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DESIGN AND CONSTRUCTION OF THE ASYMPTOTIC PAVILION

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Summary. *Digital tools have made it easy to design freeform surfaces and structures. The challenges arise later in respect to planning and construction. Their realization often results in the fabrication of many unique and geometrically-complex building parts. Current research at the Chair of Structural Design investigates curve networks with repetitive geometric parameters in order to find new, fabrication-aware design methods. In this paper, we present a method to design doubly-curved grid structures with exclusively orthogonal joints from flat and straight strips. The strips are oriented upright on the underlying surface, hence normal loads can be transferred via bending around their strong axis. This is made possible by using asymptotic curve networks on minimal surfaces [1, 2]. This new construction method was tested in several prototypes from timber and steel. Our goal is to build a large-scale (9x12m) research pavilion as an exhibition and gathering space for the Structural Membranes Conference in Munich. In this paper, we present the geometric fundamentals, the design and modelling process, fabrication and assembly, as well as the structural analysis based on the Finite Element Method of this research pavilion.*

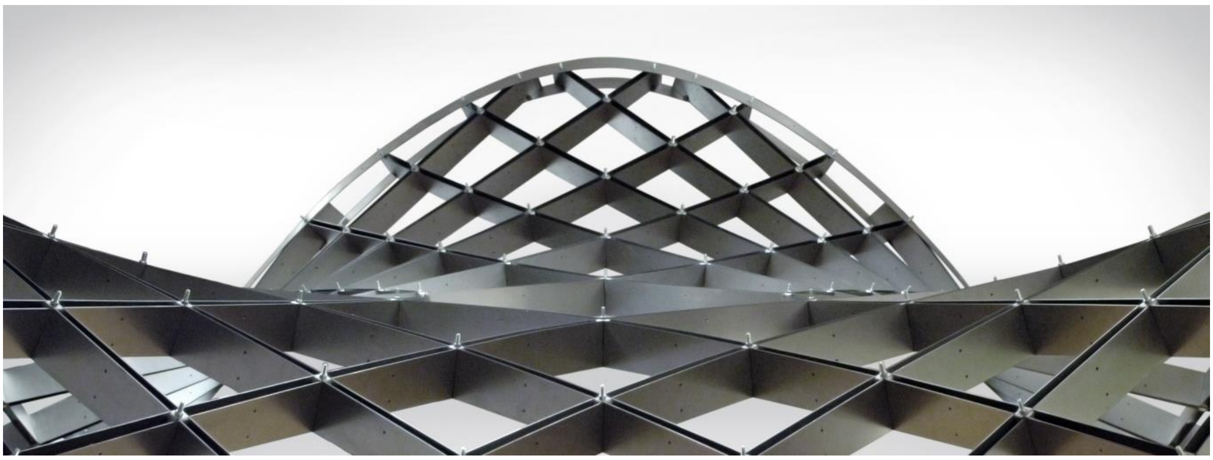

Figure 1 Prototype of an asymptotic gridshell. The structure is built from straight strips of steel. The lamellas are oriented normal to the design surface. All slot joints are identical and orthogonal. *Image*: (Eike Schling)

Figure 2: Grid structure based on asymptotic curves: The model is built from straight strips of beech veneer. All joints are orthogonal. *Image*: (Denis Hitrec)

1 INTRODUCTION

There are a number of design strategies aiming to simplify the fabrication and construction process of doubly-curved grid structures. Therein, we can distinguish between discrete and smooth segmentations [3]. One strategy, to build smoothly curved structures relies on the elastic deformation of its building components in order to achieve a desired curvilinear geometry from straight or flat elements [4]. Consequently, there is a strong interest in the modelling and segmentation of geometry that can be unrolled into a flat state, such as developable surfaces [5]. Recent publications have given a valuable overview on three specific curve types – geodesic curves, principal curvature lines, and asymptotic curves (Fig. 3) – that show great potential to be modelled as developable strips [6]. Both geodesic curves and principle curvature lines have been successfully used for this purpose in architectural projects [7]. However, there have been no applications of asymptotic curves for load-bearing structures. This is astounding, as asymptotic curves are the only type which are able to combine the benefits of straight unrolling and orthogonal nodes (Fig.2).[1, 2]

In this paper we present a method to design strained grid structures along asymptotic curves on minimal surfaces to benefit from a high degree of simplification in fabrication and construction. They can be constructed from straight strips orientated normal to the underlying surface. This allows for an elastic assembly via their weak axis, and a local transfer of normal loads via their strong axis. Furthermore, the strips form a doubly-curved network, enabling a global load transfer as a shell structure. [2]

In Section 2, we describe the geometric theory of curvature and curve networks. In Section 3 we introduce our computational design method of modelling minimal surfaces, asymptotic curves and networks. In Section 4, we implement this method in the design of a research pavilion for the Structural Membranes Conference. In Section 5, we discuss the fabrication, construction details and assembly by means of two prototypes, in timber and steel. Section 6, gives insights into the local and global load-bearing behavior, and describes the structural analysis based on Finite Element Method. We summarize our results in Section 7 and conclude in Section 8, by highlighting challenges of this method, and suggesting future investigations on structural simulation and façade development.

2 FUNDAMETALS

2.1 Curvature of curves on surfaces

To measure the curvature of a curve on a surface, we can combine the information of direction (native to the curve) and orientation (native to the surface) to generate a coordinate system called the Darboux frame (Fig.3 *right*). This frame consists of the normal vector **z**, the tangent vector **x** and their cross-product, the tangent-normal vector **y**. When moving the Darboux frame along the surface-curve, the velocity of rotation around all three axes can be measured. These three curvature types are called the geodesic curvature k_g (around **z**), the geodesic torsion t_g (around **x**), and the normal curvature k_n (around **y**) [6].

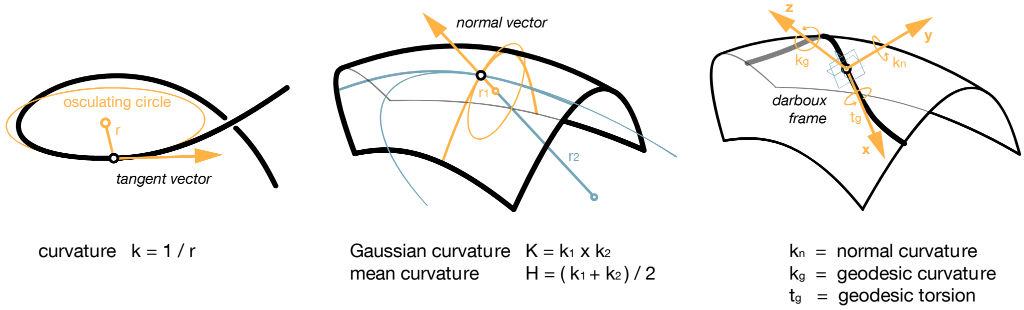

Figure 3: Definitions of curvature. *Left*: Curvature of a curve is measured through the osculating circle. *Middle*: The Gaussian or mean curvature of a surface is calculated with the principle curvatures k_1 and k_2. *Right*: A curve on a surface displays normal curvature k_n. geodesic curvature k_g and geodesic torsion t_g.

2.2 Curvature related networks

Certain paths on a surface may avoid one of these three curvatures (Fig. 4). These specific curves hold great potential to simplify the fabrication and construction of curved grid structures. **Geodesic curves** have a vanishing geodesic curvature. They follow the shortest path between two points on a surface. They can be constructed from straight, planar strips tangential to the surface. **Principle curvature lines** have a vanishing geodesic torsion — there is no twisting of the respective structural element. They can be fabricated from curved, planar strips, and bent only around their weak axis. Their two families intersect at 90 degrees. **Asymptotic curves** have a vanishing normal curvature, and thus only exist on anticlastic surface-regions. Asymptotic curves combine several geometric benefits: They can be formed from straight, planar strips perpendicular to the surface. On minimal surfaces, their two families intersect at 90 degrees and bisect principle curvature lines. [1]

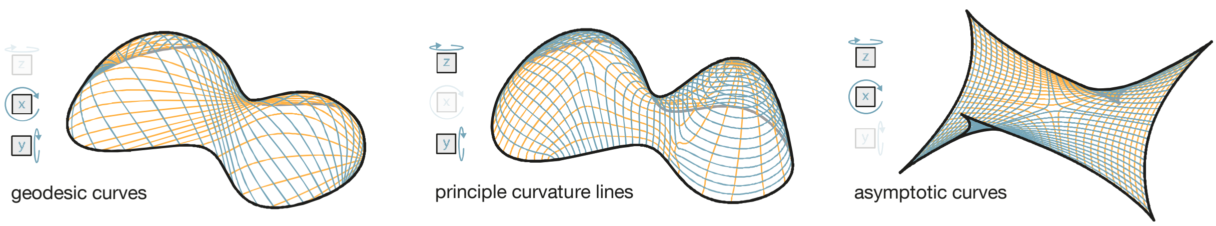

Figure 4: Surface-curves have three curvatures: Geodesic curvature (z), geodesic torsion (x), and normal curvature (y). For each of them, if avoided, a related curve type exists: geodesic curves, principle curvature lines and asymptotic curves.

3 METHOD

3.1 Minimal surface

A minimal surface is the surface of minimal area between any given boundaries. Minimal surfaces have a constant mean curvature of zero. In nature such shapes result from an equilibrium of homogeneous tension, e.g. in a soap film.

Various tools are capable of approximating minimal surfaces based on meshes, with varying degrees of precision and speed (Surface Evolver, Kangaroo-SoapFilm, Millipede, etc.). They are commonly based on a method by Pinkall and Polthier (1993) [8].

The Rhino-plugin TeDa (Chair of Structural Analysis, TUM) provides a tool to model minimal surfaces as NURBS, based on isotropic pre-stress fields [9].

Certain minimal surfaces can be modelled via their mathematical definition. This is especially helpful as a reference when testing the accuracy of other algorithms.

3.2 Asymptotic curves

Geometrically, the local direction of an asymptotic curve can be found by intersecting the surface with its own tangent plane. We developed a custom VBScript for Grasshopper/Rhino to trace asymptotic curves on NURBS-surface using differential geometry.

A detailed description of this method and the generation of accurate strip models is being published parallel at the Design Modelling Symposium 2017 [1].

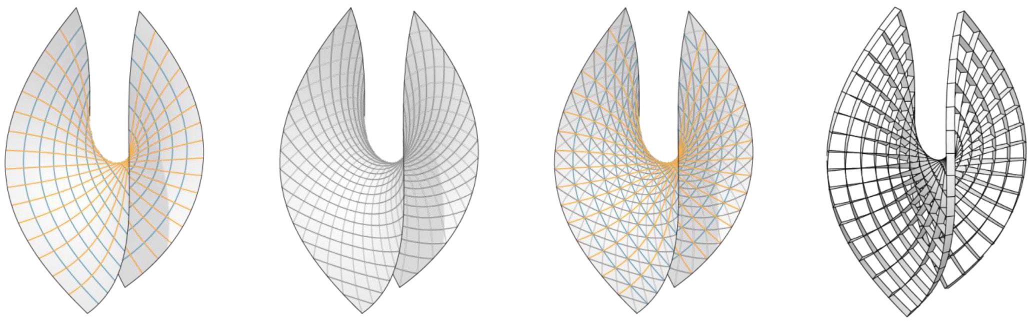

Figure 5: Enneper surface with a) asymptotic curves b) principle curvature lines c) web of both networks d) strip model of the asymptotic network

3.3 Network design

Asymptotic curve networks consist of two families of curves that follow a direction field. The designer can only pick a starting point, but cannot alter their path. If the surface is locally planar, the quadrilateral network forms a singularity with a higher valence.

To achieve a homogeneous network, we take advantage of the bisecting property between asymptotic curves and principle curvature lines (Fig. 5) [10]. By alternately drawing each curve and using their intersections as new starting point, we create an “isothermal” web with nearly quadratic cells [11].

The node to node distance, measured along the asymptotic curves, is the only variable information needed to mark the intersections on the flat and straight strips before bending and twisting them into an asymptotic support structure.

4 DESIGN

4.1 General insights

A minimal surface can be defined by one (a, b), two (c), or multiple (d, e) closed boundary-curves (Fig. 6). Symmetry properties can be used to create repetitive (a, b, c) or periodic (e) minimal surfaces. Boundary-curves may consist of straight lines (a), planar curves (d), or spatial curves (b, c, e). Straight or planar curves are likely to attract singularities (a, c, d). A well-integrated edge can also be achieved by modelling a larger surface and "cookie-cutting" the desired boundary. The Gaussian curvature of the design surface directly influences the density of the network, the position of singularities and the geodesic torsion of asymptotic curves.

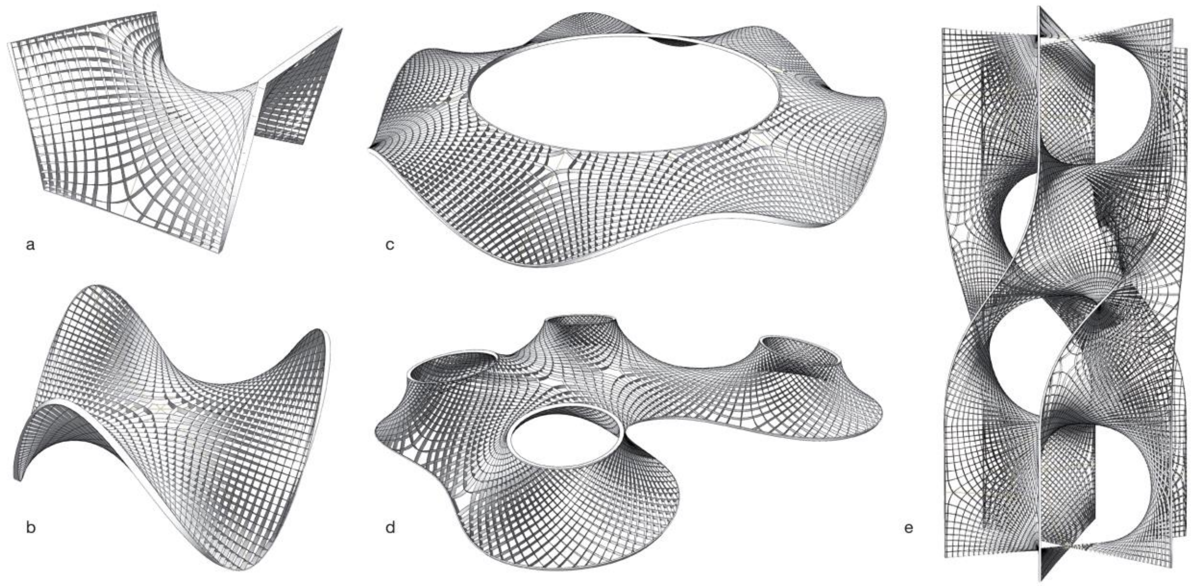

Figure 6: Examples of asymptotic strip networks on minimal surfaces. a) One polygonal boundary, creating a saddle shape with singularities appearing along the edges. b) One spatially curved boundary, creating a network with central singularity. c) Two boundary curves creating a rotational repetitive network with regular singularities. d) Multiple boundaries creating a freely designed network. e) Variation of a singly-periodic Sherk's 2nd minimal surface, with six interlinking boundaries.

4.2 Implementation

We applied this method in the design of a large scale research pavilion for the Structural Membranes Conference in Munich. The design is based on a catenoid – the minimal surface between two circles (Fig. 7). By adjusting the position and shape of these two boundary curves we created an architectural space that reacts to the specific site requirements.

The design of this self-supporting grid structure needs to fulfill **geometric requirements**, creating a homogenously curved minimal surface with well positioned singularities and an aesthetic curve network; **constructional requirements**, considering the allowable bending and torsion of all strip profiles; and **structural requirements** creating a doubly curved structure with well positioned vertical and horizontal supports and efficient arched edges.

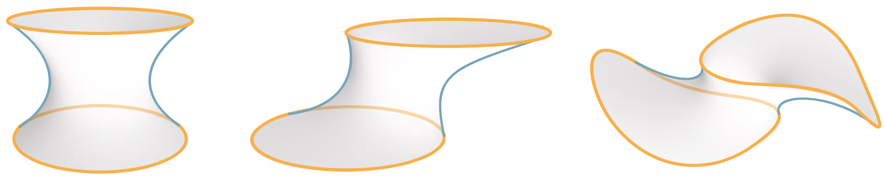

Figure 7 : Design Process : A catenoid is manipulated by shifting and shaping its boudary circles, to create an architectural space with a circular courtyard and two archways.

The pavilion has a span of approx. 9 x 12m at 5m height. The catenoid creates a circular courtyard, embracing one of the green islands on site. Two archways allow circulation to connect the campus to the conference rooms (Fig. 8). At the time of submission of this paper, we are working with sponsors to develop a finance and construction strategy.

Figure 8 : The Asymptotic Pavilion is located at the central campus of the TUM. It is fitted to the specific site requirments of this green courtyard.

5 CONSTRUCTION

5.1 Strained grid

Our construction process follows the reference of Frei Otto`s strained gridshells. This paradigm utilizes elastic deformation to create a doubly-curved lattice structure from straight wooden laths [12].

In our method, we fabricate flat strips of timber or steel and subsequently bend them into their spatial geometry. As asymptotic curves admit no normal curvature, no bending around the y-axis of the profiles is necessary during assembly. Subsequently, the grid can be constructed from straight lamellas orientated perpendicular to the surface. The geodesic curvature results in bending around the z-axis. The geodesic torsion results in twisting of the lamellas (around their x-axis). When choosing the profiles, the section modulus and thickness need to be adjusted to the maximum twist and minimal bending radii in order to keep deformation elastic. At the same time, the profiles need to provide enough stiffness to resist buckling under compression loads. These opposing factors can be solved by introducing two layers of lamellas. Each layer is sufficiently slender to easily be bent and twisted into its target geometry. Once the final geometry is installed, the two layers are coupled with a shear block in regular intervals to increase the overall stiffness similar to a Vierendeel truss. This technique was applied in the construction of two prototypes, first in timber and then in steel, each with an approx. 4 x 4m span (Fig. 9, 10).

5.2 Timber prototype – spatial construction

For the timber prototype, the two asymptotic directions were constructed on separate levels out of 4mm poplar plywood. This allowed for the use of continuous, uninterrupted profiles (Fig. 9, *bottom*). The upper and lower level were connected with a square stud, enforcing the orthogonal intersection angle. This rigid connection could only be fitted if all elements are curved in their final spatial geometry. Consequently, this prototype had to be erected spatially using framework and edge beams as temporary supports. The height of the planar edge profiles was determined by their intersection angle with the lamellas, creating a dominant frame (Fig. 9, *top*).

Figure 9: Timber prototype. The lamellas are doubled and coupled to allow for low bending radii and high stiffness. *Image*: (Eike Schling)

5.3 Steel prototype – elastic erection

The steel prototype was built from straight, 1.5mm steel strips. Both asymptotic directions interlock flush on one level. Therefore, the lamellas have a double slot at every intersection (Fig. 10, *top*). Due to a slot tolerance, the joints were able to rotate by up to 60 degrees. This made it possible to assemble the grid flat on a hexagonal scaffolding. The structure was then "eased down" and "pushed up" simultaneously and thus transformed into its spatial geometry (Fig. 10, *middle*) [13]. During the deformation process, a pair of orthogonal, star-shaped washers were tightened with a bolt at every node, enforcing the 90-degree intersection angle.

Once the final geometry was reached, the edges were fitted by attaching steel strips on top and bottom. The edge-beam locks the shape in its final geometry, generates stiffness and provides attachments for the future diagonal bracing and façade. (Fig. 10, *bottom*).

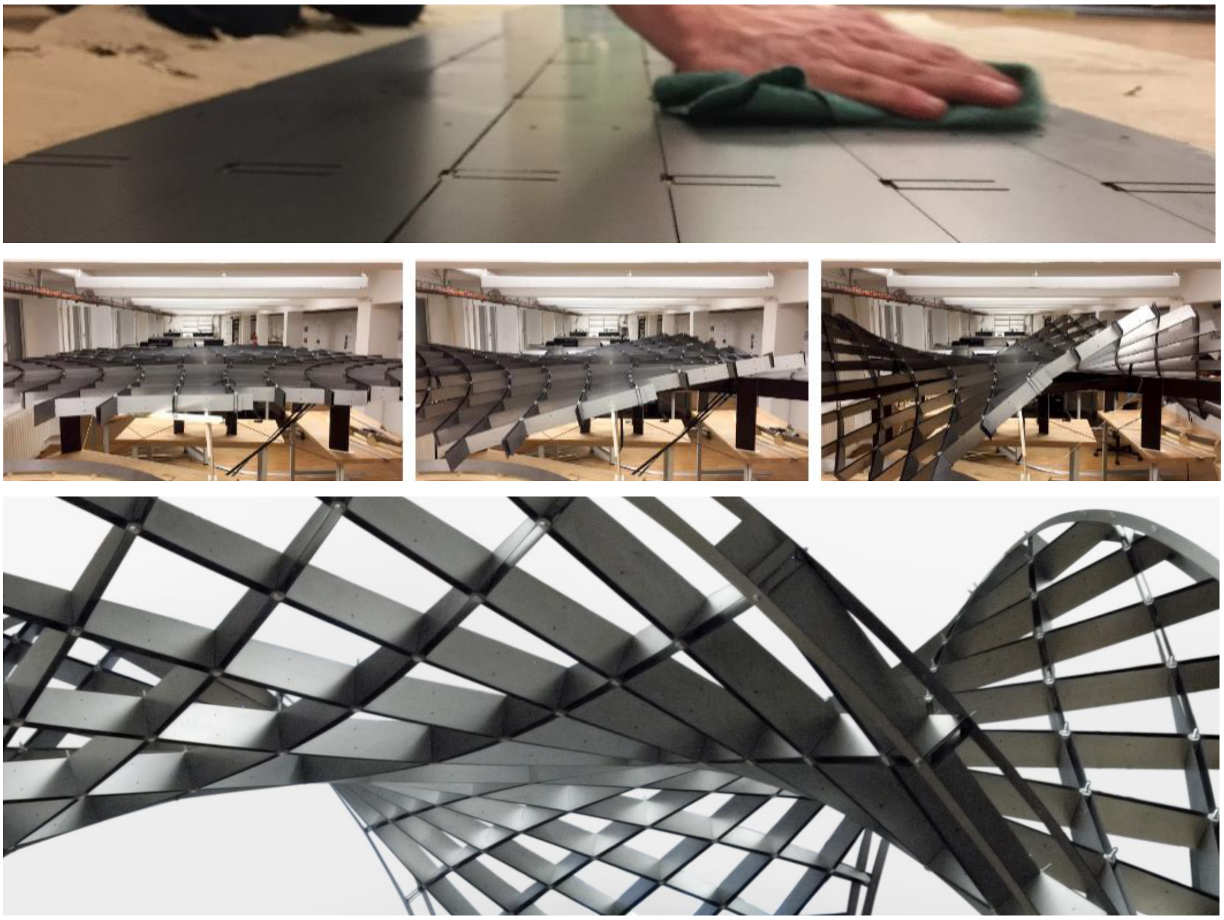

Figure 10: Steel prototype. *Top*: The grid is built from straight and flat steel strips. A double slot allows to interlock both directions in one level. *Middle*: Assembly process showing the elastic transformation from flat to curved geometry. *Bottom*: The final geometry is fixed via tangential edge strips.

6 STRUCTURAL ANALYSIS

6.1 Load-Bearing Behavior

We observed a hybrid load-bearing behavior of two competing mechanisms; a grillage and a gridshell.

The profiles are oriented normal to the surface. Due to the bending stiffness in their strong axis, the grid is able to act as a beam grillage. This is needed to account for the local planarity of asymptotic networks (due to their vanishing normal curvature) and to stabilize open edges. At the same time, the strips form a doubly-curved network. Bracing the quadrilateral network with diagonal cables and supporting it horizontally activates the form-active behavior of a gridshell.

Which of the two mechanism dominates, depends on the overall shape, the grid layout (direction, orthotropy, density and curvature), stiffness ratios (bending and axial stiffness), loads and constraints. These factors were taken into account, when designing both the pavilion and prototypes, in order to promote a shell like behavior.

The elastic erection process, results in restraint (residual) stresses inside the curved and twisted grid elements. When using suitable section dimensions, the initial bending moments stay low and have minor effects on the global behavior.

However, compression of these curved elements increases the bending moment in their weak axis. The strategy of doubling and coupling lamellas (Section 5.1) is therefore essential to control local buckling. The buckling behavior is dependent on the grid size as well as the offset and coupling interval of parallel lamellas.

6.2 FEM - Analysis

The design for the Asymptotic Pavilion (Section 4.2) was analyzed based on the Finite Element Method. The parametric line geometry was modelled in Rhino/Grasshopper and exported to RFEM. Material and joints are based on the steel prototype as described in section 5.3, including diagonal bracing and nondisplaceable supports. The grid nodes are considered rigid. To represent the weakening of the lamellas at the cross-nodes, the joints are modelled as plastic hinges.

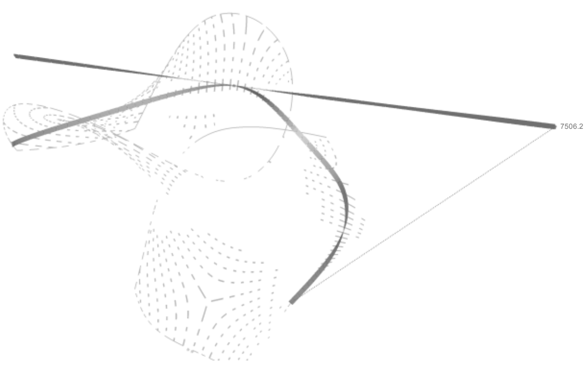

Figure 11: Due to the induced strain loads, a sample lamella will straighten, when extracted from the grid.

The final development of an FEM-Model requires some unusual strategies and is not yet fully completed. The model needs to consider high elastic deformations and their resulting residual stresses, as well as a system change due to the later coupling of parallel lamellas.

The values of geodesic curvature and torsion are measured along each curve and transferred into strain loads, which are then applied as initial load case to the curved geometry. This strategy enables us to induce the residual stresses, without modelling the assembly process.

To verify this method, a sample lamella is extracted from the grid. The applied loads straighten the lamella into its unstrained shape demonstrating correctness of the strain load values (Fig. 11).

Figure 12, shows the surface stress of the final pavilion, including restraint and self-weight. The maximum stresses are mainly caused by the initial bending process and can be adjusted by changing the stiffness in the weak axis or changing the surface curvature.

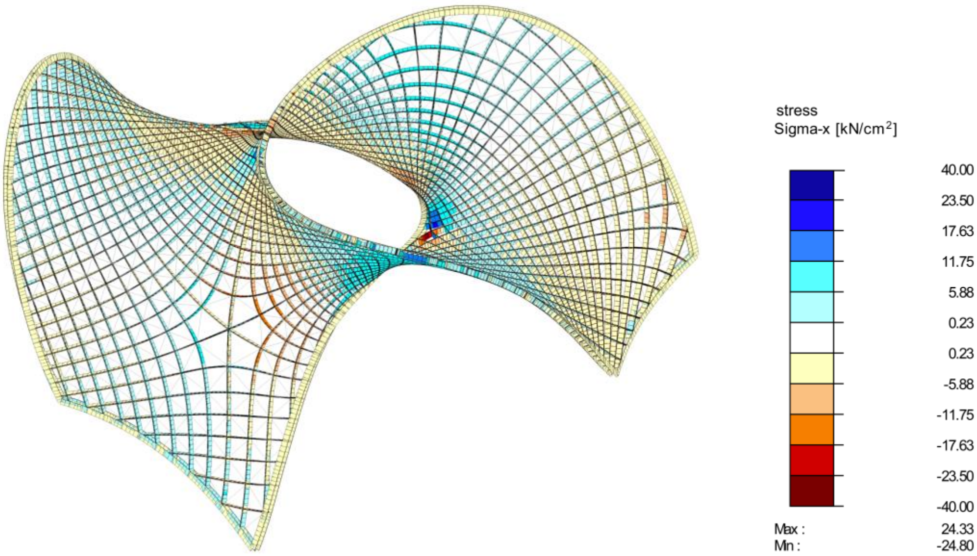

Figure 12: Surface stresses of the grid structure resulting from both the elastic erection process and self-weight.

Due to intense twisting of the lamellas, we expect further stresses according to the theory of helix torsion[14]. These effects are not yet considered in the FEM analysis and need to be quantified in further calculations.

6.3 Strain energy analysis

To evaluate and quantify the structures behaviour, the balance of strain energy is observed under self-weight, excluding the initial restraint stresses. For this, the energy due to internal forces M_x, M_y, M_z and N are compared. M_y and M_t are attributed to a grillage-like behavior, N and M_z are cathegorized as a shell-like behaviour. The calculated energy ratio, indicates a dominating shell-like load-bearing behaviour.

$$W = \frac{1}{2}\int_0^l \left[\underbrace{\frac{M_x^2(x)}{GI_P} + \frac{M_y^2(x)}{EI_y}}_{\text{Grillage behaviour } \mathbf{20\%}} + \underbrace{\frac{M_z^2(x)}{EI_z} + \frac{N^2(x)}{EA}}_{\text{Shell behaviour } \mathbf{80\%}} \right] dx$$

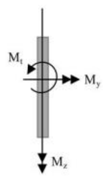

Figure 13: Formula for strain energy W[14]. Indicated below is the balance between M_x, M_y and M_z, N.

7 RESULTS

We compared the geometric properties of three specific curve networks: geodesic curves, principle curvature lines and asymptotic curves, and identified that only asymptotic curves are able to combine the benefits of straight unrolling and orthogonal nodes. They can be formed from straight strips perpendicular to the underlying anticlastic surface. This way, they resist loads normal to the surface by bending in their strong axis. On minimal surfaces, asymptotic curves intersect at 90 degrees, which allows the use of identical nodes throughout the structure. The bisecting property with principle curvature networks offers further geometric advantages for substructure and façade.

We developed a custom VBScript that can trace asymptotic curves on anticlastic surfaces with sufficient accuracy for design and construction, and implemented this method in a pavilion design for the Structural Membranes Conference 2017.

Due to an initial deformation, both twisting (geodesic torsion) and bending (geodesic curvature) have to be considered when choosing profiles for this construction. We have presented a strategy of doubling and coupling the bent structural elements to achieve sufficient stiffness of the final grid. The findings were demonstrated in the realization of two prototypes: One in timber and one in steel, each with a span of 4 x 4m.

We discussed the structural behavior based on two competing mechanism, a grillage and a gridshell and finally developed a workflow to compute the residual stress of the elastic erection process on the basis of the local geometric curvature and torsion, without simulating the assembly process.

8 CONCLUSION

An analytical approach to both geometry and material properties is required to achieve a symbiosis of form, structure and fabrication. Even though the design freedom is limited to the choice of boundary curves, there is a wide range of design solutions applicable to all scales and functions. The construction of asymptotic grids as strained grids offers advantages for both fabrication and assembly. Structurally, asymptotic gridshells show great potential, as they combine the benefits of upright sections with a doubly-curved grid. Hence, loads can be transferred locally via bending, and globally as a shell structure.

We are continuing to investigate the structural behavior of strained asymptotic structures, comparing grid orientations, shapes and supports. Another ongoing development is the implementation of constructive details: This includes cable bracing and façade systems using planar quads, developable façade strips and membranes.

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