DETERMINING THE CONSTITUTIVE PARAMETERS OF THE HUMAN FEMORAL VEIN IN SPECIFIC PATIENTS

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Abstract. This study suggests a method for computing the constitutive model for veins in vivo from clinically registered ultrasound images. The vein is modelled as a hyperelastic, incompressible, thin-walled cylinder and the membrane stresses are computed using strain energy. The material parameters are determined by tuning the membrane stress to the stress obtained by enforcing global equilibrium.
In addition to the mechanical model, the study also suggests a preconditioning of the pressure-radius signal. The preconditioning computes an average pressure-radius cycle from all consecutive cycles in the registration and removes, or reduces undesirable disturbances. In order to overcome this problem, an approach is proposed that allows constitutive equations to be determined from clinical data by means of reasonable assumptions regarding in situ configurations and stress states of vein walls. The approach is based on a two-dimensional Fung-type stored-energy function that captures the characteristic nonlinear and anisotropic responses of veins.

1 INTRODUCTION

4.5% of the population is at risk of suffering a venous thromboembolism disease, with an approximate mortality rate of 11% ([2, 3]). Our general objective consists of studying a serious pathology that has important consequences: deep vein thrombosis (DVT). The problems involved in modeling venous tissue have been largely ignored by biomechanics researchers, most of whose efforts have instead focused on determining constitutive models of the arterial tissue ([7, 11]). Venous and arterial walls have a similar structure and composition, the main difference between their respective walls being the thickness and fiber orientation of the medial zone.

In this study, we determine a constitutive model of the venous wall tissue in its real life location inside the human body. In the future, we intend to study diseased venous walls and their relation to the origins of DVP. Some studies have modulated the mechanical properties of venous walls ([1, 9]), although these studies have only looked at these properties in laboratory conditions and in non-live tissue.

Constitutive equations can be determined from experimental data regarding the diameter of a vessel segment that is subject to internal pressure and external axial force, and the load-free reference geometry of the vessel segment, including the wall thickness.
In the present study, if the membrane stresses are to be computed, two assumptions need to be made to overcome the limitations of the clinical data ([10]),
(i) The in vivo conditions, the axial stretch of the vessel and the axial external force are constant and independent from internal pressure
(ii) The ratio between the axial and circumferential stress is known at one internal pressure P.

2. METHODS

2.1. Original Data

To carry out the present study, we have used images captured by projecting ultrasound in real time, which is a typical method for clinically registering the pressure and radius. The ultrasound probe is lineal to 4.5 MHz and uses the MyLab Xview 70 high resolution image projection system (Esaote, Genoa, Italy). This new-generation ultrasound tool eliminates particles whilst preserving the information needed for diagnosis. We have used a time sequence of 10 seconds to register in a file of a healthy 40 year-old person. With this observation, we also obtain the internal pressure P.

2.2. Theoretical framework

In general, veins and arteries have similar walls and a structure in three distinct layers: the *intima*, the *media* and the *adventitia*. The media is the middle layer of the vein and consists of a complex three-dimensional network of smooth muscle cells, and elastin and collagen fibrils ([4]). The mechanical properties of venous and arterial walls are different; for example, the pulsating behaviour of the arterial walls is absent from the venous walls. In venous walls the media layer is thinner than the arterial wall. Also, the fibre orientation of venous walls is not clear. Consequently, we use a non-fibre oriented model to approximate the constitutive equation of the venous wall.

In the femoral vein, the variation range of the pressure–inner diameter and the wall thickness at mean pressure is taken from clinical data. A two-dimensional (membrane) model describing the biaxial (i.e. circumferential and axial) response is to be determined. Employing a least-squares approach this can be achieved by minimizing the sum of the squared errors W:

$$W = \sum_i \left[\left(\sigma_{\theta\theta}^{\text{mod}} - \sigma_{\theta\theta} \right)_i^2 + \left(\sigma_{zz}^{\text{mod}} - \sigma_{zz} \right)_i^2 \right] \tag{1}$$

In Eq.(1), the index i denotes the ith of the n data points, $\sigma_{\theta\theta}^{\text{mod}}$ and σ_{zz}^{mod} are the circumferential and axial Cauchy stresses predicted by the model, and $\sigma_{\theta\theta}$ and σ_{zz} are the mean circumferential and axial Cauchy stresses of the wall computed directly from experimental data by enforcing equilibrium.

Following the theory of hyperelasticity ([6]) the principal model stresses:

$$\sigma_{\theta\theta}^{\text{mod}} = \lambda_\theta \frac{\partial \psi}{\partial \lambda_\theta}; \qquad \sigma_{zz}^{\text{mod}} = \lambda_z \frac{\partial \psi}{\partial \lambda_z} \tag{2}$$

may be derived from the two-dimensional SEF $\psi = \psi(\lambda_\theta, \lambda_z)$ and can be expressed in terms of the principal stretches λ_θ and λ_z associated with the circumferential and axial directions, respectively. The circumferential and axial stretches are defined as $\lambda_\theta = d_m / D_m$ and $\lambda_z = z / Z$; whereby d_m and D_m denote the actual and the referential (unloaded) mid-wall diameter of the vessel, and *z* and *Z* denote the actual and the referential length of a vessel segment. Note that Eq. (2) is only valid if the stress tensor and the strain tensor are coaxial. This is the case in the present study, which is restricted to axisymmetric geometry and boundary conditions. For ψ a two-dimensional Fung-type SEF proposed by Von Maltzahn et al.(1984) is used as follows:

$$\psi = \frac{C}{2}(e^Q - 1), \tag{3}$$

where

$$Q = c_{\theta\theta} E_{\theta\theta}^2 + 2c_{\theta z} E_{\theta\theta} E_{zz} + c_{zz} E_{zz}^2 \tag{4}$$

The SEF ψ incorporates four constitutive parameters, C, $c_{\theta\theta}$, $c_{\theta z}$ and c_{zz}. The circumferential and axial Green–Lagrange strains $E_{\theta\theta}$ and E_{zz} can be expressed in terms of the circumferential and axial stretches, that is, $E_{\theta\theta} = \frac{1}{2}(\lambda_\theta^2 - 1)$ and $E_{zz} = \frac{1}{2}(\lambda_z^2 - 1)$, respectively. Fung-type SEFs have been used successfully to model the mechanical responses of numerous veins from various species and anatomical sites. Substituting Eq.(4) for Eq.(2) leads to explicit expressions for $\sigma_{\theta\theta}^{\mathrm{mod}}$ and $\sigma_{zz}^{\mathrm{mod}}$ as functions of the principal stretches λ_θ, λ_z and the constitutive parameters C, $c_{\theta\theta}$, $c_{\theta z}$ and c_{zz}. Note that ψ is convex if and only if $c_{\theta z}^2 < c_{\theta\theta}c_{zz}$, $c_{\theta\theta} > 0$ and $c_{zz} > 0$ (for a derivation see [6]).

The circumferential and axial mean wall stress $\sigma_{\theta\theta}$ and σ_{zz} can be determined by enforcing global equilibrium. Thus,

$$\sigma_{\theta\theta} = \frac{rP}{h}, \; \sigma_{zz} = \frac{r^2 P\pi + F}{h(2r+h)\pi} \tag{5}$$

where h is the actual wall thickness, r is the actual inner radius, P is the transmural pressure and F is the external axial force. Substituting these expressions in $k = \sigma_{zz}/\sigma_{\theta\theta}$ the external axial force F can be determined explicitly as:

$$F = r^2 P\pi(2k-1) + APk \tag{6}$$

where, according to assumption (ii), the stress ratio k is known for a particular pressure P associated with the actual radio r and $A = rh\pi$.

2.3. Calculus

The constitutive parameters C, $c_{\theta\theta}$, $c_{\theta z}$ and c_{zz} have to be determined as variables from a nonlinear zero function in order to determine the minimum error function. We consider the function W as the function of error, we want to obtain the minimum of this function or the zero of this function's derivative. To do so we will use the Levenberg-Marquardt method for the non-linear least squares problems [8]. Thus, we will considerer C, $c_{\theta\theta}$, $c_{\theta z}$ and c_{zz} as variables in the function W, that is $W(C, c_{\theta\theta}, c_{\theta z}, c_{zz})$, so that its minimum will be found in values that verify that $\nabla W(C, c_{\theta\theta}, c_{\theta z}, c_{zz}) = 0$
that is:

$$\begin{aligned}
&\frac{\partial W}{\partial C}(C, c_{\theta\theta}, c_{\theta z}, c_{zz}) = 0 \\
&\frac{\partial W}{\partial c_{\theta\theta}}(C, c_{\theta\theta}, c_{\theta z}, c_{zz}) = 0 \\
&\frac{\partial W}{\partial c_{\theta z}}(C, c_{\theta\theta}, c_{\theta z}, c_{zz}) = 0 \\
&\frac{\partial W}{\partial c_{zz}}(C, c_{\theta\theta}, c_{\theta z}, c_{zz}) = 0
\end{aligned} \tag{7}$$

where $\partial W / \partial x$ are the partial derivative of function W by the variable x. Now we need to apply the Newton-Raphson method to the function ∇W in dimension 4:

$$\nabla W'(X^{(k)})(X^{(k+1)} - X^{(k)}) + \nabla W(X^{(k)}) = 0 \qquad k \geq 1 \tag{8}$$

where $\nabla W'$ is the Jacobian 4×4 matrix. We take $X^{(0)} = (C^0, c^0_{\theta\theta}, c^0_{\theta z}, c^0_{zz})$ where $(C^0, c^0_{\theta\theta}, c^0_{\theta z}, c^0_{zz})$ are the values obtained in [10] for arteries. This is a linear system of equations for $X^{(k+1)}$, and given that $\nabla W'$ is a non-singular matrix, it can be solved using a normal lineal system method.
A simple verification shows the local minimum property of the value obtained (we apply a small perturbation to our final value).

3. RESULTS

We obtain values for the constitutive parameters C, $c_{\theta\theta}$, $c_{\theta z}$ and c_{zz}; and (using the constitutive model) we can obtain information on the unloaded referential geometry (inner diameter *D*) and the in situ boundary force *F* for each observation (see Table 1).

Table 1: Computed inner diameter D; external axial force F; constitutive parameters C, $c_{\theta\theta}$, $c_{\theta z}$ and c_{zz}.

	D(mm)	**F(N)**	**C(kPa)**	$c_{\theta\theta}$	$c_{\theta z}$	c_{zz}
Femoral Vein	8.8	1.14	14.37	2.08	1.39	1.01

With the specified constitutive parameters summarized in Table 1, the SEFs turn out to be convex, which is a crucial property that ensures mechanically and mathematically reliable behaviour. Specific constitutive equations are obtained by substituting the constitutive parameters in Eq.(2). The relations between the pressure and the inner diameter computed from the constitutive models are approximately the same as the observed values (Fig.1). The biaxial response (Fig.2) shows the anisotropy and nonlinearity between the axial and circumferential stretches.

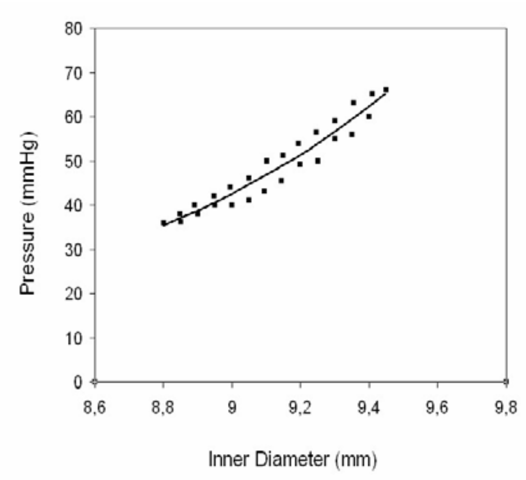

Figure 1: Pressure–inner diameter cycles (marked by squares) from the values for the femoral vein. The solid line indicates the pressure–inner diameter relation predicted by the constitutive model.

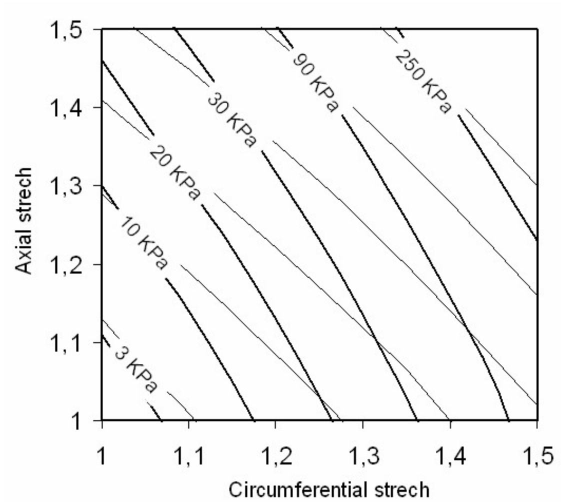

Figure 2: Circumferential (thick lines) and axial (thin lines) Cauchy stress contours in the (mid-wall) stretch plane for the femoral vein from a non pathologic subject.

4. CONCLUSIONS

This is the first attempt to provide constitutive equations for human veins and the femoral vein in particular. The mechanical behaviour of human arteries has already been described, but this is not the case for human veins. Consequently, we have used the results obtained in [9] to develop a new constitutive model for human veins.
In this study we have tried to show a simple method for determining constitutive parameters for the biaxial stretch states of human vein walls (Fig.2) in a specific subject (Fig.1).
The proposed approach is based on providing information about the axial values (axial stretch, external axial force and axial stress) that is not contained in clinical data. Evaluating the predictive capability of constitutive equations requires changes in the boundary conditions, that is, in the pressure and stretches. However, it is possible to alter the boundary conditions in a tolerable way. Obviously, future research in this area could compare several constitutive model approximations in various subjects, each one from a different risk population. Even so, the particular conditions for a specific patient can change; blood pressure can be elevated through exercise or diminished by pharmacological methods. The specificity of this method allows a specific model for each patient.
These models try to characterize the nonlinear anisotropic material responses and the in-situ boundary conditions, because the original data are measurements of stretches in-situ. This fact will promote future research into potential thrombosis risk factors and parameters for monitoring pharmacological therapies, etc. Despite inherent limitations the present approach demonstrates a reasonable way to determine constitutive equations for human veins that would otherwise not be available.

5. ACKNOWLEDGMENTS

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