XI International Conference on Computational Plasticity. Fundamentals and Applications
COMPLAS XI
E. Oñate, D.R.J. Owen, D. Peric and B. Suárez (Eds)

ANALYSIS OF PROFILE AND FLATNESS IN FLAT HOT ROLLING BASED ON NON LINEARLY COUPLED MODELS FOR ELASTIC ROLL STACK DEFLECTION AND PSEUDO-STEADY-STATE ELASTO-VISCOPLASTIC STRIP

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Key words: Flat Hot Rolling, Bulk Forming, Profile and Flatness, Elastic Roll Stack, Strip Profile Transfer, Incompatible Residual Strains.

ABSTRACT. An enhanced iterative concept for the effective numerical simulation of flat hot rolling processes is presented. The underlying physical process is the forming of metal within a flat rolling stand, i.e. between a lower and an upper roll set, each of them consisting of one or more rolls. The strip material is described elasto-viscoplastically, whereas the roll stack is deformed elastically. The accurate coupling of the strip model with the routines for the elastic roll stack deflection is a precondition to get reliable results concerning profile transfer and incompatible residual strains inside the strip, which allows the prediction of flatness defects, such as buckling. Especially for thin, wide strips and heavy plates, where the aspect ratio width over thickness is extremely unfavourable, the determination of profile transfer and flatness obviously leads to extremely high calculation times with commercial FEM-programs. Therefore, a tailor-made FEM-code for the efficient simulation of the elasto-viscoplastic material flow inside the roll gap was developed and programmed in C++. It is based on pseudo-steady-state, fully implicit stress-update approaches, where the incremental material objectivity is satisfied exactly. The developed model is well suited for systematic parameter studies to investigate flatness defects in more detail and to develop enhanced flatness criteria for thin hot and cold strips and plates.

1 INTRODUCTION AND SURVEY

For optimization and control, the development of highly sophisticated mathematical offline and online models in both hot and cold flat rolling is a vital precondition for manufacturing high quality products satisfying even the most challenging tolerance demands. Control of strip crown and shape can be considered to be among the most important technologies in flat rolling of strip and plate. Although the analysis of transient and steady-state rigid-viscoplastic and elasto-viscoplastic forming processes is not new (cf. e.g. [1–7], 13, 14), the high customer demands concerning productivity and product quality are the reason, why it is of utmost importance to attain a better understanding of the underlying process details, which requires highly sophisticated formalisms and optimized numerical simulation concepts and their application to process optimization and control purposes. During the last three decades, great efforts have been paid to competitive developments of new technologies to reduce or avoid profile errors and shape defects [11, 12]. The theoretical understanding of the underlying material flow behavior is the crucial foundation for the development of improved calculation tools for strip crown and shape evaluation in order to better meet the "offline task" of designing new machines including actuators and the "online challenge" of guiding and controlling flat rolling processes.

The accurate and reliable prediction of lateral flow and strip spread can be considered to be one of the essential objectives of (steady-state) flat hot rolling simulations. It enables the pre-calculation of strip profile (thickness over width), of profile transfer functions, and relative strip crown changes [11, 12]. For the prediction of the material flow behavior of wide strips and plates in hot and cold rolling, highly sophisticated procedures are essential, which are able to couple the deformation of the rolled stock with the elastic response of the rolls. Especially for thin, wide strips, where the aspect ratio width over thickness is extremely unfavorable, the determination of profile transfer and flatness obviously leads to extremely high calculation times of several days with commercial FEM-programs, in particular, when the elasto-plastic strip models have to be coupled with elastic roll stack deflection models. Some critical details concerning the underlying formalism of the self-developed customized simulation models will be outlined in this paper.

The tailor-made FEM-code for the efficient simulation of the elasto-viscoplastic material flow inside the roll gap is based on pseudo-steady-state and fully implicit stress-update approaches (cf. e.g. [6, 7]), where the incremental material objectivity is satisfied exactly. Special emphasis was put on the coupling of strip models with routines for the elastic roll stack deflection [8], which is a precondition to get reliable results concerning strip profile transfer and residual strain and stress distributions inside the strip [9, 10]. Such data allow the evaluation of strip-flatness based on buckling analysis and of the effectivity and adjustment ranges of profile and flatness actuators [11, 12]. The model is well suited for systematic parameter studies to investigate flatness of strips and plates in more detail and to develop enhanced flatness criteria for thin hot and cold strips. Of particular interest is the dependence of the longitudinal stress and strain distributions and of the corresponding specific rolling force-distributions across the strip width on the underlying constitutive elasto-viscoplastic laws including work hardening and softening between consecutive passes.

The basic geometry of the rolling process under consideration consists of two rotating work rolls, which are supported by backup rolls (i.e. quarto flat rolling stand) and reduce the thickness of the incoming steel strip or plate. During the rolling process, a considerable amount of force is exerted on the roll assembly, which deforms the rolls accordingly. Of particular interest is the pressure distribution between strip and work roll and also between backup roll and work roll to determine the actual elastic deformation of the roll assembly. The total elastic deformation of the roll stack can be determined very effectively and accurately by solving the 3D elastic Lame equations according to the method developed and patented by Siemens-VAI [8]. The determination is performed in cylindrical coordinates and utilizes systematic Fourier series expansions [16, 17]. As is well known [18] it suffices for hot rolling scenarios to take into account merely the radial surface displacement function u_r to describe the deformed work roll surface adequately.

2 DISCUSSION OF THE UNDERLYING COUPLING CONCEPT

Due to the high non-linearity of the whole problem, the coupling between roll stack and rolled stock has to be performed iteratively. The contact stress distribution resulting from the strip model serves as input for the determination of the deformed work roll surface, which can be performed very accurately and effectively by applying appropriate analytical and numerical methods. The new deformed work roll surface represents the *"flow channel"* for the next calculation step of the strip model. The routines for the elastic roll stack deflection have to be coupled with the modules for the strip-behavior via the a priori unknown deformed contact surface between the strip and work roll. Both the deformed contact surface and the corresponding 2D contact stress distribution have to be determined consistently and result from the coupling between the models for the strip and the roll-stack. The accurate coupling of the strip models with the routines for the elastic roll stack deflection is a precondition to get reliable results concerning profile transfer and residual stresses (cf. [9-12]) inside the strip, which allows the prediction of flatness defects, such as strip buckling.

A systematic, iterative calculation concept is taken into consideration to treat the highly non-linear coupling between deformable bodies in metal forming in an efficient and accurate manner. For prescribed contact surface geometry, the strip-model determines the contact stress distribution in real space, which serves as input distribution for the roll stack deflection model. The resulting new work roll surface contour serves as new contact surface geometry, thus, enabling the strip model to perform the next iteration step. As the roll stack model is based on the theory of linear elasticity [16, 17] (but includes the non-linear contact between work- and backup-rolls), the load is applied onto the undeformed reference configuration. This necessitates a back-transformation of the real-space contact stress distribution onto an undeformed cylindrical surface, which will be performed systematically in the next section. The transformation concept is based on the postulation that infinitesimal surface traction vectors transform covariantly, analogously to the coordinate differentials. Therefore, it suffices to determine the underlying deformation gradient [1-5], which essentially generates this highly non-linear transformation. To actually perform the contact stress transformation, the knowledge of the radial surface displacement function u_r does not suffice to determine the full deformation gradient, as the partial derivatives $\partial u_r / \partial r$ evaluated at the surface $r = R$ are required as well. The determination of both u_r and the corresponding radial strain $\partial u_r / \partial r$

at the cylindrical work roll surface is accomplished by a combination of analytical and numerical methods, the details of which will be published in a subsequent paper.

3 NON-LINEAR CONTACT MAPPING CONCEPT

The two-dimensional contact pressure distribution (both normal contact stress and tangential shear stresses) between strip and work roll, which can be calculated by an adequate strip-FE model for a given *"flow-channel"* (i.e. the deformed work roll contour), serves as input quantity for the determination of the corresponding elastic roll stack deformation. As pointed out above, a non-linear mapping of the contact stress distribution onto non-deformed (i.e. cylindrical) work roll boundaries has to be performed. This transformation of stress distributions between an Eulerian configuration (actually deformed real space scenario) and a corresponding Lagrangian (i.e. undeformed scenario) can be performed systematically by the transformation concept outlined in this section.

The undeformed reference configuration corresponding to an undeformed cylindrical work roll, which serves as Lagrangian (upper index *L*) reference configuration here, can be represented in adequate cylindrical coordinates as

$$\vec{x}^{(L)}\left(r,\vartheta,y\right)=\left(\left[x_C-r\sin\vartheta\right],\quad y,\quad\left[z_C-r\cos\vartheta\right]\right)\ , \tag{1}$$

where $r=\sqrt{(x-x_C)^2+(z-z_C)^2}$ is the distance from the axis of the cylinder, y designates the lateral coordinate (direction of the axis of the undeformed cylinder) and ϑ is the angular coordinate in azimuthal direction. Surface evaluations are performed at the value $r=R$, where R denotes the undeformed work-roll radius. Obviously, the undeformed centre-line of the cylinder is represented by $y\rightarrow(x_C, y, z_C)$.

A suitable parameterization of the actually deformed configuration, i.e. the Eulerian (upper index *E*) representation reads

$$\vec{x}^{(E)}\left(r,\vartheta,y\right)=\begin{pmatrix} x_C-\left[r+u_r\left(r,\vartheta,y\right)\right]\sin\vartheta \\ y \\ z_C-\left[r+u_r\left(r,\vartheta,y\right)\right]\cos\vartheta \end{pmatrix}\ , \tag{2}$$

where only the radial displacement field $u_r\left(r,\vartheta,y\right)$ will be taken into account to describe the deviation from the undeformed reference configuration, which suffices for hot rolling scenarios. The transformation rule between the Eulerian *(E)* and Lagrangian *(L)* coordinates is given by the deformation gradient $\vec{\vec{F}}$ $(3\,\mathrm{x}\,3$ - matrix), which is defined by utilizing coordinate differentials as follows

$$d\vec{x}^{(E)}=\vec{\vec{F}}\,d\vec{x}^{(L)}\ . \tag{3}$$

Taking into account that the cylindrical coordinate differentials $\left(dr, d\vartheta, dy\right)$ can be represented in terms of the Cartesian Lagrangian coordinates according to

$$\begin{pmatrix} dr \\ d\vartheta \\ dy \end{pmatrix}=\begin{pmatrix} -\sin\vartheta & 0 & -\cos\vartheta \\ -\cos\vartheta/r & 0 & \sin\vartheta/r \\ 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} dx^{(L)} \\ dy^{(L)} \\ dz^{(L)} \end{pmatrix} \tag{4}$$

leads directly to the explicit representation of the deformation gradient $\vec{F}$ as a function of the cylindrical coordinates

$$\begin{pmatrix} \left\{(-\sin\vartheta)\left(\frac{\partial x^{(E)}}{\partial r}\right)-\frac{\cos\vartheta}{r}\left(\frac{\partial x^{(E)}}{\partial\vartheta}\right)\right\} & \left\{\left(\frac{\partial x^{(E)}}{\partial y}\right)\right\} & \left\{(-\cos\vartheta)\left(\frac{\partial x^{(E)}}{\partial r}\right)+\left(\frac{\sin\vartheta}{r}\right)\left(\frac{\partial x^{(E)}}{\partial\vartheta}\right)\right\} \\ 0 & 1 & 0 \\ \left\{(-\sin\vartheta)\left(\frac{\partial z^{(E)}}{\partial r}\right)-\frac{\cos\vartheta}{r}\left(\frac{\partial z^{(E)}}{\partial\vartheta}\right)\right\} & \left\{\left(\frac{\partial z^{(E)}}{\partial y}\right)\right\} & \left\{(-\cos\vartheta)\left(\frac{\partial z^{(E)}}{\partial r}\right)+\left(\frac{\sin\vartheta}{r}\right)\left(\frac{\partial z^{(E)}}{\partial\vartheta}\right)\right\} \end{pmatrix}, \tag{5}$$

where the involved partial derivatives of Eulerian coordinates in Equation 5 can be represented in terms of the radial displacement field $u_r(r,\vartheta,y)$, e.g.

$$\left(\frac{\partial x^{(E)}}{\partial r}\right)=\left\{-\left[1+\left(\frac{\partial u_r}{\partial r}\right)\right](\sin\vartheta)\right\}. \tag{6}$$

The deformation gradient $\vec{F}$ directly serves as basic operator to determine the transformation behaviour of covariant vector differentials, such as Eulerian and Lagrangian surface traction vectors, denoted by $\vec{p}^{(E)}$ and $\vec{p}^{(L)}$, respectively. As covariant vectors transform analogously to coordinate differentials represented in Equation 4, the underlying transformation rule reads

$$d\vec{P}^{(E)}=\vec{F}\,d\vec{P}^{(L)}\;\rightarrow\;d\vec{P}^{(L)}=\vec{F}^{(-1)}\,d\vec{P}^{(E)}\;, \tag{7}$$

where the infinitesimal Eulerian and Lagrangian force vectors are given by

$$d\vec{P}^{(E)}=dS^{(E)}\vec{p}^{(E)}=dS^{(E)}\left\{\sigma_N^{(E)}\hat{N}^{(E)}+\vec{\sigma}_T^{(E)}\right\} \tag{8}$$

$$d\vec{P}^{(L)}=dS^{(L)}\vec{p}^{(L)}=dS^{(L)}\left\{\sigma_N^{(L)}\hat{N}^{(L)}+\vec{\sigma}_T^{(L)}\right\}, \tag{9}$$

where $dS^{(L)}$ and $dS^{(E)}$ denote the infinitesimal surface elements, on which $\vec{p}^{(L)}$ and $\vec{p}^{(E)}$ are acting. The vectors $\hat{N}^{(L)},\hat{N}^{(E)}$ are normal vectors to these surface elements, hence, $\hat{N}^{(E)}\cdot\vec{\sigma}_T^{(E)}=0$ and $\hat{N}^{(L)}\cdot\vec{\sigma}_T^{(L)}=0$.

Note that the infinitesimal surface vectors transform in a contravariant manner leading to

$$d\vec{S}^{(E)}=J\;\vec{F}^{(-T)}\,d\vec{S}^{(L)} \tag{10}$$

with the functional determinant of the deformation gradient $J=\det\left(\vec{F}\right)$.

By taking the scalar product between covariant and contravariant vector differentials, one is immediately led to the representation

$$\left(d\vec{P}^{(E)}\cdot d\vec{S}^{(E)}\right)=J\;\left(d\vec{P}^{(L)}\cdot d\vec{S}^{(L)}\right) \tag{11}$$

directly yielding the following scalar relation between the Eulerian and Lagrangian contact stress values

$$\sigma_N^{(E)}\left(dS^{(E)}\right)^2=J\;\sigma_N^{(L)}\left(dS^{(L)}\right)^2. \tag{12}$$

Equation 12 enables the determination of the Lagrangian normal contact stress distribution, provided that the corresponding Eulerian values are known. Note that the knowledge of the Eulerian normal contact stress distribution $\sigma_N^{(E)}$ suffices to determine the Lagrangian counterpart $\sigma_N^{(L)}$, i.e. this quantity is not directly influenced by the Eulerian shear stress distribution $\vec{\sigma}_T^{(E)}$. A more detailed analysis based on Equation 12, yields the following explicit representation in terms of the (inverse and transposed) deformation gradient

$$\sigma_N^{(L)}(\vartheta, y) = \left\{ J \left\| \vec{F}^{(-T)} \hat{N}^{(L)} \right\|^2 \right\} \sigma_N^{(E)}(\vartheta, y) \quad . \tag{13}$$

The partial derivatives $\partial u_r / \partial r$ evaluated at the surface $r=R$ (i.e. the radial surface strains) are required explicitly to determine the deformation gradient $\vec{F}$. By utilizing sophisticated mathematical methods, essential details of which will be published in a subsequent paper, an analytical formula could be derived, which is valid exactly for planar surfaces and still a highly satisfactory approximation for cylindrical (i.e. curved) surfaces, at least for localized normalized contact stress distributions $\sigma_N(\vartheta, y)$, as is the case for flat rolling of metal strip:

$$\frac{\partial u_r}{\partial r}\left(r = R, \vartheta, y\right) \cong \frac{(1+\nu)(1-2\nu)}{E} \sigma_N(\vartheta, y) \quad . \tag{14}$$

This formula for the determination of the radial surface strain is applied directly in numerical calculations and enables the correct non-linear mapping of the Eulerian contact stress distribution onto the Lagrangian frame of reference.

4 ELASTO-VISCOPLASTIC STRIP MODELING CONCEPT

For the numerical simulation of steady-state elasto-viscoplastic rolling processes, especially for thin wide strips, standard incremental approaches based on updated Lagrangian concepts are not very efficient and lead to very high calculation times. Therefore, an effective customized pseudo-steady-state algorithm was implemented, some basic ideas of which were proposed some years ago by Hacquin et al. [7]. The global algorithm is based on an iterative calculation of the stress and velocity fields inside the strip. The strip model is coupled with a consistent determination of the flow channel geometry (i.e. work roll surface) resulting from the deformation of the work roll surface loaded by the 2D-contact stress distribution obtained from the preceding strip calculation step.

The elasto-viscoplastic constitutive law is based on the Prandtl-Reuss decomposition of the total rate of deformation tensor $\vec{D}^{(tot)} = \left[\vec{D}^{(el)} + \vec{D}^{(pl)}\right]$ into elastic and plastic parts, where the plastic part behaves incompressible for metal forming processes (at least in very good approximation), i.e. $tr\left(\vec{D}^{(pl)}\right) = 0$. For elastic parts, isotropic linear behaviour is assumed, whereas the plastic parts are treated according to Levy-Mises. For incremental constitutive laws, the stress-update has to be performed along material streamlines. For fixed velocity-field and steady-state particle run-time values $\Delta t_{(n \to n+1)}$ between successive Gauss-integration points, denoted by n and $(n+1)$, the new Cauchy stress-values can be determined according to the fully implicit prescription in Equation 15, which satisfies exactly the required incremental material objectivity (i.e. corotational stress formulation) even for large local rotations:

$$\vec{\vec{\sigma}}_{n+1} \cong \exp\left\{\vec{\vec{W}}_{n+1} \vartriangle t\right\} \vec{\vec{\sigma}}_n \exp\left\{-\vec{\vec{W}}_{n+1} \vartriangle t\right\} + \vartriangle t \overset{\odot}{\vec{\vec{\sigma}}}_{(n\rightarrow n+1)} \quad . \tag{15}$$

The materially objective part of the Cauchy stress-rate (according to Jaumann-Kirchhoff) is denoted by the symbol $\odot$. For the case of isotropic elasticity, this constitutive law in rate representation reads

$$\overset{\odot}{\vec{\vec{\sigma}}}_{(n\rightarrow n+1)} = \left[2G \, \vec{\vec{D}}'^{(el)}_{n+1} + K \, tr\left(\vec{\vec{D}}_{n+1}\right) \vec{\vec{I}} \right] . \tag{16}$$

The local rotation-tensor $\vec{\vec{R}} = \exp\left\{\vec{\vec{W}} \vartriangle t\right\}$ with $\vec{\vec{R}}\,\vec{\vec{R}}^T = \vec{\vec{I}}$ and $\dot{\vec{\vec{R}}}\,\vec{\vec{R}}^T = \vec{\vec{W}}$ (anti-symmetric spin-tensor $\vec{\vec{W}}$) ensures the physically correct stress-update behaviour and avoids erroneous results, which occur when the infinitesimal rotation-tensor $\vec{\vec{R}} \cong \left\{\vec{\vec{I}} + \vec{\vec{W}} \vartriangle t\right\}$ is used in Equation 15. To determine the elasto-viscoplastic stress-increments along the material streamlines, operator splitting concepts (cf. Belytschko et al. [1]) are beneficial. The radial return method (cf. Simo and Hughes [2], Montmitonnet [6]) is based on the application of an elastic predictor, followed by a plastic corrector. Although the real local material rotations inside the strip forming zone (located inside the roll-gap) remain very small for flat rolling, the rotations corresponding to the elastic predictor may become pretty large when plasticity occurs, afterwards, an orthogonal back-projection onto the yield-surface is performed.

$$\begin{aligned} 0 \overset{!!}{=} & \iiint_V tr\left[\vec{\vec{\sigma}}\,\delta\vec{\vec{D}}\right] dV - \iint_{S_C} \left[\delta\left(\sigma_N \mathrm{v}_N\right) + \vec{\sigma}_T \cdot \delta\vec{\mathrm{v}}_T\right] dS \\ & -\left[\bar{\sigma}_F \, A_{out} \, \delta \mathrm{v}_{out} - \bar{\sigma}_B \, A_{in} \, \delta \mathrm{v}_{in}\right] + \left\{\delta\lambda_{OUT}\left[\left(\bar{\sigma}_{xx}\right)_{|x_{out}} - \bar{\sigma}_F\right] + \lambda_{OUT}\,\delta\left(\bar{\sigma}_{xx}\right)_{|x_{out}}\right\} . \end{aligned} \tag{17}$$

An extended variant of the principle of virtual power, Equation 17 serves as underlying weak representation for FE-discretization. For fixed geometry, both the velocity field $\vec{\mathrm{v}}$ as well as the contact stress distribution σ_N (treated as independent Lagrange-parameter-field to ensure the impenetrability condition between strip and work-roll) are determined numerically. Concerning the tangential surface traction vector $\vec{\sigma}_T$ in Equation 17, a velocity-regularized Coulomb frictional law (Kobayashi et al. [15]), in most cases truncated by the shear-yield stress, is employed. The prescribed mean back and front tensile stress values $\bar{\sigma}_B$ and $\bar{\sigma}_F$, respectively, are applied at the strip inlet (A_{in}) and outlet (A_{out}) cross sections far enough outside the roll gap. It turned out to be beneficial to apply an additional stabilization concept to match the prescribed front-tension value $\bar{\sigma}_F$ at the strip exit cross section exactly. This task was accomplished by supplementing an additional Lagrange-parameter λ_{OUT} in Equation 17.

5 CONSISTENT DETERMINATION OF RESIDUAL STRIP STRAINS

A systematic evaluation of the intrinsic (i.e. incompatible) residual strains (and stresses) is performed by employing the logarithmic strain tensor $\vec{\vec{H}}$ (Hencky strain-tensor), which is defined by

$$\vec{\vec{H}} = \ln \vec{\vec{V}} \quad \text{with} \quad \vec{\vec{F}} = \vec{\vec{V}}\,\vec{\vec{R}} \tag{18}$$

(multiplicative decomposition of the deformation gradient $\vec{\vec{F}}$ into a stretch tensor $\vec{\vec{V}}$ and a rotation-tensor $\vec{\vec{R}}$). It can be decomposed exactly into elastic and incompressible plastic contributions even for large strains

$$\vec{\vec{H}} = \left[\vec{\vec{H}}^{(el)} + \vec{\vec{H}}^{(pl)} \right] \quad \text{with} \quad tr\left(\vec{\vec{H}}^{(pl)} \right) = 0 \ . \tag{19}$$

This tensor quantity can be determined by a materially objective streamline update concept similar to that for the Cauchy-stress tensor, as the Jaumann time derivative of the Hencky strain tensor equals the rate of deformation tensor

$$\overset{\odot}{\vec{\vec{H}}} = \vec{\vec{D}} \quad \text{with} \quad \overset{\odot}{\vec{\vec{H}}} \equiv \left\{ \dot{\vec{\vec{H}}} + \vec{\vec{H}}\vec{\vec{W}} - \vec{\vec{W}}\vec{\vec{H}} \right\} \ . \tag{20}$$

For numerical purposes, discrete (i.e. finite) time increments Δt appear. Therefore, it is essential to utilize a streamline-update prescription, exactly fulfilling incremental material objectivity

$$\vec{\vec{H}}_{n+1} \cong \Delta t \vec{\vec{D}}_{n+1} + \exp\left(\vec{\vec{W}}_{n+1} \Delta t \right) \vec{\vec{H}}_n \exp\left(-\vec{\vec{W}}_{n+1} \Delta t \right) \ . \tag{21}$$

The knowledge of the Hencky-strains enables a systematic decomposition of the inhomogeneous deformation during rolling into thickness reduction, longitudinal and lateral contributions, which establishes the material flow basis for the determination of "suitably defined" transfer functions. Taking into account the plastic parts of the logarithmic Hencky-strain distributions at the strip inlet and outlet cross-sections, denoted by upper indices *(IN)* and *(OUT)*, respectively, the deviations from the respective cross-sectional mean values read

$$\delta H_{ij}^{(IN)}(y,z) \equiv \left[H_{ij}^{(pl)}(x = x_{IN}, y, z) - \left\langle H_{ij}^{(pl)} \right\rangle^{(IN)} \right] \tag{22}$$

$$\delta H_{ij}^{(OUT)}(y,z) \equiv \left[H_{ij}^{(pl)}(x = x_{OUT}, y, z) - \left\langle H_{ij}^{(pl)} \right\rangle^{(OUT)} \right] . \tag{23}$$

The strip transfer-distributions (averaged in strip-thickness direction) describing the non-uniform plastic strain-redistributions during a rolling pass can be represented as

Length transfer function:
$$L_T(y) \equiv \left\{ \delta \bar{H}_{xx}^{(OUT)}(y) - \delta \bar{H}_{xx}^{(IN)}(y) \right\} \tag{24}$$

Width transfer function:
$$W_T(y) \equiv \left\{ \delta \bar{H}_{yy}^{(OUT)}(y) - \delta \bar{H}_{yy}^{(IN)}(y) \right\} \tag{25}$$

Thickness transfer function:
$$T_T(y) \equiv \left\{ \delta \bar{H}_{zz}^{(OUT)}(y) - \delta \bar{H}_{zz}^{(IN)}(y) \right\} . \tag{26}$$

Note that due to the incompressibility constraint in Equation 19 one is led to the relation

$$\left[L_T(y) + W_T(y) + T_T(y) \right] \equiv 0 \ , \tag{27}$$

i.e. only two of these three strip transfer-functions are actually independent of each other. Further details concerning a systematic decomposition of the non-uniform thickness reduction across the strip-width into corresponding width- and length-contributions is scheduled for a subsequent paper.

6 SELECTED RESULTS FOR THE COUPLED PROBLEM

By utilizing the iterative coupling concept and formalism as outlined above, essential information concerning the actual elasto-viscoplastic material flow behavior inside the strip can be determined. In this context, special emphasis was put on the convergence properties of the coupled system, which has to be solved iteratively. The initial contact surface chosen here refers to a uniform contact pressure distribution, which suffices to obtain convergency, although the final results deviate considerably from this simple initial choice.

Iteration	*Rolling Force [MN]*	*Spread [mm]*	*C25 [μm]*	*C40 [μm]*	*C150 [μm]*
1	20.646	8.140	281.471	216.515	78.722
2	21.502	6.188	264.367	194.884	67.160
3	21.818	6.367	243.890	165.660	44.577
4	22.272	6.141	227.249	144.718	33.312
5	22.353	6.099	214.098	127.141	20.231
6	22.601	6.011	204.366	115.003	16.643
7	22.631	6.054	198.078	106.360	11.284
8	22.752	6.054	194.893	102.429	11.372
9	22.779	6.083	192.558	99.068	9.314
10	22.837	6.089	191.504	97.845	9.756
11	22.871	6.092	190.385	96.342	8.777
12	22.901	6.097	191.341	97.357	9.765

Table 1: Convergence properties of the iterative coupling loop between strip and elastic roll stack.

After each coupling iteration, a modified contact surface and a corresponding contact stress distribution (i.e. both normal and shear stresses) are determined. By systematic numerical investigations it could be shown that the initially considerable differences between results from consecutive iteration steps decrease very fast, as can be seen exemplarily from Table 1. After about nine to twelve iteration loops both the calculated geometric, velocity- and stress-distributions, and also the corresponding integral properties have converged satisfactorily in most cases considered. Beside the roll separating force, the resulting strip spread after the roll pass and the corresponding absolute strip-crown values Cxx (measured a distance xx away from the strip edge) are summarized in Table 1 for a typical test-case.

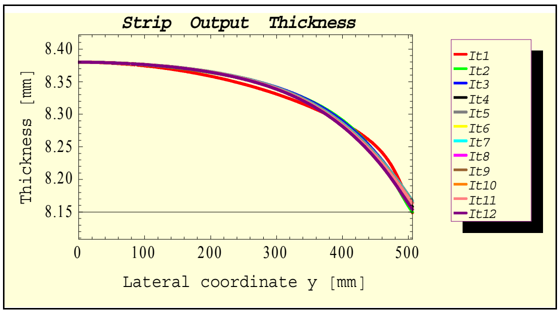

Figure 1: Deformed strip output thickness across the strip width after coupling iterations 1 to 12 (only the upper right quarter is depicted).

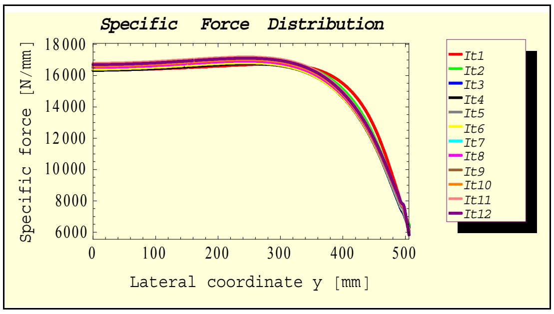

Figure 2: Specific rolling force distribution across the strip width after coupling iterations 1 to 12.

The exemplary results as represented in Figures 1 - 4 refer to a flat hot rolling test-case, where a steel strip with initial width of 1000 mm and rectangular strip entry cross section is reduced from 35 mm to 16.76 mm at the first stand of a hot finishing mill. Note that on account of assumed horizontal and vertical mid-plane symmetry properties only the upper work roll and the upper right half of the strip need to be simulated, i.e. only a quarter of the whole problem is represented here.

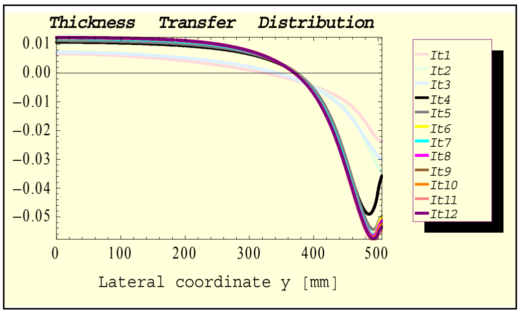

Figure 3: Intrinsic thickness transfer function, as defined by Equation (26).

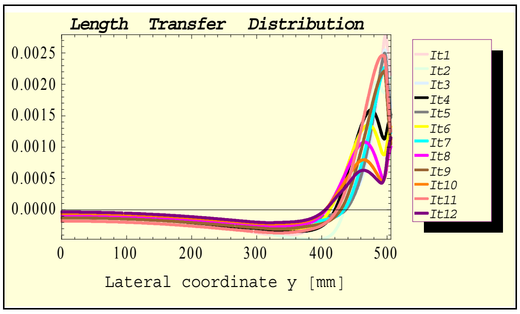

Figure 4: Corresponding length transfer function (i.e. incompatible longitudinal residual strains), as defined by Equation (24).

Of particular interest is the resulting deformed strip thickness distribution in lateral (i.e. strip-width) direction in combination with the specific rolling force distribution, the latter of which follows immediately from the two-dimensional contact stress distribution by integrating it in azimuthal work roll direction. It can be seen from the data depicted in Figure 1 and Figure 2, that the results for the strip output cross-section and the specific rolling force, respectively, meet the expectations and converge satisfactorily. Additionally, the resulting residual thickness- and length-transfer distributions, averaged over the strip thickness and determined by evaluating the logarithmic plastic Hencky strain-tensor (as outlined in section 5) about five contact lengths downstream the roll-gap, and normalized to zero mean-value, are represented in Figures 3 and 4, respectively.

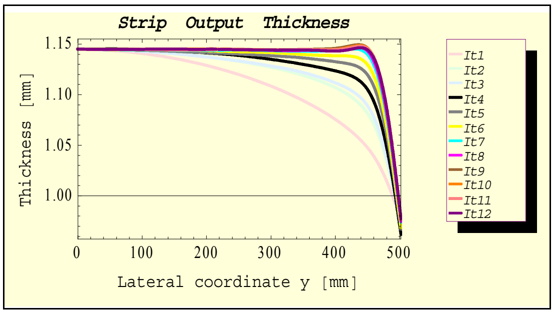

Figure 5: Deformed strip output thickness for test case 2.

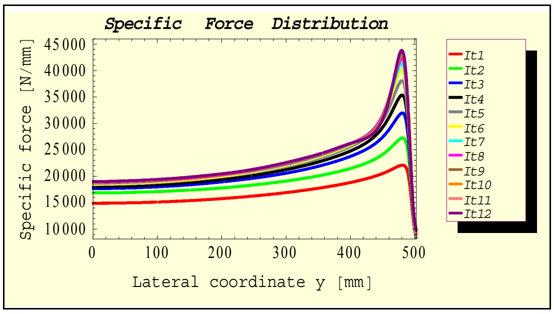

Figure 6: Specific rolling force distribution for test case 2.

The same quantities as in Figures 1 - 4 are depicted in Figures 5 – 8 for an additional finishing mill test case, where the strip is rolled from 3.26 mm to 2.29 mm, i.e. the strip aspect ratio (output width versus thickness) is now significantly higher than in the former test case. Therefore, for test case 2 the lateral material flow inside the roll gap is significantly inhibited, resulting in a drastic increase of the specific rolling force towards the strip edges, as can be seen in Figure 6. Due to the reduced support near the strip edge, the specific force sharply drops only there, which is significantly different from the long range dropping effect, as depicted in Figure 2 for the first test-case.

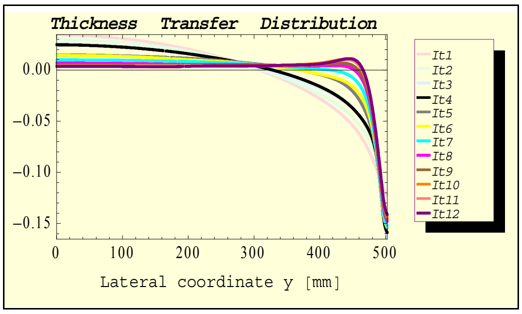

Figure 7: Intrinsic thickness transfer function, as defined by Equation (26), for test case 2.

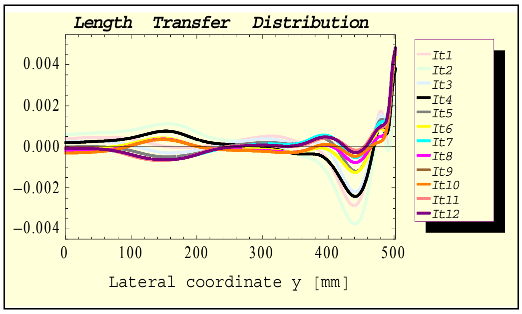

Figure 8: Corresponding length transfer function, as defined by Equation (24), for test case 2.

Note that for regions with high relative thickness reduction increased longitudinal strains are induced (corresponding to compressive residual stresses), as can be seen in Figure 4 and Figure 8. The convergence behaviour of the longitudinal strains is extremely critical due to the high sensitivity with respect to small local changes of geometric properties.

7 CONCLUSIONS

In the present study, a systematic iterative calculation concept was presented to treat the highly non-linear coupling between deformable bodies in metal forming in an efficient and accurate manner. Based on the underlying theoretical modeling concepts, as outlined in this paper, effective numerical simulation models, algorithms and tools were developed and programmed in C++. Special emphasis was put on the coupling of the strip models with the routines for the elastic roll stack deflection, which is a precondition to get reliable results concerning strip profile transfer and residual stress and strain distributions inside the strip. Such data allow the evaluation of strip-flatness and of profile adjustment ranges. The results, attained by utilizing this physically based and mechanically consistent model, were compared to data attained by commercial FEM-calculations (based on standard incremental formulations) and will be validated and calibrated against practical data from an industrial hot finishing mill. Currently, the model is already in practical use by the industrial partners to attain deeper insight into the evolution of profile and flatness in hot rolling processes. Currently, the model is used for systematic parameter studies to investigate flatness properties in more detail and to develop enhanced flatness criteria for thin hot and cold strips as well as hot rolled heavy plates. In future, the model will constitute an essential basis for enhanced metallurgical process investigations.

ACKNOWLEDGEMENT

This research project was carried out within the framework of the “Austrian Center of Competence in Mechatronics” (ACCM), a K2 center of the COMET program, which is carried out by FFG and is supported by the Austrian Federal Government (the Federal Ministry for Transport, Innovation and Technology and the Federal Ministry of Economy, Family and Youth), by the Province of Upper Austria and by the Company Partners and

Scientific Partners of ACCM. The authors gratefully acknowledge the continuous and comprehensive support of voestalpine Stahl, voestalpine Grobblech and of Siemens-VAI Metals Technologies within the frame of this research project.

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