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BENDING-ACTIVE DOME-SHAPED STRUCTURE

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Summary. *The structure, proposed in the present work, belongs to, so-called, "bending-active" systems. It comprises high-strength low-modulus initially straight beams. The beams, arranged in the radial direction, are supported by spreaders and steel cables underneath. Tensioning of cables results in substantial shape modification of the beams and turning the structure into a convex dome. The frame of the structure is investigated by means of the special computer program EASY. Uniform and non-uniform external loads are taken into account. Influence of flexible fabric membrane, attached to the upper chord of the frame, is analyzed.*

1 INTRODUCTION

Bending-active structures comprise initially straight beams, which are shaped into curved arch-like elements by means of elastic deformation[1]. The primary aim is to facilitate the process of production, transportation and installation of the construction. Complex procedure of curved beam fabrication gives way to relatively easy operations, which are performed on a construction site[2].

Flexible fabric membrane is often a constituent part of a bending-active structure[3]. The membrane provides the cladding and stabilizes curved arch-like members by means of prevention them from buckling. On the other hand, the membrane with bending elements included[4,5] becomes much more practical and cost-effective in comparison to ordinary fabric structures (e.g. saddle roofs), which often provide insufficient shading area. In addition, bending elements and flexible membrane equilibrate each other minimizing horizontal thrust. They result in effective engineering solutions due to reduction of expenditures required for foundations or supporting structures[6].

Bending-active structures are more flexible in comparison to usual systems and should include appropriate means for shape stabilization under various external loads. They are also to be made of materials allowing substantial deformations without overstress. The ratio of flexural strength (MPa) to young's modulus[2] (GPa) must not be less than 2.5. Fiber reinforced polymers[2,7] with the ratio in the range from 4.5 to 17.0, are far superior to the conventional structural steel. They, however, significantly loose their mechanical properties over time[8] and should preferably be used in temporary structures.

2 DESCRIPTION OF THE STRUCTURE

Dome-shaped structure, proposed in the present work, consists of a frame and a covering (figure 1). Although the covering may be of stiff plates or shells, flexible fabric membrane is much more appropriate due to its low weight, light translucence and possibility to withstand substantial deformations.

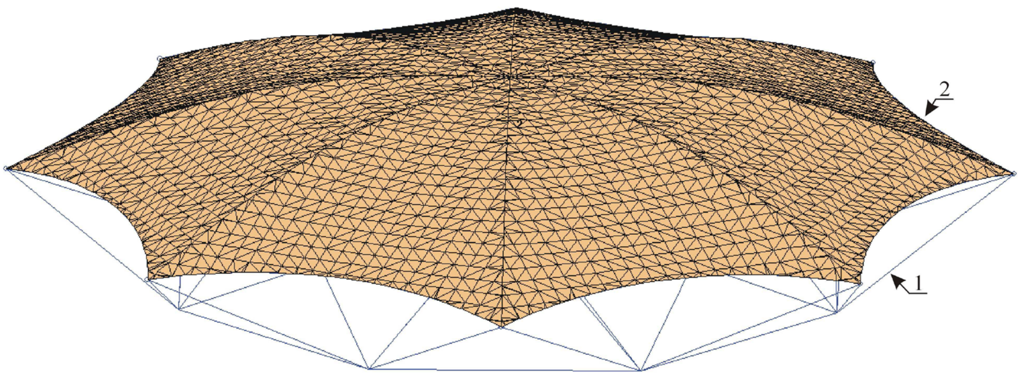

Figure 1: Dome-shaped structure, considered in the research. 1 – frame; 2 - flexible fabric membrane

The frame consists of high-strength low-modulus beams 1, supported with struts and steel cables underneath (figure 2). The beams, forming the upper chord of the frame, are initially straight. They are situated in the horizontal plane and arranged in the radial direction. The beams are made of pultruded glass-fiber reinforced polymer[7], having the modulus of elasticity $2.4 \cdot 10^4$ MPa and strength 185 MPa.

The bottom chord of the frame consists of bearer diagonal 2 and circular cables 3, outer and inner hinged struts (4 and 5) and ties. The outer struts are inclined to vertical planes of the beams. It ensures stability of the beams in so-called "out-of-plane" direction[9], especially in the event of non equal external loadings.

Tensioning of circular cables results in deformation of the upper chord of the frame and shaping the beams into curved arch-like elements. In accordance with growth of beam camber, the span of the structure diminishes, but arising bending moments ensure structural equilibrium (figure 2, b).

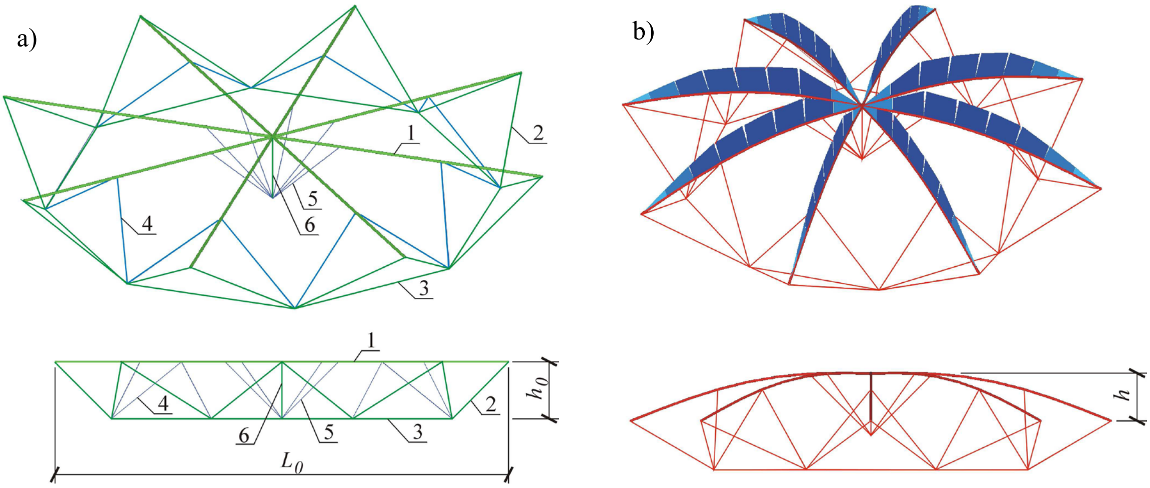

Figure 2: The frame of the structure. a – initial configuration; b – pre-stressed configuration, ready to bear external loads; 1 – beam of the upper chord; 2 – diagonal cable; 3 - circular cable; 4, 5 – outer and inner hinged struts; 6 – central tie

3 STRUCTURAL BEHAVIOR OF THE FRAME

The initial span and the height of the frame are assumed the following: $L_0 = 12$ m and $h_0 = 1.5$ m, respectively (figure 2, a). When the tensioning of circular cables is completed the upper chord of the frame takes on dome-like shape with the height in the center $h = 1.15$ m.

The upper beams are made of glass-fiber reinforced polymer tubes with the outer diameter 99 mm and the wall thickness 5 mm. The limit bending moment sustained by the beams is $M_{\lim} = 6$ kN·m. The beams are connected to each other in the center of the frame by means of pin joints.

The diameter of steel cables of the bottom chord is 10.1 mm. According to the catalogue[10] the limit tension is $N_{\lim} = 44$ kN. The struts are made of steel tubes. Their outer diameter is 45mm and the wall thickness is 3.5 mm.

External loads, influencing the frame, can be classified into installation and operational. The first ones arise during the process of frame mounting. Their values are substantially confined because the frame hasn't reached its final shape yet and the preliminary stresses have not been provided in all the elements. It has been found that the frame can sustain uniformly distributed load 0.1 kN/m^2 before the beginning of tensioning of the bottom chord.

Excessive installation loads may result in permanent downward camber of the upper chord of the frame. Thus, the structure will become not serviceable (figure 3).

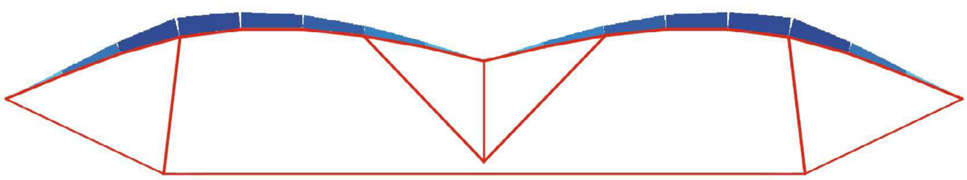

Figure 3: Influence of the vertical external load 0.3 kN/m^2 before the process of tensioning of the bottom chord is completed

Structural behavior of the frame is analyzed in two stages: pre-stress of the frame and, so-called, operational mode. Operational loads are applied after the first stage is finished. The shape of the frame and forces in its elements are to be updated before the implementation of structural analysis of the second stage. It includes the following steps:

- modification of initial coordinates $\vec{X}_0$ of all joints in accordance with their displacements $\vec{\Delta X}$ during the tensioning of the circular cables:

$$\vec{X}_1 = \vec{X}_0 + \vec{\Delta X} \tag{1}$$

- updating the initial lengths $\vec{L}_0$ of all structural elements in accordance with values, assumed in the first stage;
- implementation of angular deformations of all beams in accordance with their bending moments (figure 4) and flexural rigidity EI:

$$\Delta\psi_S = \frac{l_0}{6 \cdot EI} \cdot \left(2 \cdot M_s - M_e\right) \tag{2}$$

$$\Delta\psi_e = \frac{l_0}{6 \cdot EI} \cdot \left(2 \cdot M_e - M_s\right) \tag{3}$$

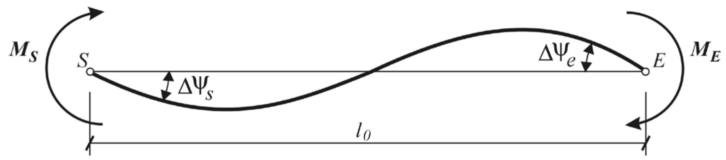

Figure 4: Angular deformations $\Delta\psi$ of a beam, influenced by bending moments M - index "S" corresponds to the starting point of the beam, and index "E" corresponds to the end point

The behavior of the frame is investigated by means of the special computer program EASY. The following load-cases are taken into account:

- LC_1: uniform load distributed on the entire surface;
- LC_2: uniform load distributed on a half of the span only;
- LC_3: wind load.

Loads LC_1 and LC_2 are directed from top to bottom, while the load LC_3 acts outwards the building. Due to the radial arrangement of the upper beams, uniform loads are transformed into, so-called, triangular ones (figure 5).

Load-case LC_1 results in relatively small deformations of the structure. The graph of axial force N in cables of the bottom chord is approximately linear (figure 6, a), while the curve of vertical displacements ΔZ of the central joint has slight downward camber. Bending moment in the top chord M tends to decrease until the load reaches 0.5 kN/m^2 and linearly increases afterwards (figure 6, b). Maximum load value 1.75 kN/m^2 may be derived from the graphs in accordance to the strength properties of structural elements.

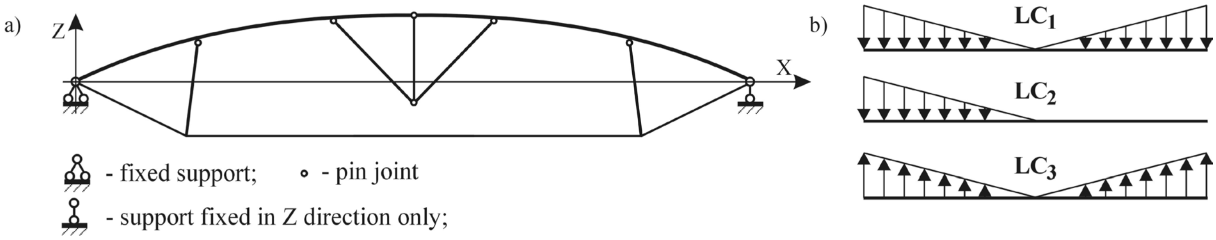

Figure 5: Plane model of the structure – a, and schematic load distribution - b

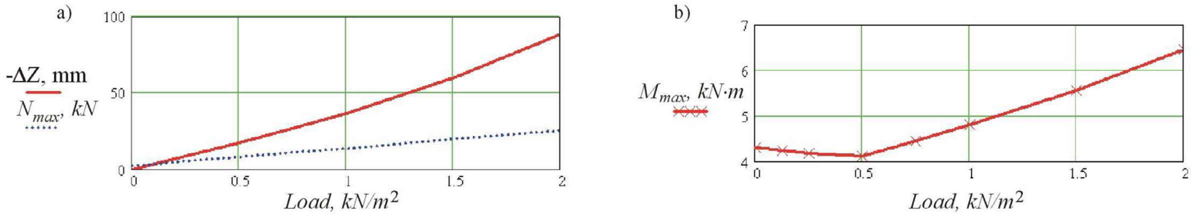

Figure 6: Graphs for the uniform load LC_1.
a - vertical displacement of the central joint (ΔZ, mm) and axial forces in the bottom chord (N_{max}, kN);
b - bending moments (M_{max}, kN·m) in the top chord

Load-case LC_2, acting on a half of the span only, results in much more substantial moments and deformations in comparison to loading of the entire span (figures 7,a and 7,b). Maximum load value 0.4 kN/m^2 may be applied on the structure without overstress of its elements. Even in this case, vertical displacements can reach 200 mm in a quarter of the span, which may be unacceptable from the operational point of view.

Additional ties, embedded in the bottom chord of the frame, result in substantial favorable effect. The ties connect the joints of the bottom chord with each other (figure 7, c). They should preferably be installed in the preliminary stressed structure (figure 2, b). If embedded into the initially undeformed frame (figure 2, a) the ties should be additionally tensioned after the structure has reached its final arch-like shape. Additional ties substantially diminish moments and displacements brought about by non-uniform loads. They ensure the behavior of the frame approximately linear (figure 8).

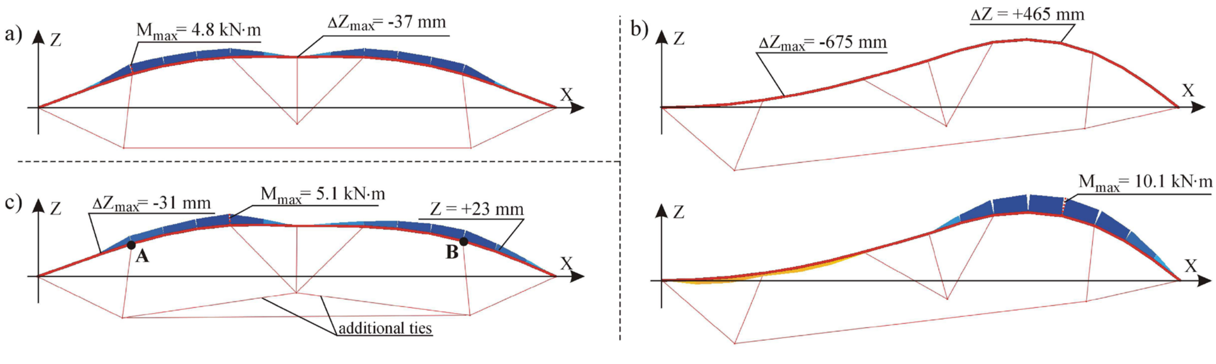

Figure 7: Deformations of the frame and diagrams of moments.
a - vertical load (1.0 kN/m^2) is distributed on the entire span;
b, c - vertical load (1.0 kN/m^2) is distributed on the left side only

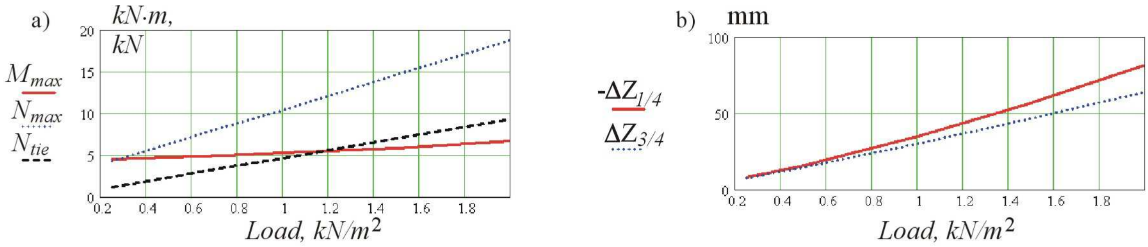

Figure 8: Graphs for the load LC_2 (additional ties are installed).
a – bending moments (M_{max}, kN·m) in the top chord and axial forces (N_{max} and N_{tie}, kN) in the bottom chord and in the additional ties; b - vertical displacement ($\Delta Z_{1/4}$ and $\Delta Z_{3/4}$, mm) in quarters of the span (points A and B in figure 7, c)

So called 'top to bottom' loads of load-cases LC_1 and LC_2 don't result in the emergence of the horizontal thrust because reactions of upper beams and bottom cables equilibrate each other. Thus, the frame needs vertical supports only. On the other hand, loads acting in the opposite direction, e.g. wind, bring about very substantial deformations of the structure (figure 9, a). In addition, the wind load results in sagging of bottom chord cables. Excessive moments and deformations arise after the complete cable slackening (figures 9,b – 9,d). So, the maximum wind load, permissible for the frame, is approximately 0.2 kN/m^2. It is usually inappropriate for most sites due to climatic conditions and regulations.

Installation of an additional horizontal support in point A (the point is marked in figure 9,a) helps to fix the problem. In case if the wind load is equal to 0.5 kN/m^2 the vertical displacement in the center of the frame is 11 mm and the moment in the upper beam is 4kN·m. These values are substantially smaller in comparison to the ones obtained without the support (figure 9). Thus, the horizontal support makes the frame functionally operative for the most conditions.

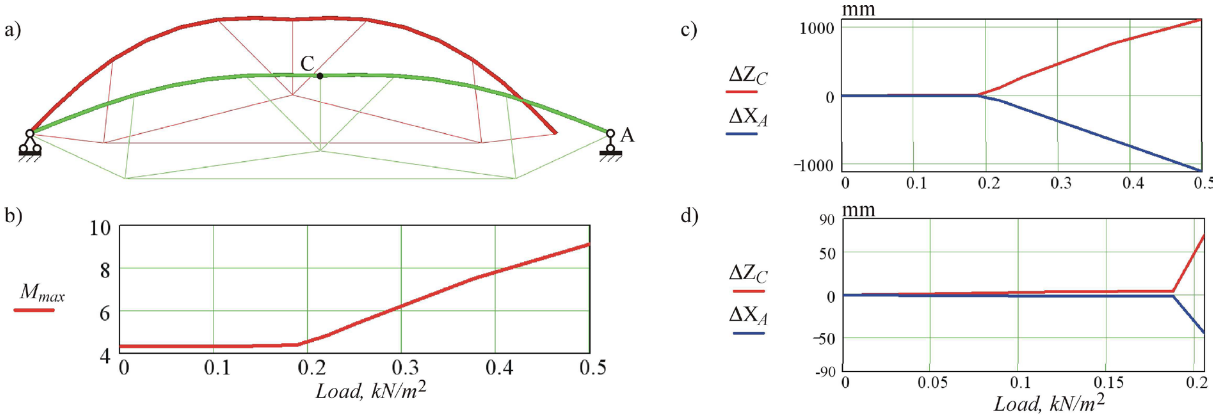

Figure 9: Behavior of the frame under load-case LC_3 (wind load). a – the frame without horizontal support in its right side; b – graph of bending moments (M_{max}, kN·m) in the top chord; c – graphs of vertical displacements ΔZ_C of joint C and horizontal displacements ΔX_A of joint A (mm); d – enlarged view of graphs in figure c

The additional horizontal supports should be installed at least in three ribs of a spatial frame. The ribs are to be arranged in different vertical planes (figure 10). Preferably they should be spaced approximately 120 degrees apart.

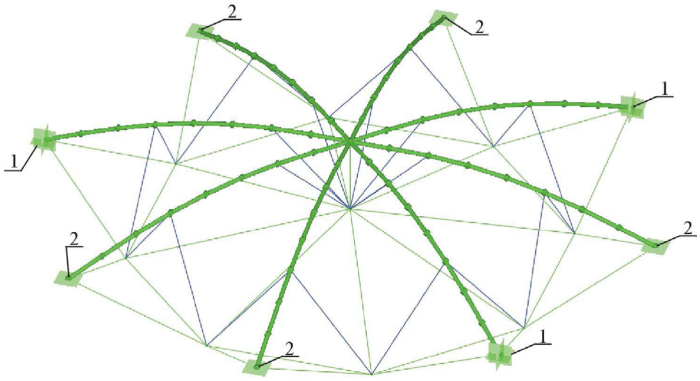

Figure 10: Spatial frame improved to sustain the wind load.
1 – fixed support; 2 – support fixed in the vertical direction only

4 FLEXIBLE FABRIC MEMBRANE

Hanging the membrane should be fulfilled after the frame has been completely installed and transformed from the initial, so called “flat”, configuration (figure 2, a) into an operational one (figure 2, b). Otherwise the membrane will sag on the entire surface.

The frame, covered with membrane, is much less susceptible to adverse effect of non-uniform loads. So-called “out-of-plane moments” M_w, which arise in the upper beams of the frame, are illustrated by diagrams (figure 11). Additional ties (shown in figure 7, c) diminish the deformations of the frame and contribute to the reduction of stresses in its beams (figure 12). They facilitate the favorable effect of inclined outer struts (shown in figure 2, a), causing negative moment (shown with the arrow in figure 11, a).

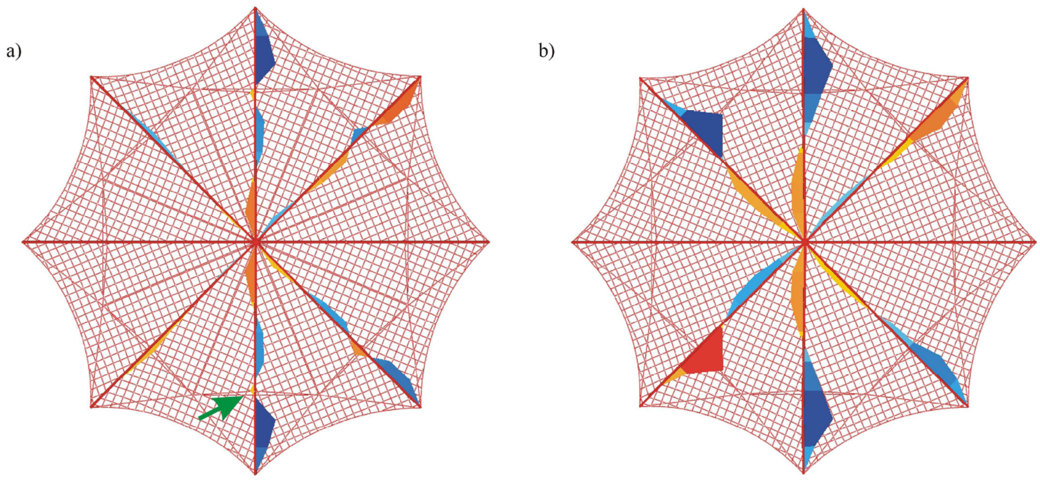

Figure 11: Diagram of out-of-plane moments, M_w.
a – the frame with additional ties (figure 7,c); b – the frame without the ties

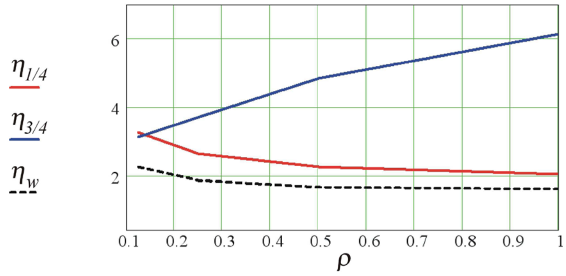

Figure 12: Ratios of displacements and moments η in relation to the fabric relative stiffness ρ

Labels in figure 12 are the following: $\eta_{1/4} = \Delta Z_{1/4} / \Delta Z^{tie}{}_{1/4}$, $\eta_{3/4} = \Delta Z_{3/4} / \Delta Z^{tie}{}_{3/4}$, $\eta_w = M_w / M^{tie}{}_w$, where $\Delta Z_{1/4}$ and $\Delta Z_{3/4}$, are displacements in quarters of the span (points A and B in figure 7, c), index "*tie*" means that the additional ties are installed. Fabric stiffness in the warp and fill directions are assumed the following: $E_{warp} = 1200 \cdot \rho$ and $E_{fill} = 800 \cdot \rho$, kN/m. Coefficient ρ varies in the range: 0.15 ... 1.0.

On the other hand, improving the frame behavior under load, the ties, however, are not inevitable in contrast to the frame without a flexible covering. Whether the ties are installed or not, the out-of-plane moments M_w in the beams (figure 13, a) are substantially smaller in comparison to so-called "in-plane of the rib moment" M_v, which is close to the value 5.7 kN·m for the same load. Judging by the relative displacements (figure 13, b), the frame, covered with the flexible fabric membrane, is a sufficiently stiff structure.

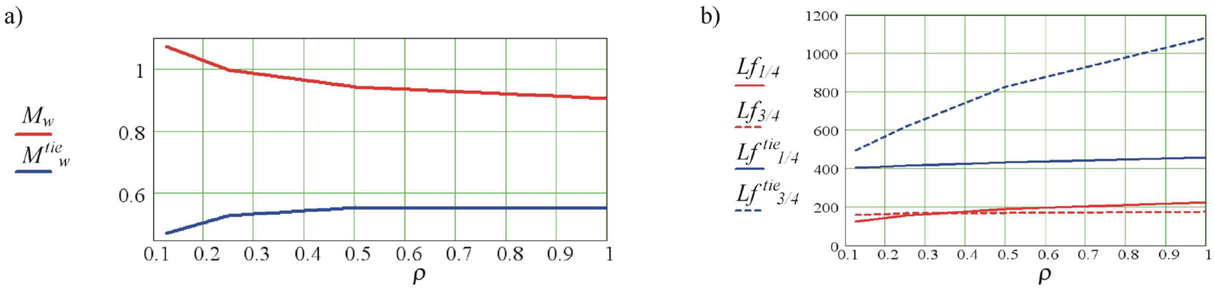

Figure 13: Behavior of the frame, covered with the membrane, under the load-case LC_2 (1.0 kN/m^2 distributed on a half of the span only).
a – graphs of moments M_w; b –graphs of relative displacements Lf

Labels in figure 13,b are the following: $Lf_{1/4} = -L / \Delta Z_{1/4}$, $Lf^{tie}{}_{1/4} = -L / \Delta Z^{tie}{}_{1/4}$, $Lf_{3/4} = L / \Delta Z_{3/4}$, $Lf^{tie}{}_{3/4} = L / \Delta Z^{tie}{}_{3/4}$, where Lf is relative displacement, L is the span of the frame.

5 CONCLUSIONS

Dome-shaped structure, proposed in the present work, comprises high-strength low-modulus beams, supported with spreaders and steel cables underneath. The beams, forming the upper chord of the dome, are initially straight, but substantial shape modification arises by means of cable tensioning.

The research is carried out with the help of the special computer program EASY. Non-uniform snow and wind loads result in substantial unfavorable effect on the frame of the structure. Installation of ties and horizontal supports is proposed in order to ensure serviceability of the frame. In addition, flexible fabric membrane, attached to the upper chord of the frame, substantially improves its structural behavior by reducing bending moments and deformations.

Bending-active dome-shaped structure, proposed in the present paper, is intended to be used for temporary covering of spaces of social occasions, entertaining events, points of retail, bus stations, etc. The structure may be installed and dismantled in a short period of time with comparatively small labor input.

REFERENCES

[1] J. Knippers, J. Cremers, M. Gabler, J. Lienhard, *Construction Manual for Polymers + Membranes*. Institut fur internationale Architektur-Dokumentation. Munchen, 2011, 296 p.

[2] J. Lienhard, H. Alpermann, C. Gengnagel, J. Knippers, "Active Bending, a Review on Structures where Bending is used as a Self-Formation Process", *International Journal of Space Structures,* Vol. **28**, No. **3** & **4**, pp. 187-196, (2013).

[3] J. Lienhard, "Bending-active membrane structure", *Tensinews*, No. **22**, pp. 9, (2012).

[4] R. Off and H. Runne, "IMS Research Project "Batsail", *Tensinews*, No. **18**, pp. 14-15, (2010).

[5] R. Off, "IMS Research Project "Bat-wing sail. A new Approach to Membrane Structures", *Fabric Architecture*, pp. 24-25, (January-February, 2010).

[6] S. Ahlquist, J. Lienhard, "Textile Hybrid M1", *Tensinews*, No. **24**, pp. 6-9, (2013).

[7] *Composite materials SMC/BMC. Pultrusion.* LLC "Tatneft-Presscomposite". Online: http://www.tnpc.ru/rus/produkciya/pultruziya/ - (19 June 2017).

[8] N. Kotelnikova-Weiler, C. Douthe, E.L. Hernandez, O. Baverel, C. Gengnagel and J.-F. Caron, "Materials for Actively-Bent Structures", *International Journal of Space Structures*, Vol. **28**, No. **3** & **4**, pp. 229-240, (2013).

[9] A.V. Chesnokov, V.V. Mikhaylov and I.V. Dolmatov, "Development of the Hybrid Dome and Research of its Behaviour under Load", *Proc. of the VII International Conference on Textile Composites and Inflatable Structures. Structural Membranes 2015*, pp. 469-476. 19-21 October 2015, Barcelona, Spain.

[10] *PFEIFER Tension Members*. PFEIFER seil- und hebetechnik GMBH, 2017, 76 p. Online: http://www.pfeifer.de/en/cable-structures/download/brochures/?dlid=4586 - (19 June 2017).