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DISCONTINUITY LAYOUT OPTIMIZATION IN UPSCALING OF EFFECTIVE STRENGTH PROPERTIES IN MATRIX-INCLUSION MATERIALS

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Abstract. The prediction of strength properties of engineering materials, which in general are time dependent due to chemical reactions and deterioration processes, plays an important role during manufacturing and construction as well as with regard to durability aspects of materials and structures. On the one hand, the speed of production processes and the quality of products may be significantly increased by improved material performance at early ages. On the other hand, the life time of materials and structures can be enlarged and means of repair and maintenance can be optimized.

For determination of strength properties of composite materials, a multiscale approach is proposed in this paper. For upscaling of strength properties, numerical limit analysis considering discontinuity layout optimization (DLO) is employed. In a first step, DLO is applied to two-phase material systems, with the matrix being represented by node clouds. In this paper, adaptive techniques regarding the spatial distribution of nodes thus the discontinuity generation are introduced in DLO, improving the computational performance of DLO within upscaling of strength properties.

1 MOTIVATION

The prediction of strength properties of engineering materials, which in general are time dependent due to chemical reactions and deterioration processes, plays an important role during manufacturing and construction as well as with regard to durability aspects of materials and structures. On the one hand, the speed of production processes and the quality of products may be significantly increased by improved material performance at early ages. On the other hand, the life time of materials and structures can be enlarged and means of repair and maintenance can be optimized.

For determination of strength properties of composite materials, multiscale approaches are often employed. Several methods for prediction of strength of materials can be found in the literature, among these are e.g. continuum micromechanics [8], the finite-element method (FEM) [9], and numerical limit analysis (LA) [6].
In this work, a two-phase composite material exhibiting a matrix-inclusion morphology is considered. For upscaling of strength properties of these two-phase material systems, numerical limit analysis [1] considering discontinuity layout optimization (DLO) [2] is employed and extended towards adaptive discontinuity layout optimization (ADLO).
First, the methodology of the employed approach is discussed which is followed by the presentation of first results obtained from ADLO. Finally, concluding remarks and an outlook on future work are given.

2 METHODOLOGY

2.1 Fundamental principle of DLO

DLO is a limit-analysis methodology for determining strength properties of materials or collapse loads of structures. Recently, this method was applied to steel frames [3], geotechnical engineering [2], concrete slabs [4] as well as masonry structures [5].
DLO requires the generation of discontinuities, of which every one may be a potential failure discontinuity and, thus, contribute to the failure mode of the material or structure. With (i) the aid of linear programming (LP), (ii) assigning of material properties to every discontinuity (Mohr-Coulomb-type material), and (iii) the definition of boundary conditions, the discontinuities contributing to the failure mechanism are obtained, when the system reaches a total internal energy minimum. This leads to an upper bound (UB) formulation with the following LP problem (see [2]):

$$min\ \lambda \mathbf{f}_L^T\mathbf{d} = -\mathbf{f}_D^T\mathbf{d} + \mathbf{g}^T\mathbf{p},$$

subject to

$$\begin{aligned} \mathbf{B}\mathbf{d} &= \mathbf{0}, \\ \mathbf{f}_L^T\mathbf{d} &= 1, \\ \mathbf{N}\mathbf{p} - \mathbf{d} &= \mathbf{0}, \\ \mathbf{p} &\geq \mathbf{0}. \end{aligned} \tag{1}$$

In Equation (1), $\mathbf{f}_L$ and $\mathbf{f}_D$ are the vector for live and dead load, respectively, $\mathbf{g}$ is a matrix containing length and cohesive shear strength of the discontinuities, $\mathbf{d}$ is the vector of discontinuity displacements, $\mathbf{B}$ is the compatibility matrix, $\mathbf{N}$ is the plastic-flow matrix, and $\mathbf{p}$ is the vector of plastic multipliers.
In the present application, DLO is used to determine the strength properties of matrix-inclusion materials. For this purpose, the dead load will be disregarded.

2.2 Current methodology and limitations

So far, a constant set of regularly distributed nodes serves as basis for the definition of discontinuities. The size of the underlying LP problem increases rapidly for larger number of nodes and discontinuities, which cannot be solved on currently available personal computers with efficiency. Instead of generating all possible discontinuities among the nodes in the model, only discontinuities existing length lower than a certain threshold length are generated within the first step. Within an stepwise calculation, additional discontinuities are gradually added to the model in zones of plastic failure of the structure (see [2]).
For the application of DLO to matrix-inclusion materials, this procedure has certain drawbacks. First, a constant (regular) set of nodes limits the number of possible failure modes to the orientation of the discontinuities. Second, in regions of the material where no discontinuities will fail, the density of the nodes remains constant, thus considering discontinuities not contributing to the failure mechanism. Third, in contrast to homogeneous materials, the node distribution at the boundary between matrix and inclusion is crucial for determination of strength properties, taking into account the influence of interface properties and the strength properties of materials.

2.3 Adaptive discontinuity layout optimization (ADLO)

With the mentioned limitations in mind, a random cloud of nodes giving the layout of the discontinuities is proposed in this paper. Hereby, the generation of discontinuities is performed with a delaundray triangulation [7]. The iterative adaptation of nodes in regions of plastic failure is illustrated in Figure 1 considering additional nodes in triangles and boundary discontinuities adjacent to discontinuities contributing to the failure mechanism. The ADLO algorithm involves the following steps:

- Preprocessing:
    - Node generation
    - Relaxation of nodes
    - Triangulation (discontinuity layout)
    - Discontinuity generation
    - Generation of compatibility matrix **B**
    - Generation of vector of plastic multipliers **p**
    - Application of external loads $\mathbf{f}_L$
- Solving LP:
    - Solving LP problem (Equation (1))
- Postprocessing:
    - Location of failed discontinuities
    - Consideration of additional nodes in zones of material failure (see Figure 1)

Figure 2 illustrates the difference between regular (203 nodes, 738 discontinuities) and random node (288 nodes, 738 discontinuities) generation. While the regular node dis-

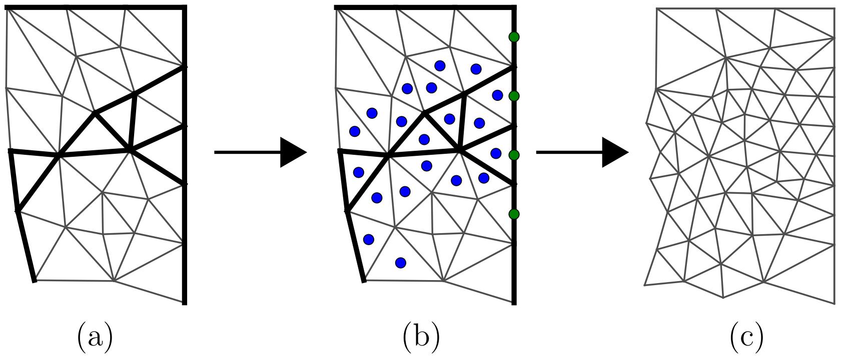
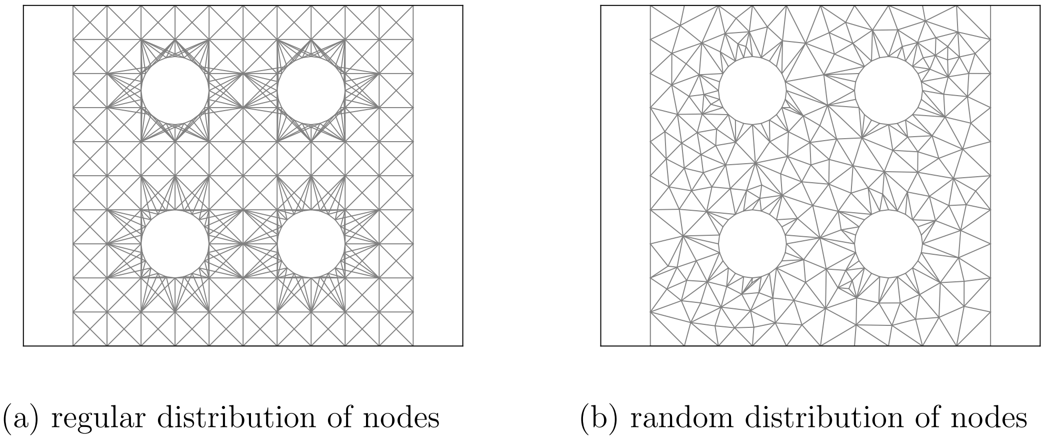


Figure 1: Illustration of node adaptation in zones of plastic failure: (a) failure mechanism obtained from current layout of discontinuities; (b) introducing additional nodes at the centroid of triangles and at the center of boundary discontinuities adjacent to this failure mechanism; (c) new layout of discontinuities

tribution leads to similar angles, the random layout of discontinuities, with subsequent adaptive refinement steps, yields a larger variety of angles and therefore a larger variety of possible failure modes.



(a) regular distribution of nodes

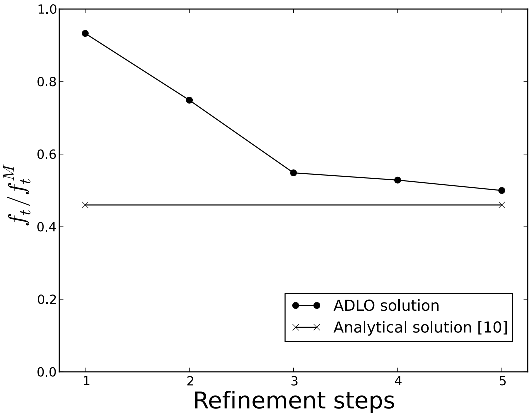
(b) random distribution of nodes

Figure 2: Effect of node arrangement on discontinuity layout

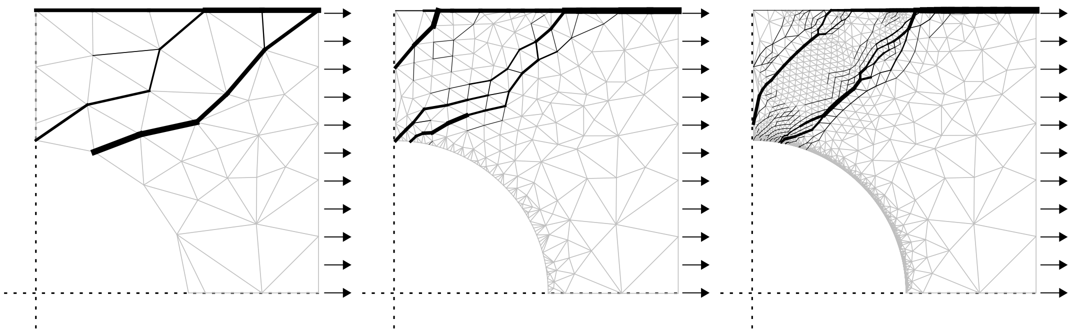
3 Results and Discussion

Process of adaptive node generation

For the demonstration of the adaptive node generation, a porous material with a single circular pore subjected to uniaxial loading is considered. Hereby, symmetry in horizontal and vertical direction are exploited. At the right boundary of the model, the load is applied. In Figure 3, thick lines indicate discontinuities which experience plastic deformation and thereby contribute to material failure. The absolute value of the velocity is illustrated by the width of the lines.



(a)



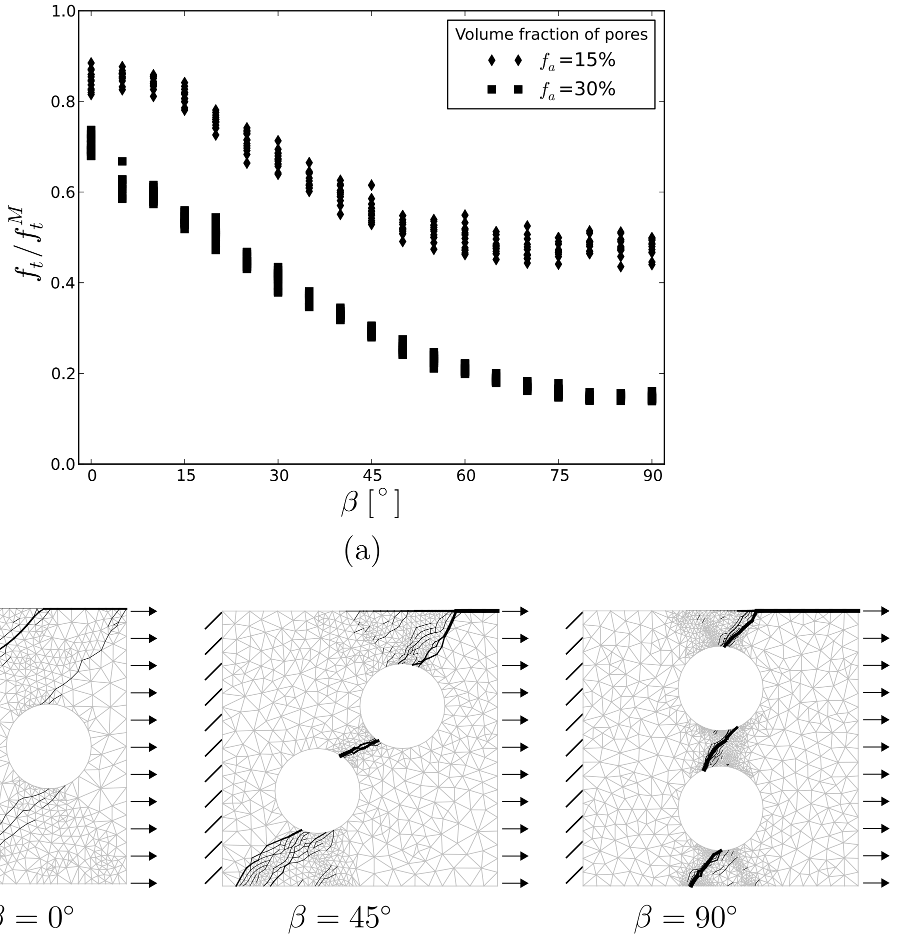
(b)

Figure 3: ADLO results obtained for porous material with one single circular pore: (a) material strength related to matrix strength as a function of refinement steps; (b) layout of discontinuities for different refinement steps

The underlying material properties of the matrix phase are chosen as: cohesion = 1, angle of friction = 0, giving an angle of the failure mode of $\pi/4$ and a failure load of $f_t/f_t^M = 0.46$ (see [10]), were f_t^M refers to the tensile strength of the matrix and f_t to the tensile strength of the porous material. Both f_t/f_t^M of 0.49 (obtained for refinement step 5) and the failure mode obtained from ADLO correlate well with the analytical solution.

Porous material with two inclusions

In the second example, a porous material with two circular pores subjected to uniaxial tensile loading is considered. By means of ADLO, the upper bound of the uniaxial tensile strength is calculated. Hereby, the two pores are rotated with respect to the loading direction from $\beta = 0°$ to $90°$ [6]. Figure 4 illustrates, the effect of β on the results obtained from different sets of nodal distributions. The different results corresponding to one angle in Figure 4(a) reflect the influence of the nodal distribution on the predicted material strength.



(a)

(b) $\beta = 0°$ $\beta = 45°$ $\beta = 90°$

Figure 4: ADLO results obtained for porous material with two circular pores: (a) effect of β on tensile strength and (b) discontinuity layout for three different angles of β (f_a volume fraction of air voids)

3.1 Conclusion and outlook

In this work, an adaptive mode of the discontinuity layout optimization (DLO) for upscaling of strength properties is proposed. Hereby the regular generation of nodes is replaced by random nodal distribution, which is enhanced in an step-wise manner in zones of plastic failure.

First results showed a good performance of the proposed node adaptation in plastic zones by ADLO. Also the improvement of the solution within increasing number of iteration step was illustrated. An example of a material with two inclusion showed the influence of the arrangement of the pores on the strength properties of the porous material.

Future work will focus on the refinement of the nodal enhancement in plastic zones. Moreover, removal of nodes in regions were no failure occurred shall be included.

3.2 Acknowledgement

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