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AN ANISOTROPIC PSEUDO-ELASTIC MODEL FOR THE MULLINS EFFECT IN ARTERIAL TISSUE

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Summary. This paper is focused on developing the theory which describes the Mullins effect in human arterial tissue. Cyclic uni-axial tensile tests were performed to obtain data characterizing the Mullins effect in arterial tissue.

In order to account anisotropy of arterial tissue, longitudinally as well as circumferentially resected samples of human aorta were tested. Each sample underwent repeated (four times) loading and unloading to a certain value of maximum stretch. This limiting stretch increased in several consecutive steps.

The arterial wall is considered as hyperelastic, locally orthotropic, incompressible material. A strain energy function is adopted in the limiting fiber extensibility form. Description of primary material response, followed by material stress softening in the repeated cycles, is based on pseudo-elastic constitutive model proposed by Dorfmann and Ogden. This theory is developed using anisotropic form of the softening variable. The primary loading curve and the fourth unloading curve of each set of cycles are chosen for regression analysis. The model with thus estimated parameters successfully fits experimental data and is suitable for application in biomedicine.

1 INTRODUCTION

Due to cardiac cycle, arteries are subjected to cyclic loading and unloading in their physiological conditions. In vitro, mechanical response of arteries is mostly realized by cyclic inflation tests and tensile tests. Some irreversible effects are observed during these tests. One of them is known as the Mullins effect (Fig.1). This softening phenomenon is characterized by the following features: when a so called virgin material (previously undeformed) is loaded to a certain value of deformation (under uniaxial tension), stress–stretch curve follows so called primary loading curve (Fig.1 – green curve). Subsequent unloading (Fig.1 – yellow and red curve) exhibits stress softening. Next reloading follows the former unloading curve until the previous maximum stretch is reached. At this moment the loading path starts to trace the primary loading curve.

Purely elastic response of soft tissues is often modeled within the framework of hyperelasticity, see [1, 2] for examples. Concerning with the Mullins effect, soft tissue is most frequently modeled within two conceptions. The first one is based on Continuum Damage Mechanics (CDM). The CDM describes the Mullins effect using a system of internal variables reflecting irreversible effects. See e.g. Peña et al. [3], who considered the internal variables corresponding to separated contribution of the matrix and the fibers in a model of arterial wall.

The second conception results from theory of pseudo-elasticity. Ogden and Roxburgh [4], Beatty and Krisnaswamy [5] and Dorfmann and Ogden [6] formulated pseudo-elastic models of the Mullins effect in rubber-like materials. Such a model describes irreversible behavior incorporating softening variable, which is included into the strain energy density function (SEDF). Peña and Doblaré [7] suggested an anisotropic extension of the pseudo-elastic model of Ogden and Roxburgh [4] with anisotropic form of softening variable. The variable is different for matrix and fibers, which are arranged in two preferred directions. This model successfully described the softening behavior of sheep vena cava under uniaxial tension.

The aim of this paper is to extend the theory of pseudo-elasticity developed by Dorfmann and Ogden [6]. The pseudo-energy function in limiting fiber extensibility form [8] is used. Contrary Dorfmann and Ogden, the anisotropic form of the softening variable is suggested.

2 METHODS

In order to illustrate the Mullins effect in human aorta, cyclic uniaxial tension tests were performed on MTS Mini Bionix testing machine (MTS, Eden Prairie, USA). Two samples of human thoracic aorta were resected from cadaveric donors with the approval of the Ethic Committee of the University Hospital Na Kralovskych Vinohradech in Prague. Respecting anisotropy of the aorta, samples were resected in the circumferential and longitudinal directions. The arteries were stored in physiological solution at a temperature of about 5°C till the beginning of the experiment. Post mortem interval was about 40–48 hours. The temperature during the test was 23°C.

An extension and loading force were measured by MTS testing machine. Five levels of maximum stretch were performed during the tests: $\lambda_m = 1.1$, $\lambda_m = 1.2$, $\lambda_m = 1.3$, $\lambda_m = 1.4$ and $\lambda_m = 1.5$, where λ_m is the maximum ratio between current length l and referential length L. Recorded data are shown in Fig. 1.

Each level represented four-cycle of loading. Considering the incompressibility of the tissue, loading stresses were obtained according to the following relation:

$$\sigma = \frac{F}{s} = \frac{F \cdot l}{L \cdot B \cdot H} \tag{1}$$

where F denotes applied force and s the current cross-section. B and H denote width and thickness of a sample in the reference configuration.

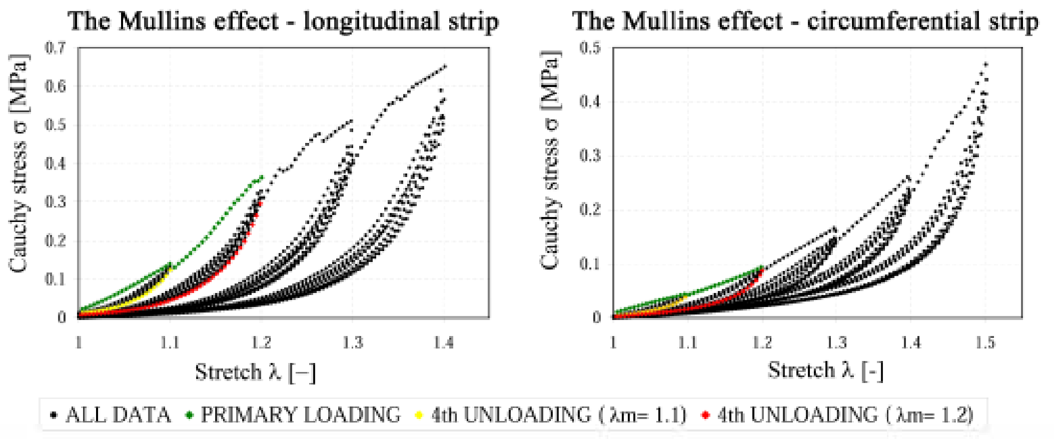

Figure 1: Stress–strain response of the human thoracic aorta under cyclic uniaxial tension . Maximum stretch has increased after 4 cycles due to stabilizing mechanical response of the aorta. Colored points correspond to cycles used within regression analysis

3 MODEL

Primary response of the artery was modeled as an incompressible, hyperelastic, locally orthotropic continuum. Deformation was described with the deformation gradient $\boldsymbol{F}$, which was assumed in the form of:

$$\boldsymbol{F} = diag\left[\lambda_1, \lambda_2, \frac{1}{\lambda_1 \lambda_2}\right] \tag{2}$$

where λ_i are principal stretches. Strain energy density function for incompressible rectangular sample embodied in x_1x_2 plane of Cartesian coordinate system $x_1x_2x_3$, is expressed in form:

$$W_0 = W_{0iso}(I_1) + W_{0aniso}(I_4) \tag{3}$$

which reflects the microstructure of an transversally isotropic material, composed of a ground isotropic matrix and fibrous network. I_i are the principal invariants of the right Cauchy-Green tensor.

SEDF is incorporated in limiting fiber extensibility form as follows [8]:

$$W_0 = \frac{c}{2}\left(\lambda_1^2 + \lambda_2^2 + \frac{1}{\lambda_1^2\lambda_2^2} - 3\right) - \mu J_f \ln\left(1 - \frac{(\lambda_1^2 \cos^2\beta + \lambda_2^2 \sin^2\beta - 1)^2}{J_f^2}\right) \tag{4}$$

where μ and c are stress-like material parameters, J_f +1 is the limiting stretch of the reinforcing fibres, β is an angle of enforcing fibres with coordinate axis x_1.

Let us assume that sample is loaded in the direction of coordinate axis x_α (α = 1,2). Corresponding Cauchy stresses in the direction of (unidirectional) loading are:

$$\sigma_\alpha = \lambda_\alpha \frac{\partial W_0}{\partial \lambda_\alpha} - p_0, \ \alpha = 1, 2, \tag{5}$$

where p_0 denotes a Lagrange multiplier associated with the incompressibility constrain $\lambda_1\lambda_2\lambda_3$=1. The Eq. (5) describes stresses at the "virgin" material (primary loading). Within unloading a stress softening occurs, and stresses should be reduced by a factor η_α:

$$\sigma_\alpha = \eta_\alpha \lambda_\alpha \frac{\partial W_0}{\partial \lambda_\alpha} - p, \ \alpha = 1,2, \tag{6}$$

The softening variable η_α may be active or inactive and this change from inactive to active state is induced when unloading is initiated:

$$\begin{aligned} \lambda_\alpha = \lambda_{\alpha\max} &\rightarrow \eta_\alpha = 1 \\ \lambda_\alpha < \lambda_{\alpha\max} &\rightarrow \eta_\alpha = \eta_\alpha(\lambda_\alpha) \end{aligned} \tag{7}$$

The reduction of stresses (Mullins effect, $\eta_\alpha < 1$) occurs as soon as the actual energy W_0 is less than maximum value $W_{m\alpha}$ attained during the whole previous deformation history. The stress reduction increases with the increasing difference $W_{m\alpha}$ - W_0 and is approximated by the following empirical formula:

$$\eta_\alpha = 1 - \frac{1}{r} f\left(\frac{W_{m\alpha} - W_0(\lambda_1, \lambda_2)}{k_\alpha s}\right) \tag{8}$$

where $f(t)$ can be any monotonically increasing and bounded function, e.g. Error function $Erf(t)$ [4]. Resulting model has 6 parameters: c, μ, β, J_f, r, s that should be identified by experiments.

We suggest material parameter k_α in the form which incorporates material anisotropy, in the meaning of the Young modulus of the material in the initial (virgin) reference configuration. Its advantage is in not increasing number of material parameters:

$$k_\alpha = \left. \frac{\partial^2 W_0(\lambda_1, \lambda_2)}{\partial \lambda_\alpha^2} \right|_{\lambda_1 = \lambda_2 = 1} \tag{9}$$

4 RESULTS

Due to the testing machine design, only displacements in loading directions (α =1 and α=2) were measured. Displacements in the transversal direction were eliminated using the boundary conditions of zero transversal stresses. Parameters p_0 and p were determined from Eq. (5, 6) considering $\sigma_3 = 0$.

Primary loading curve and fourth unloading curve for λ_m = 1.1 and λ_m = 1.2 were included in the regression analysis. Assuming the idealized Mullins effect, following reloading was identified with previous unloading. Primary material responses of loading with λ_m > 1.2 were not considered because of their non-convexity in longitudinal samples (see Fig. 1). Regression analysis was performed using weighted least square method in Maple (Maplesoft, Waterloo, Canada).

Using form of the softening variable expressed by Error function, we estimated parameters J_f, c, μ, $sin^2\beta$, r and s that are summarized in Table 1. Experimental and numerical results for loading and unloading of the thoracic aorta are shown in Fig. 2. Regression results were also checked on the condition I_4 >1. Because I_4 models reinforcement with collagen fibers they may contribute to the stored energy only in tensile strains. It was found that this condition was satisfied in all data points.

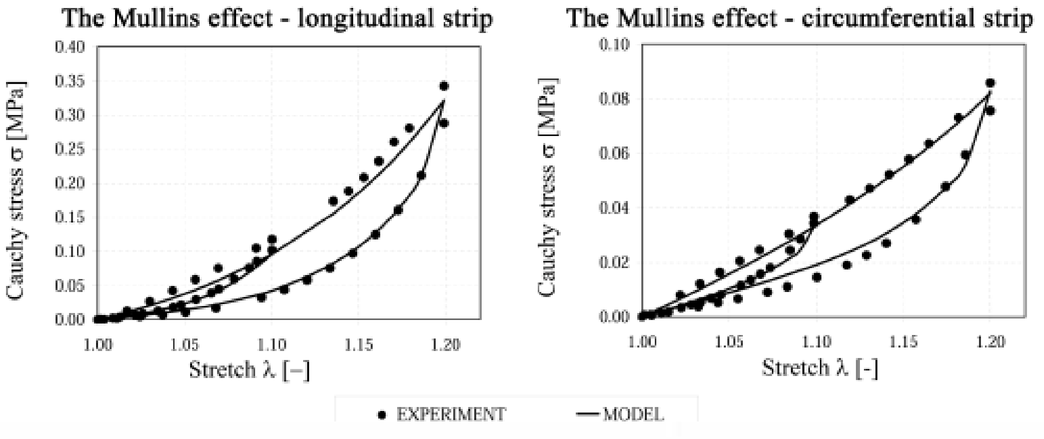

Figure 2: Comparison of the experiment and numerical model of loading and unloading curves in human thoracic aorta with maximum stretches of λ_m = 1.1 and λ_m = 1.2. Numerical simulations have been performed using the pseudo-elastic model with anisotropic form of the softening variable. The softening variable has been designed in form incorporating Error function.

Table 1: Material parameters of the pseudo-elastic model

sample	Material parameter	
Thoracic aorta	J_f [1]	0.0786
	c [Pa]	96401
	μ[Pa]	116744
	$sin^2\beta$ [1]	0.5863
	s[1]	2.47E-5
	r[1]	3.2058

5 DISCUSSION

The strain-induced stress softening in human aorta has been described by means of the stress reduction factor η. Particular mathematical form of η has been adopted from the pseudo-elasticity theory introduced by Dorfmann and Ogden [4] who successfully described the Mullins effect in particle-reinforced rubber. We used anisotropic form of the softening variable in contrast of these authors. It means that model is able to reflect dependence of the stress softening on the direction in which the tension is applied.

The main advantage of the present model is a small number of material parameters. Only 2 of the proposed 6 parameters belong to the pseudo-elastic theory. The anisotropic model suggested by Peña and Doblaré [7] fits the data using 7 pseudo-elastic material parameters.

However present study has some limitations. The first one is due to the limited number of experimental data. Also the design of the experiment does not enable to measure transversal stretches. Finally, the model in the present form is not able to describe permanent strains usually observed during cyclic experiments.

In spite of all these limitations, experimental and numerical simulations show good agreement.

6 CONCLUSIONS

Under cyclic loading conditions, large strain-induced softening (known as the Mullins effect) was observed during uniaxial tension of human thoracic aorta. Purely elastic response of arterial tissue was successfully fitted using SEDF in limiting fiber extensibility form. The Mullins effect was modeled within the theory of pseudo-elasticity. The pseudo-elastic model of Ogden and Roxburgh [4] has been extended by applying anisotropic form of the softening variable. This has been suggested in the form incorporating Error function. The model described experimental data successfully and is applicable to model the Mullins effect in arterial tissue.

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