VIII International Conference on Textile Composites and Inflatable Structures
STRUCTURAL MEMBRANES 2017
K.-U.Bletzinger, E. Oñate and B. Kröplin (Eds)

CABLE ROOF STRUCTURE WITH FLEXIBLE FABRIC COVERING

V.V. MIKHAYLOV[ᵞ], A.V. CHESNOKOV[*]

[ᵞ] The Faculty of Civil Engineering
Lipetsk State Technical University
Moskovskaya street 30, 398600 Lipetsk, Russian Federation
e-mail: mmvv46@rambler.ru, web page: http://ixserv.stu.lipetsk.ru/

[*] The Faculty of Civil Engineering
Lipetsk State Technical University
Moskovskaya street 30, 398600 Lipetsk, Russian Federation
e-mail: andreychess742@gmail.com, web page: http://ixserv.stu.lipetsk.ru/

Key words: Cable Roof, Fabric, Flexible Membrane, Tensegrity

Summary. *The proposed roof structure consists of a frame covered with a flexible fabric membrane. The frame comprises steel cable members linked together by struts and ties. Calculation formulas, needed for static analysis of the proposed roof structure, are given. The technique for estimation of its basic parameters is offered. The proposed roof structure is intended to be used for public and industrial buildings having large column spacing. Its primary advantages are reduced overall height and possibility to tension the entire structure by means of small number of cables.*

1 INTRODUCTION

Cable roof, covered with a flexible fabric membrane, is a low-weight translucent structure. Unlike conventional constructions, comprising steel and concrete elements, fabric covering is set into place in form of large-scale sheets. It reduces amount of site joints and results in diminishing of labor input of the project[1].

On the other hand, fabric membrane must be preliminary tensioned in order to bear external loads without wrinkles. Appropriate supporting members, e.g. single cables or cable trusses, should be provided so as to confine sagging of the membrane and to ensure its structural ability.

Flexible fabric covering is generally used in buildings having significant overall height of the roof. It is suitable for improving architectural appearance but sometimes results in excessive internal space of the construction[2]. An approximately flat roof is often much more appropriate for a lot of public and industrial buildings, than the roof having substantial difference between elevations of its ridge and valley. Under this condition the distance between supporting members of the roof becomes comparatively small in order to ensure required curvature of the membrane. It results in reduction of column spacing and may be unacceptable for buildings, which require large free spans.

In order to overcome this problem, the fabric covering is proposed to be supported with, so called "ordinary" members. They are arranged comparatively closely to each other and lean on “primary” members which are, in turn, connected to columns of the building. The primary and ordinary members, united with struts and ties, form the frame of the proposed roof structure[3,4] (figure 1). Thus, the flexible membrane, covering the frame, can be properly tensioned so as to bear external loads without excessive sagging.

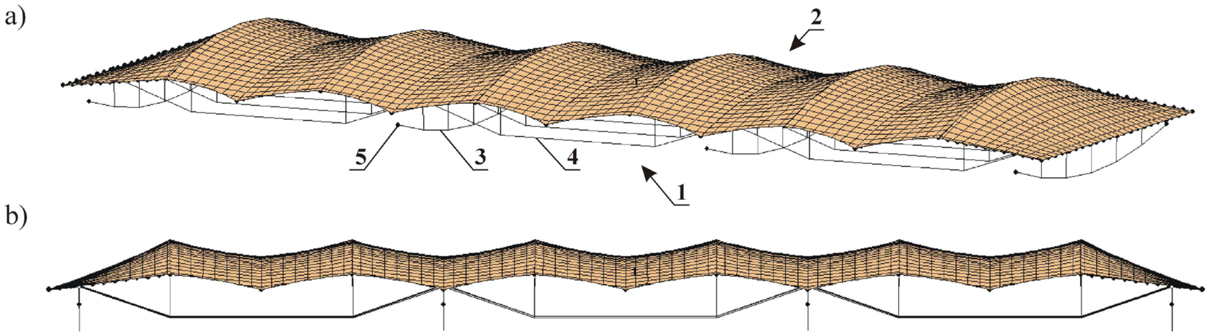

Figure 1: The proposed roof structure[3,4]. a - axonometric view; b - front view; 1 – the frame of the structure; 2 – flexible covering (membrane); 3 - primary member; 4 - ordinary member; 5 – fixed support (e.g. column of the building)

2 THE FRAME OF THE PROPOSED ROOF STRUCTURE

The frame of the proposed roof structure belongs to, so-called, “tensegrity” systems[5,6] of class 1. It consists of continuous cables and multitude of discontinuous struts, which are not directly coupled. The frame comprises a number of sections (figure 2). The sections are connected with each other successively. Longitudinal thrust, brought about by a section, is either equilibrated by an adjacent one, or transmits to horizontal supports provided in the ends of the frame.

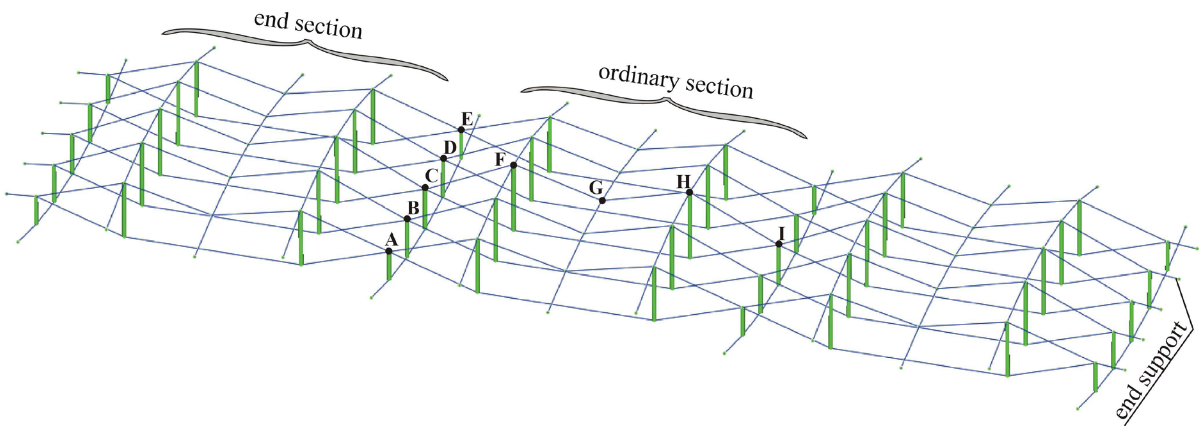

Figure 2: The frame of the roof structure

A section of the frame (figure 3) consists of preliminary stressed cables 1, 2 and 4. The cables are arranged in vertical planes. They are linked together by means of struts 5 and 6. The cables are subdivided into bearer and backstay ones. Backstay cables 2, to which the flexible fabric covering is attached, are convex upwards. Together with flexible ties 3, 7 and 8 they form the upper chord of the frame. Bearer cables 1 and 4 are convex downwards. They are arranged in mutually-perpendicular directions and may be classified into primary and ordinary ones.

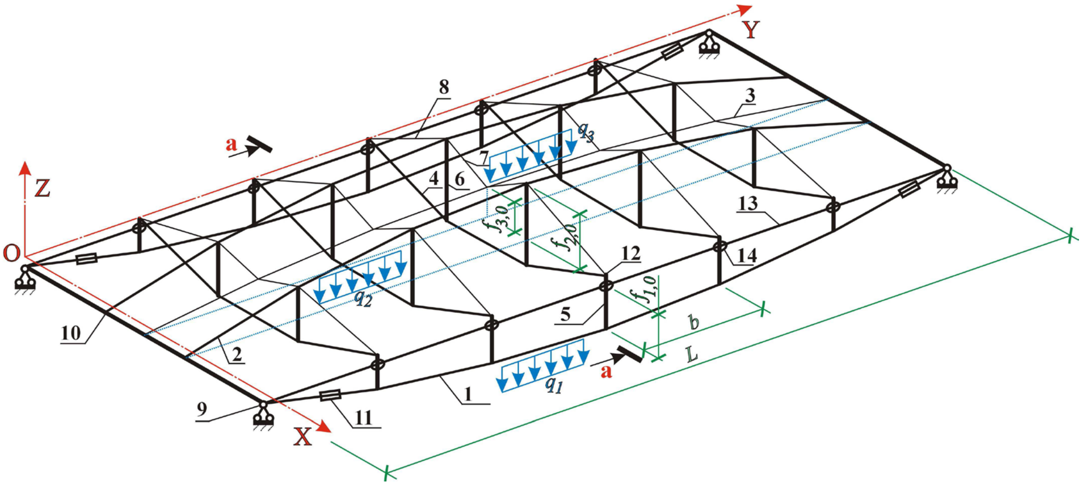

Figure 3: Axonometric view of a section of the frame

Primary bearer cables 1, equipped with tensioning appliances 11, are connected to columns of the building at points 9. Ordinary cables 4 are supported by primary ones by means of struts. The struts 5 are fixed in the direction along the Y-axis by means of ties 13 (figure 3). The ties are equipped with special connections 14, allowing the struts to freely pass in the longitudinal direction only.

The connections 12 of cables 4 to struts 5 are proposed to be arranged in a horizontal plane[4] (XOY) which is situated above the supports 9. The plane also should be arranged between upper and lower joints of the struts 6. It ensures the bearer cables 1 and 4 to keep their vertical position due to the emergence of stabilizing forces.

Backstay cables 2 are attached fixedly to supports 10. The thrust, brought about by cables 2, is much smaller in comparison to the thrust of bearer cables 1. It allows simplifying the construction of supports 10 in contrast to supports 9, which should be capable to sustain substantial forces.

3 ANALYSIS OF THE FRAME

In the assumption, that the roof structure is influenced by uniformly distributed external loads, calculation formulas, needed for its static analysis, are derived as follows.

Model of structure is shown in figures 3 and 4. It is assumed, that external loads (q_1, q_2 and q_3) are uniformly distributed along the spans of the cables 1, 2 and 3. Positive directions of displacements and loads are indicated in the figures.

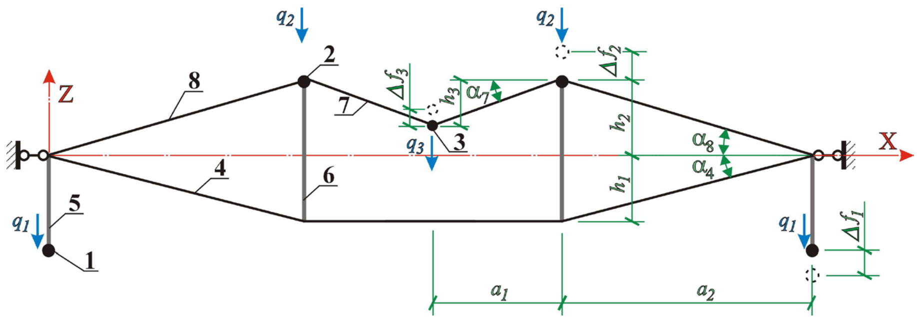

Figure 4: View of the frame along the line a-a in figure 3

Cables 1, 2 and 3, situated along the span of the frame, are loaded with approximately uniformly distributed vertical forces p_1, p_2 and p_3. The forces are the sums of external loads q and vertical reactions of struts 5 and 6, ordinary bearer cables 4 and ties 7 and 8.

The equilibrium of a uniformly loaded cable i, where i = *1, 2* and *3*, may be written as follows:

$$\frac{f_{i,0} + \Delta f_i}{p_i} = \frac{\rho_i}{Lc_i - Lc_{i,0}} \tag{1}$$

where $f_{i,0}$ is the initial sag of the cable i in the middle of its span L; Δf_i is the deflection of the cable i; p_i is the resulting load, influencing the cable; Lc_i and $Lc_{i,0}$ are the lengths of the cable in deformed and initial states, respectively; ρ_i is the coefficient:

$$\rho_i = \frac{L^2 \cdot Lc_{i,0}}{8 \cdot EA_i} \tag{2}$$

where EA_i is the stiffness of the cable i, equal to the product of the modulus of elasticity by its cross section area.

The length of the cable is calculated according to the expression:

$$L_{cab} = \Psi_2 \cdot f^2 + L \tag{3}$$

where Ψ_2 is the coefficient: $\Psi_2 = 8/(3 \cdot L)$.

The initial length of the cable is calculated as follows:

$$Lc_{i,0} = Lc_{i,0,0} - \Delta L_{p,i} \tag{4}$$

where $Lc_{i,0,0}$ is, so called, geometrical length, calculated from (3) for the initial sag $f_{i,0}$; $\Delta L_{p,i}$ is tensioning of the cable by means of a turnbuckle or another appropriate equipment in order to ensure required pre-stress.

The length of the cable in deformed state Lc_i is derived from (3) as a function of the deflection $Lc(\Delta f_i)$ using the following expression: $f = f_{i,0} + \Delta f_i$. Substituting the function into (1) yields the cubic equation:

$$f_{i,0} + \Delta f_i = \frac{\rho_i \cdot p_i}{Lc(\Delta f_i) - Lc_{i,0}} \qquad (1')$$

In order to convert it into the linear one, the simplification is proposed by the Taylor series:

$$\Delta f^r \approx r \cdot \Delta f_a^{(r-1)} \cdot \Delta f + (1-r) \cdot \Delta f_a^{\,r} \qquad (5)$$

where Δf_a is an approximation to the value Δf.

Substituting (5) into (1') for each cable i yields a set of three simultaneous equations from which unknown cable deflections are to be calculated:

$$\Delta f_i \cdot \mu_i + \lambda_i = p_i(\Delta f_1, \Delta f_2, \Delta f_3) \qquad (6)$$

where μ_i, λ_i are the following coefficients:

$$\mu_i = \frac{\Delta L_{p,i} + \Psi_2 \cdot \left[3 \cdot \left(\Delta f_{a,i} + f_{i,0}\right)^2 - \left(f_{i,0}\right)^2\right]}{\rho_i} \qquad (7, a)$$

$$\lambda_i = \frac{\Delta L_{p,i} \cdot f_{i,0} - \Psi_2 \cdot \left(\Delta f_{a,i}\right)^2 \cdot \left(2 \cdot \Delta f_{a,i} + 3 \cdot f_{i,0}\right)}{\rho_i} \qquad (7, b)$$

and p_i is a linear approximation to the load, written as follows:

$$p_i(\Delta f_1, \Delta f_2, \Delta f_3) = \Omega_{i,0} + \Omega_{i,1} \cdot \Delta f_1 + \Omega_{i,2} \cdot \Delta f_2 + \Omega_{i,3} \cdot \Delta f_3 \qquad (8)$$

where coefficients Ω are calculated as follows:

$$\Omega_{3,0} = -q_3 \qquad (9, a)$$

$$\Omega_{3,1} = \frac{\sin(2 \cdot \alpha_8) \cdot \cos(\alpha_7)}{2 \cdot \tau_3} \qquad (9, b)$$

$$\Omega_{3,2} = \frac{\sin(\alpha_7 + \alpha_8) \cdot \cos(\alpha_8)}{\tau_3} \qquad (9, c)$$

$$\Omega_{3,3} = \frac{-\sin(\alpha_7) \cdot \cos(\alpha_8)^2}{\tau_3} \qquad (9, d)$$

and

$$\Omega_{2,0} = -q_2 \qquad (10, a)$$

$$\Omega_{2,1} = \tau 1 \cdot \Omega_{3,1} + \tau_2 \qquad (10, b)$$

$$\Omega_{2,2} = \tau 1 \cdot \Omega_{3,2} + \tau_2 \qquad (10, c)$$

$$\Omega_{2,3} = \tau_1 \cdot \Omega_{3,3} \tag{10, d}$$

and

$$\Omega_{1,0} = q_1 \tag{11, a}$$

$$\Omega_{1,1} = \chi \cdot \left(\Omega_{2,1} + 0.5 \cdot \Omega_{3,1}\right) \tag{11, b}$$

$$\Omega_{1,2} = \chi \cdot \left(\Omega_{2,2} + 0.5 \cdot \Omega_{3,2}\right) \tag{11, c}$$

$$\Omega_{1,3} = \chi \cdot \left(\Omega_{2,3} + 0.5 \cdot \Omega_{3,3}\right) \tag{11, d}$$

where χ is the coefficient equal to 1.0 if the primary bearer cable 1 is situated at the end of the frame and $\chi = 2.0$ if the cable 1 is situated between two adjacent sections of the frame; and τ_i ($i = 1, 2$ and 3) are the following coefficients:

$$\tau_1 = \frac{-sin\left(\alpha_7 + \alpha_8\right)}{2 \cdot sin\left(\alpha_7\right) \cdot cos\left(\alpha_8\right)} \tag{12, a}$$

$$\tau_2 = \frac{-1}{b} \cdot \frac{EA_4 \cdot sin\left(\alpha_4\right)^2}{L_4 + a_1 \cdot cos\left(\alpha_4\right)^2} \tag{12, b}$$

$$\tau_3 = \frac{1}{2} \cdot \frac{b}{sin\left(\alpha_7\right)} \cdot \left(\frac{L_7}{EA_7} \cdot cos\left(\alpha_8\right)^2 + \frac{L_8}{EA_8} \cdot cos\left(\alpha_7\right)^2\right) \tag{12, c}$$

where a_1, b, α_4, α_7, α_8 are initial dimensions and angles, designated in figures 3 and 4; L_4, L_7, L_8 are initial lengths of the cables; EA_4, EA_7, EA_8 are stiffness of the cables.

Solution of (6) is derived as follows:

$$\vec{\Delta f} = [M]^{-1} \cdot \vec{V} \tag{13}$$

where

$$\vec{\Delta f} = \begin{pmatrix} \Delta f_1 \\ \Delta f_2 \\ \Delta f_3 \end{pmatrix}, \quad M = \begin{pmatrix} \Omega_{1,1} - \mu_1 & \Omega_{1,2} & \Omega_{1,3} \\ \Omega_{2,1} & \Omega_{2,2} - \mu_2 & \Omega_{2,3} \\ \Omega_{3,1} & \Omega_{3,2} & \Omega_{3,3} - \mu_3 \end{pmatrix}, \text{ and } \quad \vec{V} = \begin{pmatrix} \lambda_1 - \Omega_{1,0} \\ \lambda_2 - \Omega_{2,0} \\ \lambda_3 - \Omega_{3,0} \end{pmatrix}.$$

Loads p_i are obtained from (8), and forces in frame members are calculated as follows:

– cables $i = 1, \ 2, \ 3$, situated along the span:

$$N_i = \frac{p_i \cdot L^2}{8 \cdot \left(f_{i,0} + \Delta f_i\right)} \tag{14, a}$$

– cables, arranged in transverse direction:

$$N_4 = \left[\left(q_2 + p_2\right) - \left(q_3 + p_3\right) \cdot \tau_1\right] \cdot \frac{b}{sin\left(\alpha_4\right)} \tag{14, b}$$

$$N_7 = \frac{b}{2 \cdot sin(\alpha_7)} \cdot (q_3 + p_3) \quad (14, c)$$

$$N_8 = N_7 \cdot \frac{cos(\alpha_7)}{cos(\alpha_8)} \quad (14, d)$$

Analysis of the roof structure is proposed to be implemented in two stages. In the first stage, approximations $\Delta f_{a,i}$ (see expression (5)) to the deflections Δf_i are equal to zero for all three cables (i = *1, 2* and *3*). The second stage uses results Δf_i, found in the previous stage, as initial approximation $\Delta f_{a,i} = \Delta f_i$, in order to improve tolerance of the solution.

4 ESTIMATION OF THE FRAME PARAMETERS

The frame of the roof structure exhibits complex behavior under load. Cables, which constitute the top and bottom chords, should be in tension. Otherwise they sag and the frame will become unstable. It may substantially complicate computer simulation of the proposed roof due to divergence of the iteration process. In contrast to ordinary constructions, the main parameters of the frame of the proposed roof should be estimated before implementation of the static analysis. The parameters include geometrical dimensions, strength properties, pre-stresses and stiffness values of structural elements.

Expressions (14) allow estimating structural parameters by means of the following conditions:

$$\Theta > 0.1 \quad (15, a)$$

$$\Theta < 1.0 \quad (15, b)$$

$$\Theta = N_{cab} / (A \cdot R) \quad (16)$$

where N_{cab} is the force in the cable; A – is the cable cross section area; R is the cable strength.

Let's consider the frame with the following dimensions (figures 3 and 4) expressed in meters:

$$L = 12 \quad b = 2 \quad a_1 = a_2 = 3 \quad h_1 = 1 \quad h_2 = 1.5 \quad h_3 = 0.8 \quad f_{1,0} = f_{2,0} = 1 \quad f_{3,0} = 0.7$$

Cable mechanical properties are adopted from the catalogue[7]: $E = 1.3 \cdot 10^8$ kN/m^2 and $R = 7 \cdot 10^5$ kN/m^2. Uniformly distributed external load acts on the roof. The load is equal to 1.8 kN/m^2. In the assumption, that the membrane is attached to the backstay cables 2, the area load is converted into the line distributed one: $q_2 = 10.8$ kN/m, and $q_1 = q_3 = 0$. Pre-stress of the frame is implemented by means of tensioning $\Delta L_{p,1}$ of the bottom chord only. Thus, elongations of cables 2 and 3 are equal to zero: $\Delta L_{p,2} = \Delta L_{p,3} = 0$.

Estimation of cross section areas of frame members and the value $\Delta L_{p,1}$ is proposed to be executed in the step by step manner. Having started from an initial approximation, the parameters to be determined are being modified in order to fulfill conditions (15).

So-called, targeted search is performed by using graphs of impact (figure 5). The graphs are plotted according to the expression (16). They illustrate the influence of frame parameter variation on values Θ. Cross section areas of bearer cables (A_1 and A_4), main backstay cables (A_2) and also elongation $\Delta L_{p,1}$ of the bottom chord are chosen due to their greatest influence on (16).

Indexes, shown in figure 5, correspond to element numbers (see figures 3 and 4). Each curve is obtained as a result of two calculations of the frame: without external loads (pre-stress only) - $Value_0$ and with loads - $Value_1$. The maximum of these values is used for every curve, except Θ_3, for which the minimum value is chosen. Every graph (figure 5) allows specifying permissible sub-range of parameter variation according to conditions (15).

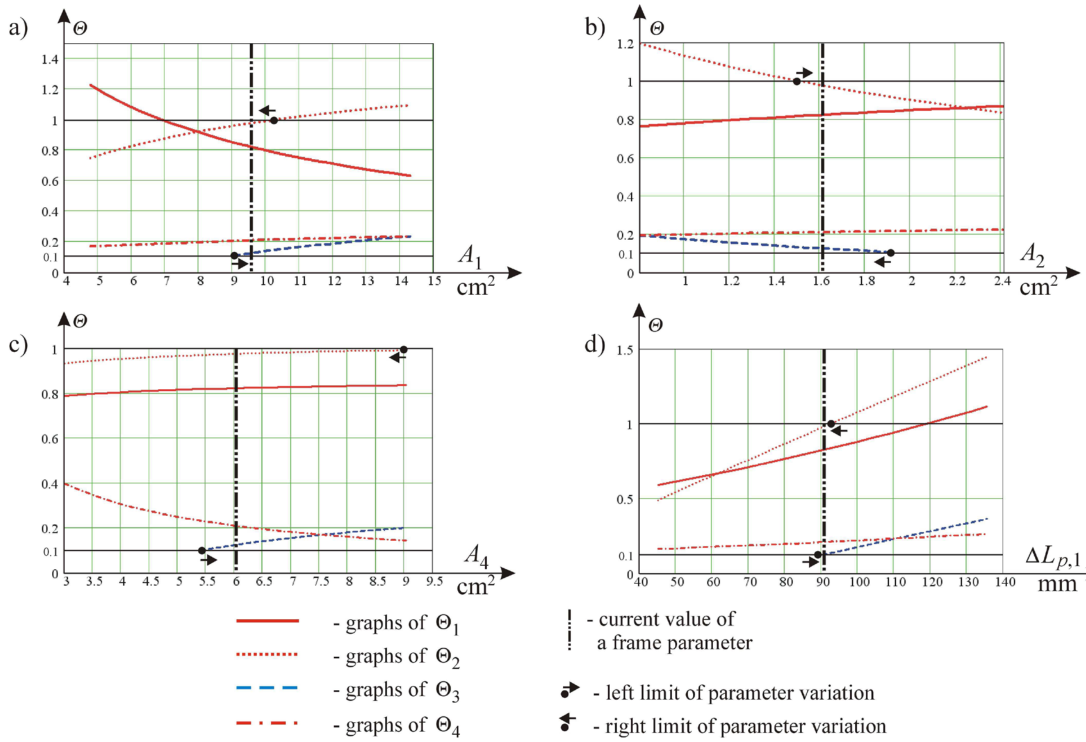

Figure 5: Graphs of impact of frame parameters

According to the catalogue[7] the following cables are chosen: primary bearer cable 1 is made of two strands $d_s = 28.6$ mm with total cross section area $A_1 = 9.54$ cm^2, ordinary cable 4 is made of a strand $d_s = 32.1$ mm with cross section area $A_4 = 6.02$ cm^2, backstay cable 2 is made of a strand $d_s = 16.6$ mm with cross section area $A_2 = 1.61$ cm^2. Flexible ties 3, 7 and 8 are made of the smallest strand in the catalogue[7] - $d_s = 6.1$ mm with cross section area $A_2 = 0.22$ cm^2.

Comparison of results, obtained by the proposed formulations (index "p") and by means of the special computer program Easy (index "e"), are in tables 1 and 2. Discrepancies ϖ are indicated beneath the corresponding data.

Table 1: Deflections of the roof structure

Load-case	Deflections, mm					
	$\Delta f_{1,p}$	$\Delta f_{1,e}$	$\Delta f_{2,p}$	$\Delta f_{2,e}$	$\Delta f_{3,p}$	$\Delta f_{3,e}$
Pre-stress only:	-147	-154	136	144	102	115
$q_1 = q_2 = q_3 = 0$	ϖ = 4.4 %		ϖ = 5.5 %		ϖ = 11.8 %	
Pre-stress and vertical load:	-87	-91	66	67	24	20
$q_2 = 10.8$, $q_1 = q_3 = 0$	ϖ = 5.0 %		ϖ = 1.8 %		ϖ = 18.2 %	

Table 2: Forces in elements of the structure

Load-case	Forces, kN							
	$N_{1,p}$	$N_{1,e}$	$N_{2,p}$	$N_{2,e}$	$N_{3,p}$	$N_{3,e}$	$N_{4,p}$	$N_{4,e}$
Pre-stress only:	302.5	298.2	110.7	108.0	8.0	7.9	47.2	46.5
$q_1 = q_2 = q_3 = 0$	ϖ = 1.4 %		ϖ = 2.4 %		ϖ = 2.2 %		ϖ = 1.5 %	
Pre-stress and vertical load:	547.7	536.0	51.7	47.0	1.8	1.8	88.3	84.5
$q_2 = 10.8$, $q_1 = q_3 = 0$	ϖ = 2.2 %		ϖ = 9.4 %		ϖ = 0 %		ϖ = 4.3 %	

Tables 1 and 2 show that formulas, proposed for static analysis of the frame, well simulate its structural behaviour. Discrepancies of forces N_2 and deflections Δf_3 may be explained by divergence of backstay cables 2 and ties 3 from their initial parabolic shape. These inaccuracies negligibly affect estimation of frame parameters, and the proposed formulas may successfully be used for structural behaviour investigation.

5 THE FLEXIBLE FABRIC MEMBRANE

Flexible fabric membrane is supported by the upper chord of the frame. It is also attached to catenary cables, restrained by backstay cables and connected to the ends of the roof structure (figure 6).

The membrane should be preliminary stressed not less than 1.3% of the average tensile strength[2] in order to be able to sustain external loads. The following ways of its tensioning are considered: by means of special additional equipment, and with the help of appliances already used for pre-stressing the frame of the roof structure.

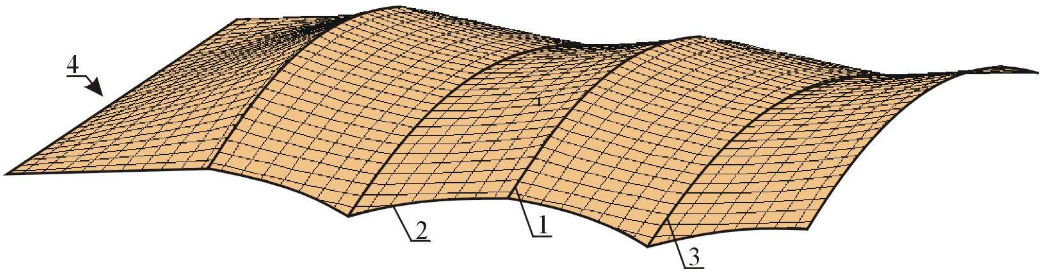

Figure 6: A fragment of the membrane.
1 - upper chord of the frame; 2 - catenary cable; 3 - backstay cable; 4 – end of the roof

In the first case the frame is assumed to be fully completed before hanging the membrane. Catenary and backstay cables are tensioned by means of turnbuckles, while membrane edges are connected to ends of the roof using lashing strap systems[1]. This method results in approximately uniform stress distribution in the membrane due to the possibility of adjusting any inaccuracies.

On the other hand, tensioning the frame and the membrane by appliances embedded in the bottom chord of the roof structure (elements 11 shown in figure 3) allows substantial reducing of labor input of the project. The frame should have been tensioned halfway in order to be able to sustain loads brought about by the membrane. Then, the membrane is attached to supports and the process continues simultaneously. Resulting stress distribution is substantially uneven in comparison to the previous tensioning technique (figure 7).

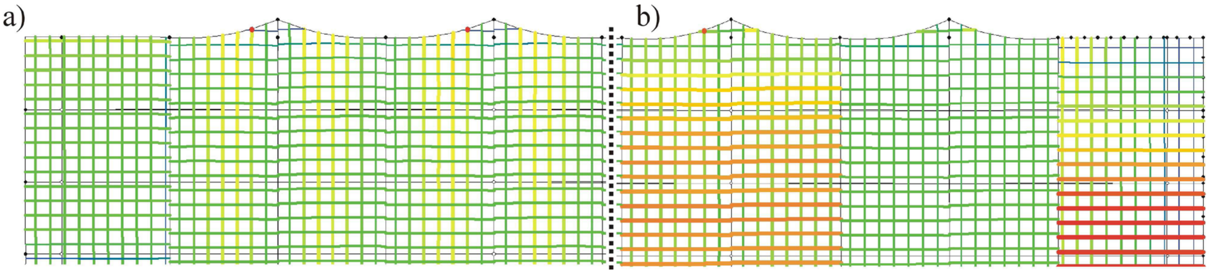

Figure 7: Diagrams of stresses in the flexible fabric membrane.
a – the membrane is tensioned by means of additional appliances after the completion of the frame; b – the membrane is tensioned together with the frame

The roof structure is analyzed with the help of computer program EASY. The following loads are taken into account: vertical load $S_1 = 1.5$ kN/m^2, acting on area “1” of the roof structure and load $S_2 = S_1 \cdot \vartheta$, acting on area “2”. Layout of the areas is in figure 8. Load 1.5kN/m^2 is adopted in the assumption, that vertical reaction of the fabric membrane is approximately 0.3 kN/m^2. Resultant load, equal to 1.8 kN/m^2, corresponds to the value, for which parameters of the frame are determined. Coefficient ϑ varies in the range $\vartheta = 0...1$. It allows simulating non-uniform impacts. Vertical deflections of the roof structure are shown in figure 9.

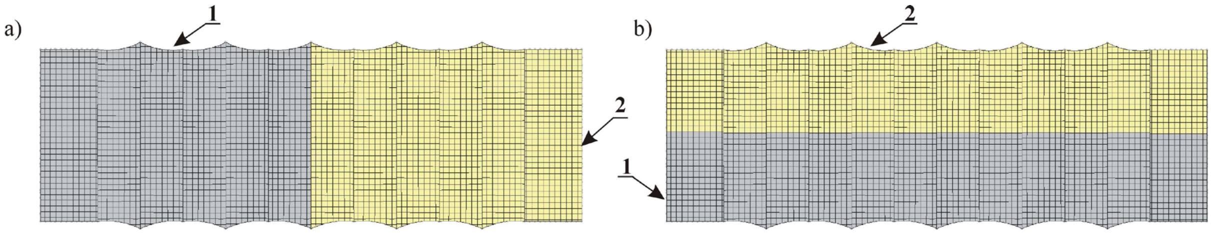

Figure 8: Variants of load distribution. 1, 2 – number of area

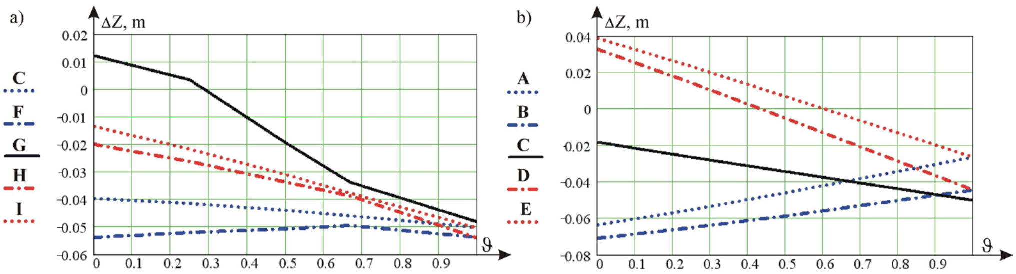

Figure 9: Graphs of deflections of the roof structure, influenced by non-uniform external loads. a, b - variants of load distribution according to figure 8; **A** ... **I** are points, marked in figure 2

Figure 9 shows that deflections of the roof structure under non-uniform external loads are approximately linear, except point G, which belongs to tie 3. This non-linear behaviour may be explained by, so-called, "kinematic" movements of the tie and by horizontal displacements of joints F and H. On the other hand, the proposed roof structure behaves stable. It may successfully withstand external effects in real conditions.

6 CONCLUSIONS

The proposed cable roof structure is intended to be used for public and industrial buildings with enlarged column spacing. It consists of the frame covered with the flexible fabric membrane.

The frame comprises so called "primary" and "ordinary" members, made of steel cables. The cables are linked together by struts and ties. The ordinary members, arranged comparatively closely to each other, support the fabric membrane. They lean on the primary ones, connected to columns of a building. The roof has substantially lower overall height in comparison to usual membrane structures of the same span. Resulting in diminishing of unused internal space of the building, it is more attractive from an economic point of view.

Calculation formulas for static analysis of the roof structure are proposed. They are verified by comparison with results provided by the computer program EASY. The technique for estimation of basic parameter of the proposed roof structure is offered.

The roof structure well behaves in case of potential external loads, including non-uniformly distributed effects.

REFERENCES

[1] M. Seidel, *Tensile Surface Structures: A Practical Guide to Cable and Membrane Construction*. Ernst & Sohn, 2009, 229 p.

[2] B. Forster, M. Mollaert, *European design guide for tensile surface structures*. TensiNet, 2004, 354 p.

[3] Patent for Invention RU 2567588. *Cable roof* / A.V. Chesnokov, V.V. Mikhaylov.

[4] Utility Model Patent RU 169612. *Cable roof structure* / A.V. Chesnokov, V.V. Mikhaylov.

[5] Snelson, K. “The Art of Tensegrity”, *International Journal of Space Structures*, Vol. **27**, № 2-3 (2012), pp. 71-80.

[6] R.E. Skelton and M.C. de Oliveira, *Tensegrity Systems*. Springer, 2009, 216 p.

[7] *PFEIFER Tension Members*. PFEIFER seil- und hebetechnik GMBH, 2017, 76 p. Online: http://www.pfeifer.de/en/cable-structures/download/brochures/?dlid=4586 - (19 June 2017).