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# NEURAL NETWORK BASED MATERIAL DESCRIPTION OF UNCURED RUBBER FOR USE IN FINITE ELEMENT SIMULATION

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**Abstract.** The finite element method (FEM) is widely used for structural analysis in engineering. In order to predict the behaviour of structures realistically, it is important to understand and to describe the material behaviour. Therefore, extensive material tests have to be conducted. For highly inelastic materials, such as uncured rubber, the characterisation of the behaviour requires a quite complex rheology. Rheological models are used to describe time-dependent mechanical material behaviour (stress-strain-time dependencies). The mapping of the real material behaviour by such models is only possible with restrictions. However, the evaluation of these models at each integration point within the FEM needs time consuming internal iterations in most cases. In order to describe the material behaviour without model restrictions and to reduce computational cost, the aim of this work is the development of a procedure which enables structural analyses without a specific constitutive material model. In this paper, a neural network is used in order to describe uncured rubber behaviour as a model-free approach.

## 1 INTRODUCTION

Rubber products are essential components of technical systems for example in mobility. Tires, braking systems and engines are not imaginable without rubber parts. Elastomeric material mainly consists of natural or synthetic rubber. Other ingredients are e.g. sulphur, carbon black, softener and antioxidants. All these components influence the properties and the behaviour of rubber material significantly. Rubber material is temperature dependent. For low temperature, the constitutive behaviour is nearly elastic whereas for rising temperature, rubber offers a viscoelastic feature. Irreversible deformation occurs for higher temperature or for large deformations. Beside, rubber material withstands large deformations with little damage effects. It is a thermal and electrical isolator and has a huge damping capacity. Hence, rubber material is a useful and flexible applicable material.

Rubber develops its intended properties after curing under high pressure and temperature. Before the curing process starts, the uncured rubber is formed into a mould in order to obtain the designated shape of the product. Mostly, this forming process of uncured rubber takes place in a curing press. A monitoring of this process is difficult and quite expensive.

Using simulation tools based on the finite element method (FEM) enables a visualisation of such forming processes. In order to predict the behaviour of forming procedures within the FE simulation realistically, it is important to understand and describe the behaviour of green rubber material. Uncured rubber is characterised by elastic, plastic and viscous constitutive response. The constitutive behaviour can be analysed by appropriate material tests. Viscoelasticity is characterised by time- and history-dependent behaviour (see [1, 2, 3, 4]). If a specimen of viscoelastic material is loaded by a constant stress, the strain within the specimen will grown over time (creep). The stress within the same specimen will decrease over time (relaxation) if it is loaded at a constant strain. For a realistic description of the elastomer, extensive compression, tensile and pure shear tests are conducted to generate different states of loading. The viscous effects are characterised by means of relaxation and cyclic tests. Furthermore, the irreversible part of the deformation is determined with the help of a special kind of tensile test.

Usually, specific material models are developed, which are able to represent the constitutive behaviour. The selection and validation of adequate material models as well as the development of new formulations is often time consuming. Highly inelastic materials, such as uncured rubber, are characterised by a quite complex rheology. The time-dependent mechanical behaviour (stress-strain-time dependencies) is described by rheological models. In most cases, these approaches require time consuming internal iterations at each integration point within the FE procedure. After the selection of a suitable model, its parameters have to be identified. The identification is realised by parameter fitting procedures which are in fact an optimisation process.

The selection and development of a material model as well as the identification of material parameters require a lot of computational cost and manpower. Furthermore, the material models are an idealisation of reality and, therefore, this representation is always a reduction of properties and information. Additionally, the computational cost due to internal iteration within the rheology of the material model slow down FE simulations significantly. Hence, the goal of the authors' work is the development of a procedure which enables structural analyses without an explicit constitutive approach. Such a procedure can be used for different types of materials and, hence, the development of different models for different materials is no longer required. In this approach, the material behaviour is described by an artificial neural network (ANN). Commonly, ANN are used for different tasks in engineering [5], e.g. response surface approximation, parameter and system identification. Here, the representation of material behaviour by ANN and the implementation into the FEM are of interest (see e.g. [6]). For the description of time-dependent material behaviour, the authors' decided to use a recurrent neural network (RNN) [7]. The architecture of these ANN enable the consideration of inelastic time-dependent material features. The network parameters are identified using directly data (representing stresses and strains) obtained from tests. One benefit of this approach is the reduction of computational cost, because no internal iteration at the integration points is required. RNN can be adapted to different kinds of constitutive behaviour, which reduces the development time for material representations. Additionally, a higher accuracy is expected because of the direct implementation of material test results into the FE model with no reduction of information.

## 2 EXPERIMENTAL INVESTIGATIONS

The material properties of an uncured rubber can be observed by appropriate material tests. For an intensive study of the following material tests, it is referred to [8]. A realistic description of material properties requires extensive testing. Therefore, tensile, compression, and shear tests are carried out. In Figure 1, the specimens of the different material tests are shown. All types of specimen are punched and cut out of rubber plates

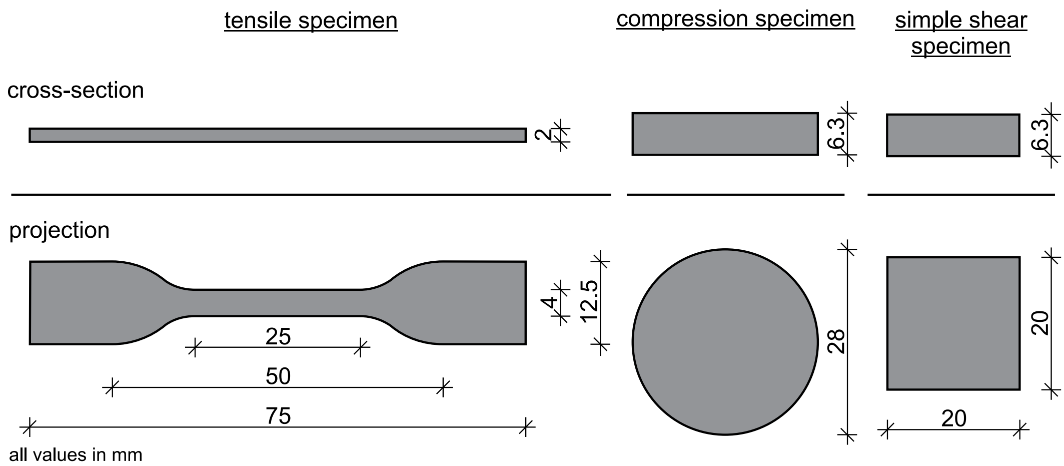


**Figure 1**: Cross-section and projection of the specimens

with a thickness of 2 mm and 6.3 mm, respectively.

The tensile tests are conducted using a standardised specimen [9]. In the middle of this specimen, the cross-sectional area is smaller than at its edge, in order to preserve a uniaxial state of stress. The displacements of the undisturbed and uniform deformation region of the specimen are measured by an external displacement transducer. The reference length $l_0$ of the undisturbed region is 15 mm. The complete test equipment is shown in Figure 2a. In order to obtain the temperature dependency of the material, all tests are pursued at three different temperatures ($T$ = 25°C, 65°C and 105°C). At first, single stage tensile tests are conducted until breaking elongation is achieved. Inelastic properties can be obtained under consideration of cyclic and relaxation tests. Every cyclic test consists of five load cycles. One load cycle consists of a loading phase and a following unloading period. The unloading is completed when the measured force is equal to zero. After every cycle, the strain is increased by 20%. Hence, the maximum strain of the specimen within the cyclic test is 100%. The relaxation tests take place in three load steps. At the beginning of every load step, the strain is increased by 30%. After achieving the current load level, the strain is kept constant for 900 s. Within that time, stress relaxation takes place. Finally, a cyclic test with a holding time after the unloading of every cycle is conducted. The holding time is 240 s. Within this time, the viscoelastic part of deformation is reversed and only the irreversible deformation is stored in the material. Under consideration of these material tests, the irreversible and viscoelastic part of deformation can be quantified.

The compression tests are carried out with cylindrical specimen. The diameter is 28 mm and the total height is 18.9 mm. The contact surface between the specimen and the testing machine is rough. By means of physical and numerical tests, it is shown that within the specimen a uniaxial state of stress exists for a strain up to −40%. The test equipment is presented in Figure 2b. Also for the compression tests, single stage and

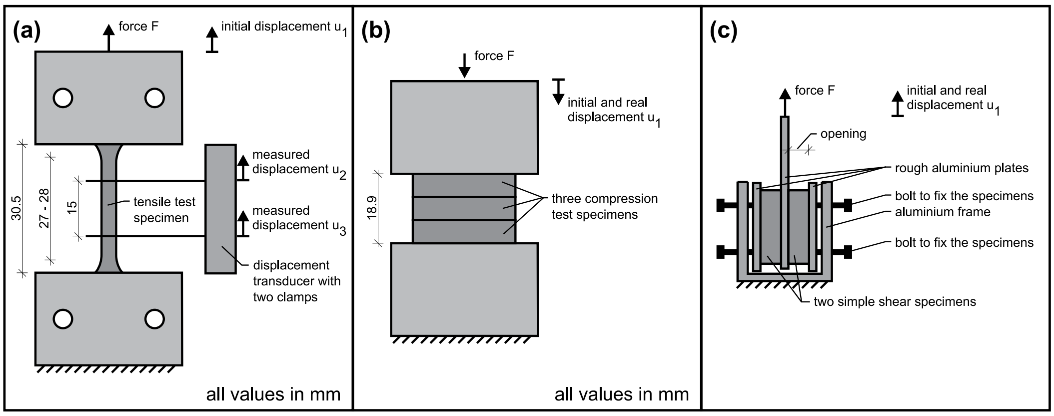


**Figure 2**: Test equipment for: (a) tensile test; (b) compression test; (c) simple shear test

cyclic tests are carried out. The single stage tests have a maximum strain of −75%. The cyclic tests consist of seven load cycles. In every load cycle, the strain increases by −10% and, hence, the maximum strain within this test series is −70%.

Finally, simple shear tests are developed in order to gain information on the uncured rubber material behaviour for a different state of stress. The used specimen is a cuboid with a square base area (20 mm × 20 mm) and a height of 6.3 mm. Two of these specimen are fixed in the frame of the test equipment (compare Figure 2c). Between the two specimen, a rough aluminum plate is placed. The frame is fixed and the aluminum plate is loaded. The distance between the frame and the aluminum plate is defined as the opening. During the different shear tests, the opening has a value of 4 mm and 5 mm, respectively. For this kind of material test, also single stage tests and a cyclic test are carried out. Within the single stage tests, the aluminum plate is driven by 5 mm and the total displacement of the plate within the cyclic test is 5.3 mm. The cyclic test consists of five load cycles and, hence, the displacement of the aluminum plate increases by 1.06 mm for every load cycle.

## 3 MATERIAL DESCRIPTION BY ARTIFICIAL NEURAL NETWORKS

### 3.1 Recurrent neural networks

Recurrent neural networks enable, among others time-dependent approximations of different classes of structural analysis. The computation is time efficient and, hence, it can be used especially for the approximation of time-dependent structural behaviour (see e.g. [10, 11]). Here, a RNN is used instead of a material model in a FEM code.

For the formulation of stress-strain-time dependencies, a network architecture according to [7] is used. These RNN consist of $M$ layers (input, $M-2$ hidden and output layer). Each layer $m$ has a number of neurons, which are linked by synaptic connections to the neurons of the following layer $m+1$. The number of input and output neurons is defined by the number of strain and stress tensor components. For 3D material formulations, six input and six output neurons are required. The experimentally obtained data series are discretised into equidistant time steps $n$. In each time step, the $j = 1, \ldots, 6$ strain components are transferred to input signals ${}^{[n]}x_j^{(1)}$ and the output signals ${}^{[n]}x_k^{(M)}$ of the network define the $k = 1, \ldots, 6$ stress components. The hidden and the output neurons are connected additionally to context neurons in order to capture time-dependent material behaviour. This approach enables to consider all $j = 1, \ldots, 6$ current strain components and the whole strain history for the computation of each stress component of time step $n$.

The signals of RNN are computed layer by layer. The output signal of neuron $k$ in layer $m$ is obtained by

$$ {}^{[n]}x_k^{(m)} = \varphi\left(\sum_{j=1}^{J}\left[{}^{[n]}x_j^{(m-1)}\cdot w_{kj}^{(m)}\right] + \sum_{i=1}^{I}\left[{}^{[n]}y_i^{(m)}\cdot c_{ki}^{(m)}\right] + b_k^{(m)}\right) \; . \tag{1} $$

The argument of the activation function $\varphi$ contains the sum of all input signals ${}^{[n]}x_j^{(m-1)}$ of the previous layer multiplied by the weights $w_{kj}^{(m)}$, the sum of all context signals ${}^{[n]}y_i^{(m)}$

multiplied by the context weights $c_{ki}^{(m)}$ and the bias value $b_k^{(m)}$. Various types of activation functions can be used (see e.g. [12]). In this application, the area hyperbolic sine activates the hidden neurons and a linear function is selected for the output neurons.

The output signals of the context neurons are computed by the previous output signal of the hidden or output neuron $i$ multiplied by the memory factors $\gamma_i^{(m)}$ and the previous context signal ${}^{[n-1]}y_i^{(m)}$ multiplied by the feedback factor $\lambda_i^{(m)}$ summarised by

$$ {}^{[n]}y_i^{(m)} = {}^{[n-1]}x_i^{(m)} \cdot \gamma_i^{(m)} + {}^{[n-1]}y_i^{(m)} \cdot \lambda_i^{(m)} \, . \tag{2} $$

The memory factors and the feedback factors are defined as values of the interval [0, 1]. A feed forward network is obtained as a special case of the presented RNN, if all memory factors are zero.

The analytical determination of the algorithmic tangential stiffness for the element stiffness matrix requires to evaluate the partial derivatives

$$ {}^{[n]}C_{kj} = \frac{z_k^{sc}}{x_j^{sc}} \cdot \frac{\partial^{[n]} x_k^{(M)}}{\partial^{[n]} x_j^{(1)}} \tag{3} $$

of the output signals ${}^{[n]}x_k^{(M)}$ with respect to the input signals ${}^{[n]}x_j^{(1)}$. The parameters $x_j^{sc}$ and $z_k^{sc}$ are scaling factors of the input and output signals, respectively. The partial derivatives are computed by multiple applications of the chain rule (see [12]). They can be computed layer by layer similar to the output signals.

### 3.2 Modification of neural network output quantities for application in FEM

Within the RNN approach, input quantities are mapped onto system responses (output quantities). For the introduced application of the RNN, the elements of the strain tensor (input quantities) are mapped onto the elements of the stress tensor (output quantities). The usage of a RNN instead of a material model requires a modified strain and stress tensor as input and output quantities. This adaptation of the strain and the stress tensor is necessary to ensure an interaction between FEM and RNN. In order to solve the non-linear and inelastic FE problem, an incremental iterative solution is employed.

The used FE code needs the Cauchy stress tensor ${}^{[n]}\underline{\boldsymbol{\sigma}}$. All stress and strain tensors of the following derivation are tensors at the time step $n$. For simplification, the index of the time step ${}^{[n]}\underline{\bullet}$ is omitted in the tensor notation.

In classical continuum mechanics, the Cauchy stress tensor $\underline{\boldsymbol{\sigma}}\,(\underline{\mathbf{b}})$ can be given in dependency on the Finger tensor $\underline{\mathbf{b}} = \underline{\mathbf{F}}\,\underline{\mathbf{F}}^T$. The dilatation $J = \det \underline{\mathbf{F}}$ is an indicator for the compressibility of the material. For $J \approx 1$, the material is nearly incompressible. The dilatation is given in dependency on the deformation gradient $\underline{\mathbf{F}}$. The Cauchy stress tensor $\underline{\boldsymbol{\sigma}}$ is mainly calculated by the Finger tensor $\underline{\mathbf{b}}$ and, therefore, the Finger tensor should be used as input of the RNN and the scaled output signals $z_k^{sc} \cdot {}^{[n]}x_k^{(M)}$ deliver Cauchy stresses $\underline{\boldsymbol{\sigma}}$. In order to preserve the functionality of the RNN, the inputs $x_j^{sc} \cdot {}^{[n]}x_j^{(1)}$ are substituted by the difference of Finger tensor and identity tensor $\underline{\mathbf{b}} - \underline{\mathbf{1}}$.

The tangent moduli

$$\underline{\underline{\mathbb{c}}} = 2\,J^{-1}\underline{\mathbf{b}}\frac{\partial\underline{\boldsymbol{\sigma}}\,J\,\underline{\mathbf{b}}^{-1}}{\partial\underline{\mathbf{b}}}\underline{\mathbf{b}} \tag{4}$$

are defined as the derivatives of the Cauchy stress tensor $\underline{\boldsymbol{\sigma}}$ with respect to the Finger tensor $\underline{\mathbf{b}}$. Finally, the tangent moduli in the current configuration

$$\underline{\underline{\mathbb{c}}} = 2\,J^{-1}\underline{\mathbf{b}}\cdot\left(J\,\underline{\mathbf{b}}^{-1}\cdot\frac{\partial\underline{\boldsymbol{\sigma}}}{\partial\left(\underline{\mathbf{b}}-\underline{\mathbf{1}}\right)}:\underline{\underline{\mathbb{I}}}+\underline{\boldsymbol{\sigma}}\cdot\left[\frac{J}{2}\left(\underline{\mathbf{b}}^{-1}\otimes\underline{\mathbf{b}}^{-1}\right)-J\,\underline{\mathbf{b}}^{-1}\cdot\underline{\underline{\mathbb{I}}}\cdot\underline{\mathbf{b}}^{-1}\right]\right)\cdot\underline{\mathbf{b}} \tag{5}$$

are derived in dependency on the partial derivatives of the network outputs with respect to the network inputs $\partial_{\underline{\mathbf{b}}-\underline{\mathbf{1}}}\,\underline{\boldsymbol{\sigma}}$, the Cauchy stress tensor, the Finger tensor and the forth order identity tensor $\underline{\underline{\mathbb{I}}}$. The derivative $\partial_{\underline{\mathbf{b}}-\underline{\mathbf{1}}}\,\underline{\boldsymbol{\sigma}}$ is equivalent to the tangential stiffness ${}^{[n]}C_{kj}$ of the RNN. The Cauchy stress tensor and the derivatives are obtained from the RNN algorithm. These output quantities can be used directly to calculate the tangent moduli $\underline{\underline{\mathbb{c}}}$ in Eq. (5) which is consistent with the FEM.

### 3.3 Verification of approach by FE simulations

The described approach has to be verified via FE simulations. If the RNN is able to represent the behaviour of a real rubber material, then it has to represent also the behaviour of a non-linear elastic material model. Here, the Yeoh material description [13] is chosen as one typical model. The typical upturn at large strain can be represented by the Yeoh formulation.

Within a training procedure, the unknown network parameters, i.e. the weights, the bias values and the context weights, have to be determined. These unknowns are chosen by the comparison of training data and network responses. The aim of the training is to find network parameters which provide a minimised difference between training data and network response. A more detailed explanation of the training algorithm is given in Section 3.5.

In this example, 2000 different states of strain are randomly found. The corresponding stresses are calculated by the chosen Yeoh material model. Hence, 2000 different states of strain and corresponding stresses define the data set of 2000 strain-stress dependencies. Subsequently, the data set is divided into two parts. 1000 strain-stress dependencies are used for the training and the other 1000 are used in order to validate the quality of the identified network parameters. This kind of validation is a part of the training procedure.

After the implementation of the authors' approach, explained in Section 3.1 and 3.2, two different examples are carried out for verification. The conducted uniaxial tensile and shear test are pictured in Figure 3. The geometry of the uniaxial test sample is a cuboid with a square base area of 4 mm × 4 mm and a height of 20 mm. It is discretised by 40 solid elements with 8-nodes per element. The boundary conditions enable a uniaxial behaviour of the test sample. The load is applied in two steps. In the first step, the top of the test sample is driven by 20 mm in y-direction (see Figure 3) and, in the second step,

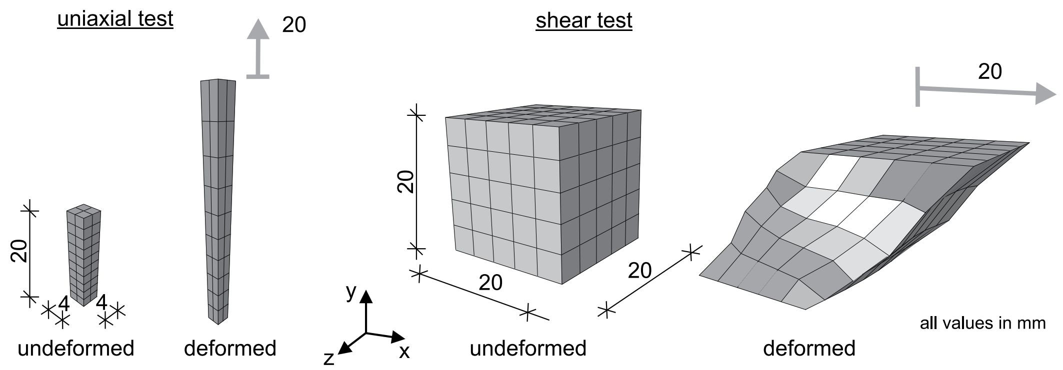


**Figure 3**: FE simulations: uniaxial and shear test

the top is deformed into the initial condition. The geometry of the shear test sample is a cube with an edge length of 20 mm. Here, likewise 8-node solid elements are used for discretisation. The shear test sample consists of 125 elements. The nodes at the top and at the bottom of the shear test sample are fixed in all three directions. The shear test also consists of two load steps. In the first step, the top of the test sample is deformed by 20 mm in x-direction (see Figure 3) and, in the second step, the top is driven back into the initial condition.

In Figure 4, results of the simulation using Yeoh material formulation and the RNN are shown for the tensile and shear test. The pictured stress-time dependencies represent the stresses in the middle of the test samples. For the uniaxial test, the stress $\sigma_{22}$ in loading

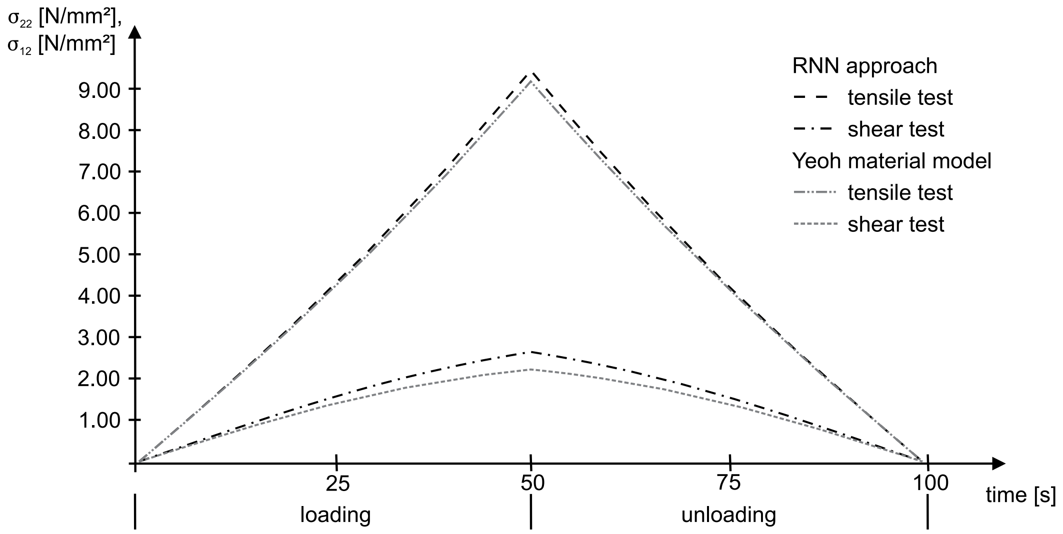


**Figure 4**: Comparison of Yeoh material model and RNN approach

direction is chosen for the comparison of the two approaches. The shear component $\sigma_{12}$

is used to compare both shear test results. The differences between the two formulations are marginal in both cases. The behaviour of the Yeoh model can be represented by the RNN.

The training of the RNN delivers quite good results. The consideration of additional state of strains, especially with a focus on shear strains, even would improve the training and, therefore, the simulation results. For this example, the quality of the RNN is acceptable. It could be shown, that the approach is ready for implementation into a FE code.

### 3.4 Preparation of material test results for training of neural network

The usage of the RNN instead of a material model for a real material requires the training of the network parameters in dependency on the material behaviour. All training data are based on the results of the material tests (compare with Section 2). Therefore, the material test results have to be prepared for training of the neural network.

The testing machines deliver time-dependent displacements $u$ and forces $F$. For all uniaxial tests, the strain $\varepsilon = u/l_0$ is equal to the ratio of displacement and reference length. The stretch $\lambda$ is equal to the sum of 1 and the strain $\varepsilon$. The stress component $P_u$ is defined as the ratio of the force $F$ and the reference cross-sectional area $A_0$ of the specimen. For the interpretation of shear tests, the shear angle and the shear stress are necessary. The required shear angle $\gamma = u/Sb$ is the ratio of the displacement $u$ of the test machine and the opening $Sb$ (see Figure 5). The shear stress $P_s$ is calculated by dividing the measured force $F$ by the square base area (20 mm $\times$ 20 mm) of the specimen.

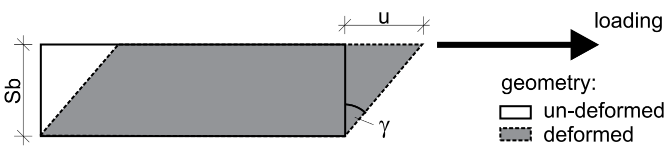


**Figure 5**: Shear angle in dependency of displacement $u$ and opening $Sb$

Uncured rubber material is nearly incompressible. Hence, the deformation gradient is defined as

$$\underline{\mathbf{F}} = \begin{bmatrix} \lambda & \gamma & 0 \\ 0 & \lambda^{-1/2} & 0 \\ 0 & 0 & \lambda^{-1/2} \end{bmatrix}. \tag{6}$$

Based on this deformation gradient, the Finger tensor can be calculated. For the uniaxial tests, the shear angle $\gamma$ is set to zero and for the shear tests, the stretch $\lambda$ is equal 1. The required Cauchy stress tensor $\underline{\boldsymbol{\sigma}} = J^{-1}\,\underline{\mathbf{P}}\,\underline{\mathbf{F}}^T$ is a function of the deformation gradient $\underline{\mathbf{F}}$ and the first Piola-Kirchhoff stress tensor $\underline{\mathbf{P}}$. The component $P_{11}$ of the first Piola-Kirchhoff stress tensor $\underline{\mathbf{P}}$ is equal to the force in uniaxial direction $P_u$ and the component $P_{12}$ is equal to the shear stress $P_s$.

### 3.5 Training and validation results

The determination of the unknown network parameters is achieved by a training procedure. This training procedure is an optimisation approach, comparable to identification tools for material parameter fitting. Therefore, the differences between test results and output values of the RNN are computed for all output neurons. The sum of these differences is equal to the objective function which has to be minimised by the optimisation procedure. Before the optimisation and training, respectively, starts, test data are divided into two parts. One part of data is used for the training and the other data set is used to validate the network quality.

In Figure 6, results of the network training and validation are shown for a uniaxial state of stress at a temperature of 25°C as an example. The stresses $\sigma_{11}$ of the main

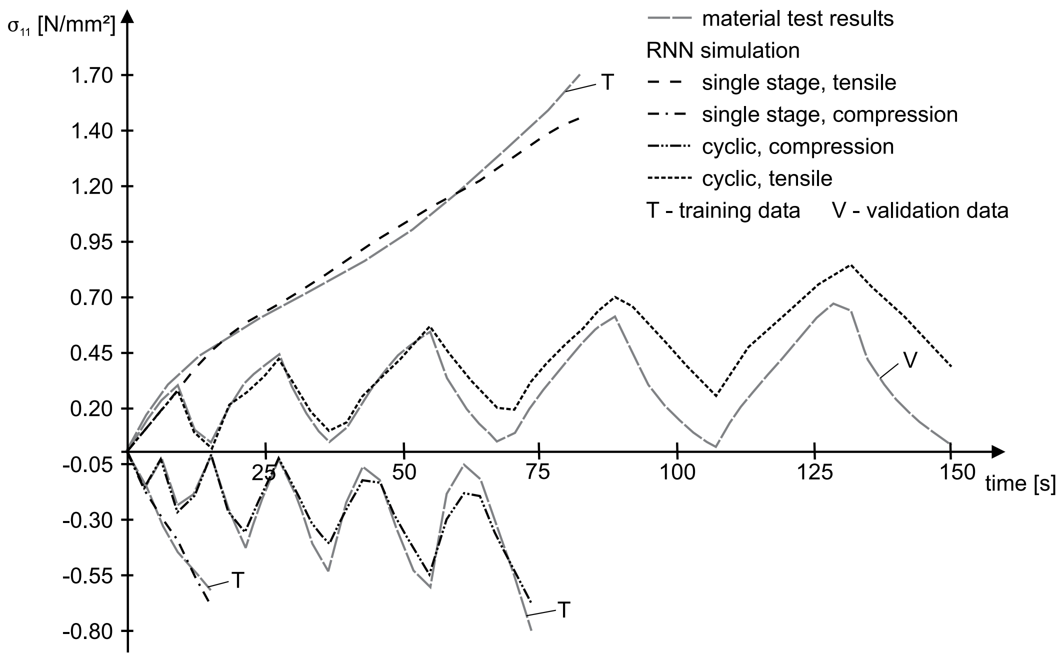


**Figure 6**: Stress-time dependency: material tests, training and validation results

loading direction versus time are plotted for four different tests. One single stage and one cyclic tensile test as well as single stage and one cyclic compression test are used. The cyclic tensile test has been used for validation. For the training, the other three test results are employed. The three tests which have been used for the training deliver quite good results. Especially, the time dependency of the cyclic tests can be represented by the RNN in an adequate way. Whereas the first three cycles of the validation curve match very well, it differs for the last two cycles.

The presented training and validation are first results. The improvement of the training results and the application of the RNN in complex FE simulations will be shown.

## 4 CONCLUSIONS

- The characterisation of material behaviour of uncured rubber requires an intensive study and extensive material tests. For each structural analysis (e.g. FE simulation), a realistic description of the material is fundamental in order to obtain realistic and consistent results.

- The feasibility of the usage of an RNN to substitute a classical continuum mechanical material model within the FEM is shown. A required modification in order to gain consistent tangent moduli and the appropriate preparation of the material test results for the training are explained. The application of the RNN instead of a material model could be shown by first FE simulations.

- Finally, first results of the functionality of the RNN are presented. Training and validation are shown for a representative state of strain.

- The neural network approach can be extended further to consider temperature dependencies of the uncured rubber.

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