ is, so called, geometrical chord length in unloaded state and ΔL_p is tensioning of the chord by means of a turnbuckle or another appropriate equipment in order to ensure required pre-stress of the truss.

Geometrical chord lengths L_c and $L^0{}_{c,0}$ are found from (25), using chord ordinates in the middle of the span. Coefficients Ψ_2 and Ψ_4, given by (26), may be approximated as follows:

$$\Psi_2(\eta) = \frac{\eta^2 + 4}{1.5 \cdot L} \tag{42, a}$$

$$\Psi_4(\eta) = -\frac{\eta^4 + 16 \cdot \left(\eta^2 + 1\right)}{2.5 \cdot L^3} \tag{42, b}$$

The coefficient $\eta = 0$ should be applied in expressions (42) to get cable length $L^0{}_{c,0}$. For the length of the cable under load L_c, the coefficient η is defined by (38).

According to (25), (36), (38), (41) and (42), elongations of the chords ΔL^b, ΔL^r and load ratios η_b, η_r are the functions of the deflections: $\Delta f_{1/4}$ and $\Delta f_{1/2}$. Thus, there are three simultaneous equations: (34), (40, a) and (40, b). They allow deriving uniformly distributed part of the "link load" p_{Eq} and deflections ($\Delta f_{1/4}$, $\Delta f_{1/2}$) of the truss in the quarter and in the middle of the span.

Inverse-symmetric part of the "link load" p_{Inv} is obtained from (31, b) and (33), written for the restraining chord:

$$p_{Inv} = p_{Eq} \cdot \eta_r \tag{43}$$

6 EXAMPLES

6.1 Single cable

The cable is illustrated in figures 1 and 4. Its span is $L = 12$ m. In the assumption, that the shape of the cable obeys parabola, the initial ordinates are the following: $f^0{}_{1/2} = 1.5$ m – in the center of the span and $f^0{}_{1/4} = 1.125$ m – in quarters of the span. Modulus of elasticity of the cable is $E = 1.3 \cdot 10^4$ kN/cm^2. Cross section area is 3 cm^2.

The following load-cases are considered (figure 4): uniformly distributed load on the entire span – Ld_1, load on a half of the span – Ld_2 and triangular load - Ld_3. The magnitude of load is $q = 10$ kN/m.

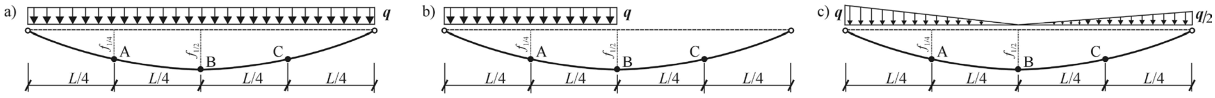

Figure 4: Flexible cable under load. a – load-case Ld_1, b – load-case Ld_2, c – load-case Ld_3

Comparison of results, obtained by the proposed formulations (index "*p*") and by means of the special computer program Easy (index "*e*"), is in the table 1. Indexes "*A*", "*B*" and "*C*" relate to the points, marked in figure 4. The force is shown in the middle of the span. Discrepancies ϖ are indicated beneath the corresponding data.

Table 1: Comparison of results for the single cable

Load-case	Deflections, mm						Force, kN	
	$\Delta f_{A,p}$	$\Delta f_{A,e}$	$\Delta f_{B,p}$	$\Delta f_{B,e}$	$\Delta f_{C,p}$	$\Delta f_{C,e}$	N^p	N^e
Ld_1	44.3	44.3	61.3	60.7	44.3	44.3	115.4	115.3
	ϖ = 0 %		ϖ = 0.9 %		ϖ = 0 %		ϖ = 0.1 %	
Ld_2	232.8	236.7	-105.8	-113.3	-402.3	-412.9	64.7	64.2
	ϖ = 1.7 %		ϖ = 6.9 %		ϖ = 2.6 %		ϖ = 0.8 %	
Ld_3	158.8	150.8	-198.1	-207.9	-143.5	-141.9	33.8	34.1
	ϖ = 5.2 %		ϖ = 4.8 %		ϖ = 1.1 %		ϖ = 0.9 %	

6.2 Two-chord cable truss

Model of structure is shown in figure 2. The span of the truss is $L = 12$ m. Initial ordinates of the chords of the truss are the following: $f^0{}_{b,1/2} = 1.5$ m, $f^0{}_{b,1/4} = 1.125$ m, $f^0{}_{r,1/2} = 1.0$ m, $f^0{}_{r,1/4} = 0.75$ m. The chords are made of steel cables. Modulus of elasticity is $E = 1.3 \cdot 10^4$ kN/cm^2. Cross section areas are the following: bearer cable $A_b = 3$ cm^2 and restraining cable $A_r = 1$ cm^2. The bearer cable is preliminary tensioned in order to ensure pre-stress in the truss: $\Delta L^b{}_p = 0.1$ m.

The following load-cases are considered (figure 5): uniformly distributed load on the entire span – Ld_1, load on a half of the span – Ld_2 and quasi inverse-symmetric load - Ld_3. The magnitude of load is $q = 10$ kN/m.

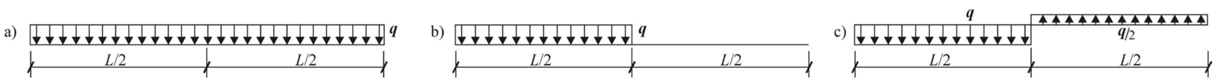

Figure 5: Loads on two-chord cable truss, considered in the example.
a – load-case Ld_1, b – load-case Ld_2, c – load-case Ld_3

Comparison of results, obtained by the proposed formulations (index "*p*") and by means of the special computer program Easy (index "*e*"), is in the table 2. The forces are shown in the middle of the span of the bearer (N_b) and restraining (N_r) chords.

Table 2: Comparison of results for the two-chord cable truss

Load-case	Deflections, mm				Forces, kN			
	$\Delta f^{p}{}_{1/2}$	$\Delta f^{e}{}_{1/2}$	$\Delta f^{p}{}_{1/4}$	$\Delta f^{e}{}_{1/4}$	$N^{p}{}_{b}$	$N^{e}{}_{b}$	$N^{p}{}_{r}$	$N^{e}{}_{r}$
Ld_1	83.0	80.2	62.3	58.9	157.5	155.3	39.4	37.0
	ϖ = 3.5 %		ϖ = 5.8 %		ϖ = 1.4 %		ϖ = 6.5 %	
Ld_2	120.7	119.0	-26.5	-27.3	122.1	119.8	70.1	68.4
	ϖ = 1.4 %		ϖ = 2.8 %		ϖ = 1.9 %		ϖ = 2.5 %	
Ld_3	144.0	143.3	-54.7	-55.1	111.6	110.2	93.0	93.6
	ϖ = 0.5 %		ϖ = 0.6 %		ϖ = 1.3 %		ϖ = 0.6 %	

7 CONCLUSIONS

The present work contributes to the structural analysis of cable systems. Trigonometric sine-series is used to deal with the differential equation of equilibrium. Cable shape function is represented by a multitude of coefficients. It is shown that they depend on each other and on the type of load. The technique for finding these coefficients and deflections of the cable is given. This approach may be used for analysis of single cables and cable structures, influenced by substantially non-uniform external loads. It allows to gain precise analytical results, proved by the comparison with data, provided by special computer program EASY.

The present work also contributes to the structural analysis of flexible membrane systems, simulated by a number of cables, arranged in mutual-perpendicular directions.

The proposed results are intended to be used for purposes of structural optimization. They facilitate elaboration of analytical models in order to estimate system parameter values. The paper allows additional tools for verification of numerical results, obtained by computer systems of static analysis.

The technique considered in the present work is able to be extended on complex cable structures, comprising several tiers or chords. It can, also, be generalized to non-shallow cables, having the sag comparable to the span.

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