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ESTIMATION OF RESPONSE OF STEEL SHEET PLATED WITH THIN HARD LAYER

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Abstract. Elastic and elastic-plastic responses were examined of cantilevers made from a cold rolled steel sheet and made from the same sheet plated with a thin hard layer. Tension test of these sheets showed a non-linear behaviour even in the area of small strain and conventional linear theory of cantilever had to be modified. By extending this theory to a sheet plated with a thin hard layer Young's modulus of plated layer was estimated. The range of estimated Young's modulus was similar to those in previous works but material non-linearity, especially on the compression side, must be measured more precisely.

1 INTRODUCTION

Metallic materials such as steels are deemed to be a linear elastic body at the beginning stage of its deformation and to deform plastically as soon as the stress reaches the intrinsic yield limit of the material [1]. However, precise observation showed that microscopic plastic deformation influences the macroscopic elastic response even after annealing and the elastic stress-strain curve becomes slightly non-linear [2]. Numerical investigation was carried out on this phenomenon by using homogenization method [3]. It is known for a metallic material that the Young's modulus decreases after plastic deformation [4] and for inverse loading this phenomenon is known as the Bauschinger's effect [5]. The response after plastic deformation is seemingly linear elastic but precise observation shows that the response is non-linear [6,7]. In the present work influence of this intrinsic non-linearity of material is evaluated on the response of bulk material with and without a thin hard coated layer taking an example on a bending experiment of a cantilever. Finally the Young's modulus of a thin hard coated layer is estimated on an assumption that the response of base material is non-linear.

2 BASIC EXPERIMENT

2.1 Theory

It is assumed in tension test that specimen of uniform cross sectional area in the axial direction elongates uniformly under the tensile force *F*. When the loading is on the linear elastic stage Young's modulus *E* is the coefficient that links stress σ, which is the force *F* divided by cross sectional area, and strain ε, which is the elongation per unit length in the axial direction. Schematic illustration is given in Figure 1. The stress-strain relationship is written by equation (1).

$$\sigma = E \cdot \varepsilon \qquad (1)$$

If the specimen has a thin plated layer of a different material on the surface the force undertaken by two materials are F_b and F_p as it is illustrated in Figure 1. The distributions of stress for these two cases are illustrated in Figure2. If the materials of plated layer and the base are the same the distribution of stress is uniform in the cross section but if the Young's modulus of the hard layer is higher the stress is higher in the hard layer than in the base.

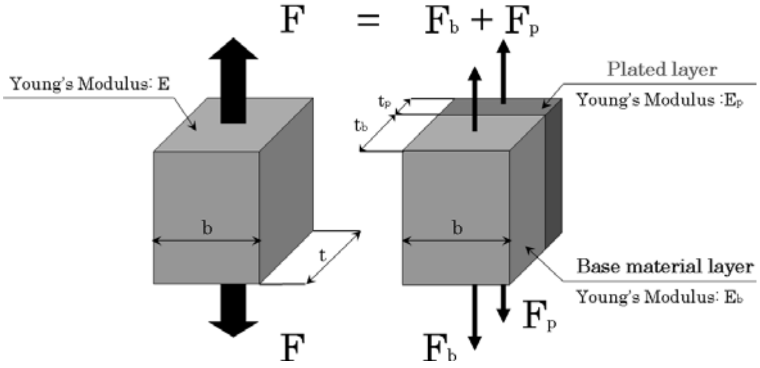


Figure 1: Schematic illustration of responses of specimens with and without thin hard plated layer

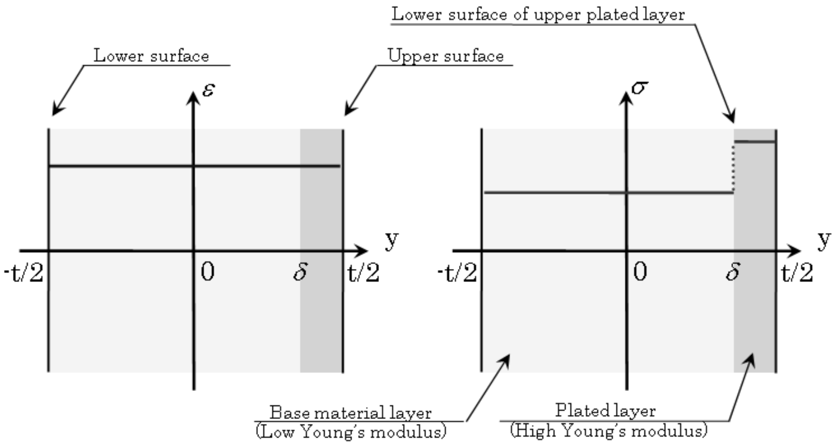


Figure 2: Schematic comparison of distribution of stress in specimens with and without thin plated layer

The average Young's modulus E_a of the whole material with a plated layer can be calculated by equation (2)

$$E_a = (E_b + E_p (t_p / t_b)) / (1 + (t_p / t_b)) \tag{2}$$

where E_b, E_p, t_b, t_p are the Young's module and the thicknesses of the base and the hard plated layer respectively. It is easy to understand that E_a is nearly equal to E_b if t_p is much smaller than t_b, and the value E_a obtained by tension test may give a considerably precise value of E_b regardless of the Young's modulus E_p of the hard plated layer.

2.2 Tension test

Tension tests were carried out on small specimens with and without a thin hard plated layer in a manner shown in Figure3. A couple of strain gauges of which gauge length was 0.3mm were placed at the centre of specimen. The major concern was the elastic response or a nearly elastic response and the tension test was stopped when the stress was estimated to reach the yield limit. The thickness of base sheet was a cold-rolled steel sheet with 0.8mm in thickness and the thickness of plated CrN layer was 2μm, and the value of t_p / t_b is 0.0025, which is small enough to grant that Ea is nearly equal to E_b in equation (2). The results of base sheet and plated sheet are shown in Figure 4.

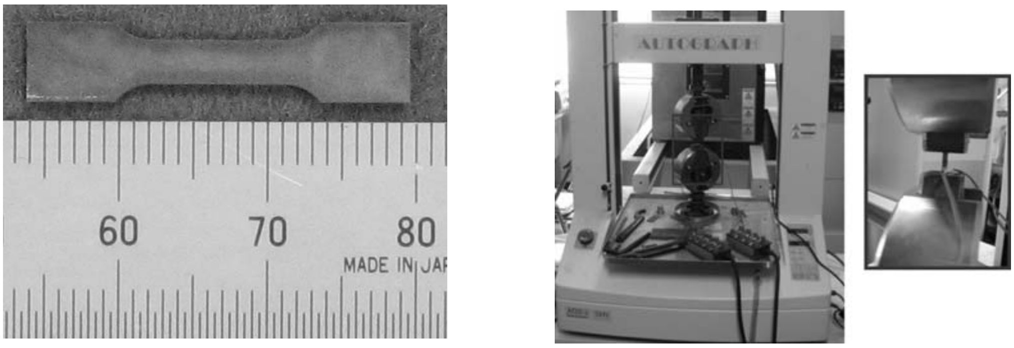


Figure 3: Specimen geometry and view of tension test

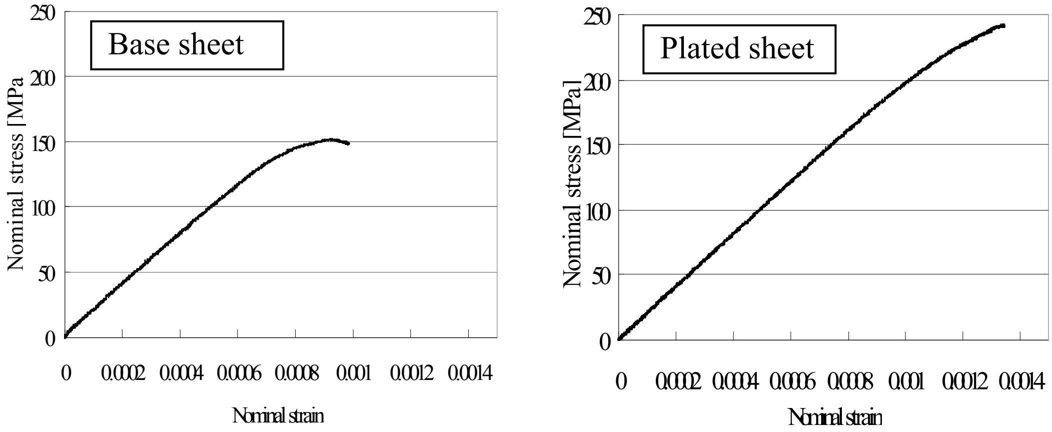


Figure 4: Stress-strain curves of specimens with and without thin hard plated layer

The stress-strain curve of the base sheet in Figure 4 can be deemed linear but detailed observation shows that it is non-linear. Piecewise measurement of the tangent of this curve showed that the tangent gradually decreases according to the increase in strain. The result is indicated in Table 1. After annealing treatment the Young's modulus of steel may lie usually between 200 and 210 MPa and the tangent for the lowest strain range in Table 1 may be an appropriate value. It is known that the Young's modulus decreases according to the increase in plastic strain [4,5] and the tendency of the change in the tangent value in Table 1 suggests that similar phenomenon can be observed depending upon the range of strain adopted for calculating the Young's modulus.

Table 1: Piecewise tangent values of stress-strain curve of base sheet

Range of strain	0.0000-0.0002	0.0002-0.0004	0.0004-0.0006	0.0006-0.0008
Tangent (MPa)	207	189	180	144

The response of the base sheet is not that of linear elastic material but it is non-linear when a wide range of strain is taken. It is only a phenomenological approach but it is curious to note that the stress-strain relationship is well expressed by equation (3)

$$\sigma = a \ (\varepsilon)^2 + b \ \varepsilon \tag{3}$$

where σ and ε are stress and strain, and *a* and *b* are coefficients [7]. Examples of the values of *a* and *b* are shown by equations (4) and (5) for a case such that the range of strain is 0.0 and 0.0008. The maximum difference between the measured stress and the calculated stress by equation (3) was 1 MPa and the approximation may be fairly good.

$$a = -4.48 \times 10^{7} \tag{4}$$

$$b = 2.16 \times 10^{5} \tag{5}$$

The base sheet can be granted a non-linear elastic material, but it can be granted an elastic-plastic material as it was suggested by using the homogenization method [8]. The base sheet can be granted this kind of material, i.e. linear elastic material under the strain of 0.0002 and plastically deforms after that with an extremely high work-hardening ratio until the value of strain reaches 0.0012. Linear elastic approach may be useful when one simulates the behaviour of base sheet under loading, but elastic-plastic approach may be useful depending upon the problem that one simulates.

Compared to the stress-strain curve for base sheet the curve for a plated sheet seems to be straight in Figure 4. The piecewise tangent values of this curve is indicated in Table 2. The plated sheet shows harder response than the base sheet. It is assumed that the plating operation might have given heat to the base sheet and the heat affected the recovery of Young's modulus [4]. It depends upon the accuracy required on the predicted result but it can be possible to grant the plated sheet a linear elastic body.

Table 2: Piecewise tangent values of stress-strain curve of plated sheet

Range of strain	0.0000-0.0002	0.0002-0.0004	0.0004-0.0006	0.0006-0.0008
Tangent (MPa)	209	199	199	196

3 THEORY OF BENDING

3.1 Linear elastic body

In the conventional theory of elastic bending of a cantilever strain is defined by equation (6) [9], where y, η and ρ are coordinate value in the thickness direction, the coordinate value of neutral plane and the radius of curvature of the neutral plane respectively. If the cantilever is a linear elastic body the axial stress σ is related to strain ε as given by equation (7) via the Young's modulus *E*.

$$\varepsilon = (y - \eta)/\rho \tag{6}$$

$$\sigma = E \cdot \varepsilon \tag{7}$$

By solving two equations (8) and (9) for the equilibriums of axial force and moment in a cross section the values of η and ρ are determined, where h, *A* and *w* are half thickness, cross sectional area and width of cantilever, and *M* is the moment by the external force exerting at the top of cantilever. The deflection of cantilever the can be calculated by using the values of *E*, η and ρ.

$$\int_{-h}^{h} \sigma \, dA = \int_{-h}^{h} (E \cdot (y - \eta)/\rho)\, w\, dy = 0 \tag{8}$$

$$\int_{-h}^{h} \sigma \cdot (y-\eta) \, dA = \int_{-h}^{h} (E \cdot (y-\eta)^2/\rho)\, w\, dy = M \tag{9}$$

The moment is calculated by equation (10) where *F*, *L* and *x* are the external force exerting on the free end of cantilever, the length of cantilever and the axial position of cross section after taking the origin at the fixed end of cantilever respectively.

$$M = F \cdot (L - x) \tag{10}$$

For a plated sheet equations (8) and (9) are modified as shown in (11) and (12), where *k*, *Eb* and *Ep* are the y-coordinate value of the boundary of the base and the plated layer and the elastic module of the base and the plated layer. In a manner similar to that for a linear elastic material the deflection is calculated.

$$\int_{-h}^{k} (E_b \cdot (y - \eta)/\rho) \cdot w\, dy + \int_{k}^{h} (E_p \cdot (y - \eta)/\rho) \cdot w\, dy = 0 \tag{11}$$

$$\int_{-h}^{k} (E_b \cdot (y - \eta)^2/\rho) \cdot w\, dy + \int_{k}^{h} (E_p \cdot (y - \eta)^2/\rho) \cdot w\, dy = M \tag{12}$$

When the Young's modulus E_p of the plated layer is unknown and that of the base sheet E_b is known, it is possible to determine the value by carrying out a bending experiment to measure the deflection of cantilever.

3.2 Non-linear body

If the base material reveals a non-linear behaviour described by equation (3), the equilibrium equations of axial force and moment corresponding to equations (8) and (9) are given by equations (13) and (14). What is important here is that it is not specified whether the material is an elastic body or not. Similarly to the elastic material η and ρ are determined by solving these two equations, and the deflection of cantilever is calculated.

$$\int_{-h}^{h} (a \cdot (y-\eta)^2/\rho^2 + b \cdot (y-\eta)/\rho)\, w\, dy = 0 \tag{13}$$

$$\int_{-h}^{h} (a \cdot (y-\eta)^3/\rho^2 + b \cdot (y-\eta)^2/\rho)\, w\, dy = M \tag{14}$$

4 BENDING TEST

Bending tests of a cantilever was carried out by using the base sheet of which thickness was 0.8mm and the sheet with a plated layer of CrN of which thickness was 2 μ m. The size of specimen was 5mm in width and 150mm in length. Two pieces of cantilever of the same dimensions were sectioned from the parent sheet to check the repeatability. End portion of 20mm in length was sandwiched in between heavy steel blocks and weight was loaded on the other end in an incremental manner. The view of experiment is shown in Figure 5.

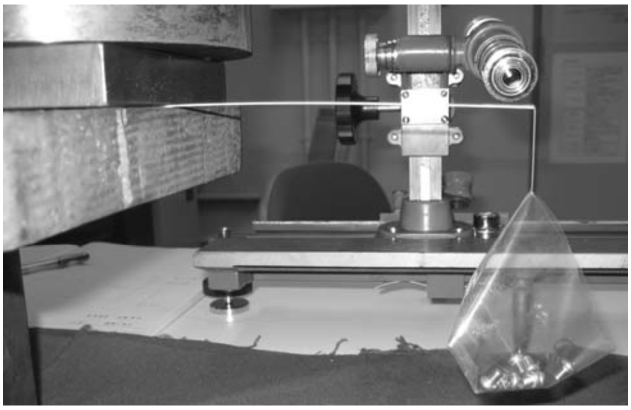


Figure 5: View of bending experiment of cantilever

5 RESULTS

The deflection of cantilever is plotted against the weight in Figure 6. Good repeatability is observed between the two cantilevers and between the cases when the front side of the sheet was used upward and when the back side was used upward. If the basic equations (12) and (13) of linear elasticity are used and the Young’s modulus is 209 GPa in Table 2 the maximum deflection of the top end must be 6.2 mm which is much less than the measured value in Figure 6.

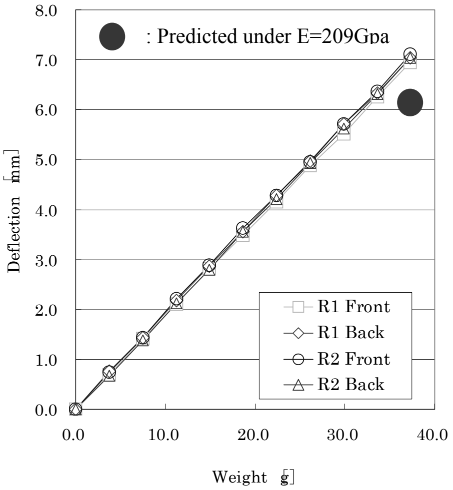


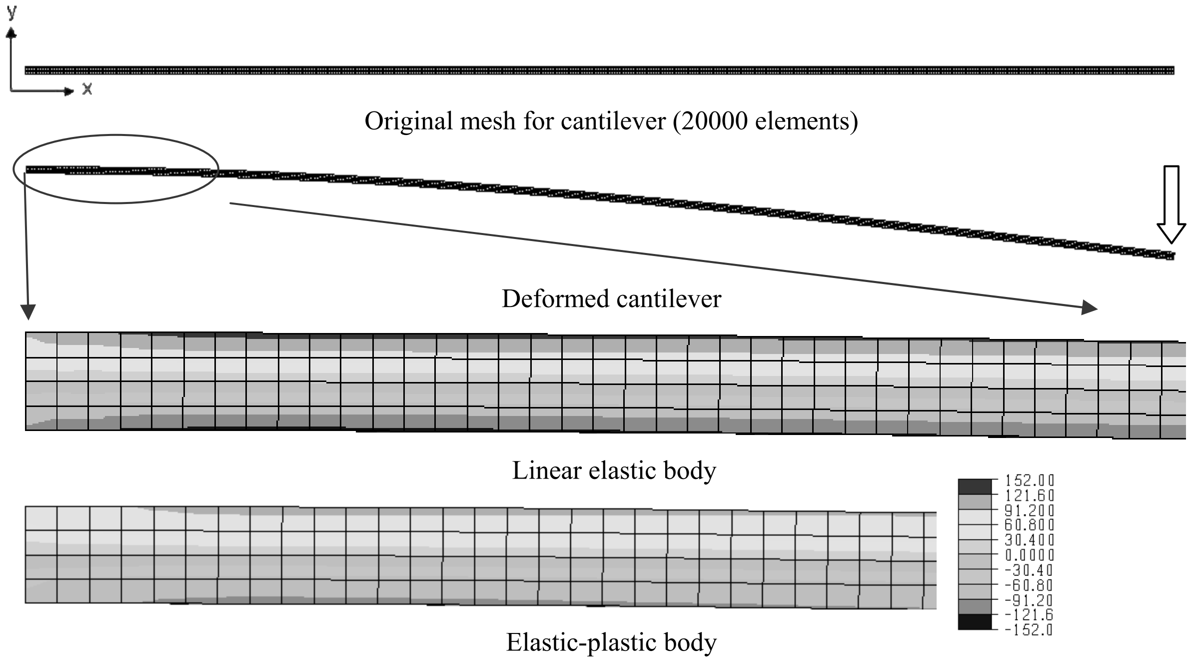
Figure 6: Responses of parent sheet

It is shown in the references [4, 5, 6] the Young's modulus of a plastically deformed material on the reverse loading side is smaller. If smaller value of Young's modulus is adopted on the compression side the discrepancy between the predicted and the measured deflections becomes smaller. The value of Young's modulus on the compression side may be slightly below 160GPa, but it is recommended to measure it by experiment.

For the sheet with a thin hard layer of CrN bending experiment was carried out in the same manner. The measured Young's modulus was 209 GPa as it was shown in Table 2 and the predicted deflection of the top end of cantilever never meets the measured deflection when the maximum weight of 37.3g was applied. If a value about 160 GPa is given to the Young's modulus on the compression side of the base steel of the plated sheet the estimated Young's modulus may lie within a range of 300 and 650 GPa. In order to determine the precise value of the Young's modulus of thin layer of CrN it is necessary to carry out a compression test of plated sheet to know the Young's modulus of the plated sheet.

6 DISCUSSIONS

If the material shows a non-linear behaviour use of equations (3), (4), (5), (13) and (14) may be recommended, but in the present work elastic-plastic FEA was carried out on the bending of a cantilever of base sheet assuming that the material behaves elastically when the strain is under 0.0002 and plastically deforms after that as it was indicated in Table 1. The results are shown in Figure 7. The software used for the analyses was ELFEN [10] developed at University of Wales. Basically the stress-strain curves on tension and compression sides are regarded as point symmetry and the Young's modulus has the same values both on the tension and compression sides. The difference in the deflection at the top was 14% smaller for linear elastic body than elastic-plastic body, but the difference was still large. Measurement of compressive S-S curve is necessary.



Original mesh for cantilever (20000 elements)

Deformed cantilever

Linear elastic body

Elastic-plastic body

Figure 7: Comparison of axial stress levels in response of bending of base sheet

7 CONCLUSIONS

The stress-strain curve of a cold-rolled steel sheet was measured by a tension test and the response of cantilever made of the sheet was predicted assuming the sheet as linear elastic and elastic-plastic bodies. Assumption of linear elastic body gave smaller deflection at the top of the cantilever than that of the experimental result. Even when the deflection is predicted on the assumption such that the sheet is an elastic-plastic body with extremely high work-hardening ratio the difference was still large. It was assumed that the key to the precision of prediction was the measurement of the Young's modulus or the stress-strain curve on the compression side. When a smaller value of Young's modulus is given on the compression side than the tension side the predicted value came closer to the measured value. By using this technique for predicting the deflection the Young's modulus of a thin plated layer of CrN on the steel sheet was estimated to give a moderate value.

NOTE

Some part of this research work was carried out by Mr. Tetsuya Yamamoto, the second author when he was a student in Kyoto Institute of Technology and this paper has nothing to do with his present work at his present affiliation Canon Machinery Inc..

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