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LIMIT ANALYSIS FOR POROUS MATERIALS APPLIED FOR SLOPE STABILITY ANALYSIS

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Abstract. Slope stability analysis is a classical geotechnical problem. It aims to characterize the potential failure surface and the slope safety factor for a given geometry and loading conditions. From a mechanical point of view the analysis is carried out to check the condition which soil structures failure. Failure condition can be analyzed by analytical, experimental and numerical methods. Griffiths and Lane point out some methods used in the literature as the equilibrium method, modified Bishops method, Morgenstern and Price among others.The objective of this work is to evaluate the slope stability problem, considering the soil as a porous media obeying the yield function proposed by Ulm, Cariou and Gathier.

1 INTRODUCTION

Slopes are any soil inclined surface made of earth, rock or a mix of both materials. Due to overloads, landslides may occur and the determination of the limit load of the slope is a key point in order to evaluate its occurrence.

This problem is analyzed under plasticity theory, since plastic flow occurs and the part of soil mass slides in relation to the fixed one by shearing. Some analytical solutions are proposed as in [13] and [16]. It is common to evaluate this problem by limit equilibrium method, where a failure surface is proposed: they may be a straight surface failure (Culmann Method), a circular surface (Swedish circle method) or a Log-spiral surface. Furthermore, a slice method is also applied in stability of slopes studies and it is based on assumption that the failure soil mass is divided into slices and the equilibrium requirements must be satisfied. In [13], the limit analysis approach is presented under assumption of these failure surface shapes.

Meanwhile, the shape of the sliding surface is taken as an hypothesis and some errors may be induced. Therefore, the finite element method and limit analysis method are important tools in order to analyze such problems and among other results, the slip lines are determined.

Another issue concerns the material modeling: soil are heterogeneous materials and are composed of many particles and off course, porous. Traditional yield criteria such Mohr-Coulomb and Drucker-Prager functions take into account soil friction angle and cohesion. Recent works of Ulm, Cariou and Gathier [7, 8] have inferred the influence of porosity on yield function. This Ulm-Gathier function embodies properties that are dependent of porosity, friction angle and cohesion.

In order to solve the stability problem, the body is divided into 2-D finite elements applied into limit analysis method. Under plane strain hypothesis, the Ulm-Gathier yield function is applied to the model. The finite element/limit analysis results will be compared to those in [13] and [16].

Slope stability simulation can also be performed by finite element technique using incremental plasticity [14]. In this paper some simulations were carried out using Drucker Prager model for incremental plasticity through the computational code CodeBright [15]. Then, this problem is modeled under plasticity theory, where the limit load is determined by limit analysis method and compared with the results achieved by incremental analysis.

2 SLOPES STABILITY

This problem deals with the determination of a collapse factor and the critical height of soil. As external load, in this model is considered only the self-weight.

The stability of slopes involves plasticity theory since the concernment is the determination of limit load, under its own weight, in order to avoid failure. In this context, failure is the sliding of a part of soil relative to another that remains fixed. Then, this problem is modeled under plasticity theory, where the slope limit load is determined by limit analysis method or incremental analysis. Schematically, Figure 1 shows a slope:

The nondimensional number called Stability number found in [13] and [16] relates the load due to soil weight and the soil cohesion, defined as in Equation (1)

$$N = \alpha \frac{\gamma H}{c} \tag{1}$$

where: α is the collapse load, γ is the soil specific weight, H is the height of the slope and c is the soil cohesion.

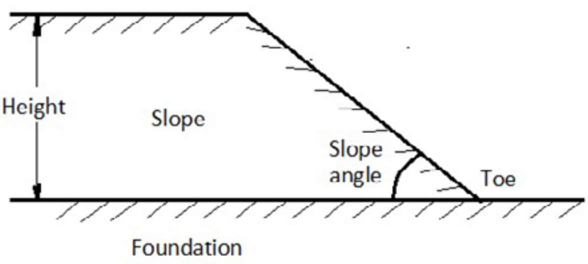

Figure 1: Slope e some terminology.

3 LIMIT ANALYSIS THEORY

Limit analysis method aims the determination of the loads that will cause the phenomenon of incipient plastic collapse, in a body made of elastic ideally plastic material as seen in [12],[1] and [6]. The body is submitted to a load reference F and limit analysis consists in finding the load factor α that will amplify this reference load F and cause plastic collapse. This phenomenon is also characterized by finding an admissible stress field T in equilibrium with external proportional loads, a kinematically admissible velocity field v and the strain rates D^p and the plastic multiplier λ.

Thus, the limit analysis problem consists in finding $\alpha \in I\!R$, a stress field $T \in W'$, a plastic strain rate field $D^p \in W$ and a velocity field $v \in V$ such that

$$D^p = \mathcal{D}v \qquad v \in V \tag{2}$$

$$T \in S(\alpha F) \tag{3}$$

$$T \in \partial X(D^p) \tag{4}$$

The meaning of these relations and the notation are explained in the following. Equation (2) imposes that the collapse plastic strain rate D^p is related to a kinematically admissible velocity field v by means of the tangent deformation operator $\mathcal{D}$ (V and W are the space of admissible velocities and strain rate fields).

The equation (3) expresses the equilibrion condition

$$\langle T, D\rangle = \alpha\langle F, v\rangle \tag{5}$$

where T is the statically admissible stress field in equilibrium with the proportional load. That is, considering a body that occupies a region $\mathcal{B}$ with regular boundary Γ and let V the function space of all admissible velocity fields v complying with homogeneous boundary conditions prescribed on Γ_u of Γ. The strain rate tensor denoted by D relates with v by a linear operator and the duality product between stress fields T and strain rate D belonging respectively from spaces $W^{'}$ and W is written as:

$$\langle T, D \rangle = \int_{\mathcal{B}} T.D \, d\mathcal{B} \tag{6}$$

The load system is represented by an element F from space V', dual of V. The duality product is denoted as:

$$\langle F, v \rangle = \int_{\mathcal{B}} b.v \, d\mathcal{B} + \int_{\Gamma_t} \tau.v \, d\Gamma \tag{7}$$

where b and τ are body and surface forces respectively, Γ_t is a part of boundary Γ where external loads are prescribed.

Aditionally, the stress field T is constrained to fulfill the plastic admissibility condition, belonging to the set P defined as:

$$P = \{T \in W' | f(T) \leq 0\} \tag{8}$$

where the inequality above is then understood as imposing that each component f_k, which is a regular convex function of T, is non-positive. Then, at any point of $\mathcal{B}$, equation (4) is equivalent to the normality rule $D^p = \nabla f(T) \, \dot{\lambda}$, where $\nabla f(T)$ denotes the gradient of f, and $\dot{\lambda}$ is the $\widehat{m}-$vector field of plastic multipliers. At any point of $\mathcal{B}$, the components of $\dot{\lambda}$ are related to each plastic mode in f by the complementarity condition $\dot{\lambda} \geq 0$, $f \leq 0$ and $f \cdot \dot{\lambda} = 0$ (these inequalities hold componentwise).

The classical extremum principles of limit analysis, that is the kinematical, statical and mixed formulations, can be derived from the optimality conditions (2-4) [11, 12, 1]. The discretized versions of these formulations lead to a single type of finite dimensional problem, which can be cast in four strictly equivalent forms, namely the statical, mixed and kinematical discrete formulations, and the set of discrete optimality conditions.

4 YIELD FUNCTION FOR POROUS MATERIALS

As it is well known, soils are biphasic materials composed by porous and solid particles. Porous, eventually, may be totally or partially filled with water. In these conditions there are a large number of possible loading and boundary conditions in geotechnical problems.

From a microscopic point of view, soils are composed of a mix of lots of different particles, creating a very heterogeneous material. Because of its heterogeneity and presence of porous, the determination of mechanical properties of such materials becomes a difficult task. In order to solve this, the development of a predictive model to determine the strength of porous materials will make extensive use of the theory of strength homogenization. Recent advances on homogenization techniques are found in [7].

Porous materials with a dominating matrix-pore inclusion morphology are well represented by the Mori-Tanaka and Self-Consistent schemes, as seen in [7] and [8]. In Mori-Tanaka scheme some material parameters such as $\alpha_d, \sigma_0, \alpha_m$ are calculated and they include the soil cohesion, porosity and friction angle effects. The called Ulm-Gathier yield function is written in Equation (9):

$$F(\mathbf{\Sigma_d}, \Sigma_m) = \left(\frac{\Sigma_m + \sigma_0}{\alpha_m}\right)^2 + \left(\frac{J_2}{\alpha_d}\right)^2 - 1 \tag{9}$$

where $\alpha_d, \sigma_0, \alpha_m$ are material parameters and calculated as seen in [7] and [8], the second invariant J_2 is associated to deviatory stress and Σ_m is mean stress.

Depending on density packing value η defined in Equation (10), the Uln-Gathier yield function may assume and elliptical or an hyperbolical shape:

$$\eta = \frac{V_s}{V_t} \tag{10}$$

where V_s is the solid volume and V_t is the material total volume, including pores.

Mori-Tanaka morphology is applied in this work and according to it, there is a density packing critical value that defines two distinct regimes: below the critical value, the yield function assumes the elliptical shape; otherwise, it assumes the hyperbolical shape. This critical density packing η_{crit} is function of friction angle α_s and calculated as in Equation (11):

$$\eta_{crit} = 1 - \frac{4\alpha_s^2}{3} \tag{11}$$

Moreover, under certain special conditions, the Ulm-Gathier function may reach asymptotically either to von Mises or Drucker-Prager criteria: von Mises criterion is obtained if friction angle $\alpha_s \to 0$ and packing density $\eta \to 1$; however, if $\alpha_s \neq 0$ and $\eta = 1$, Drucker-Prager criterion is obtained.

Figure 2 shows an example of Ulm-Gathier function, plotting J_2 (deviatory) versus σ_m (mean stress) for a friction angle $\alpha_s = 0.4$. In this case, $\eta_{crit} = 0.786$ and it means that any packing density below this critical value, the yield surface has an elliptical shape and otherwise, the criterion is hyperbolical. It is also observed that when $\eta \to 1$ the cone-shaped Drucker-Prager criterion is reached. If von Mises citerion were represented, it would be an horizontal line parallel to mean stress axis since Mises is independent of the mean stress, as expected.

5 RESULTS

Some examples of slope stability analysis using limit analysis and finite elements are presented. At first, the limit analysis/fem solution is compared to classical solutions found in [13] and [16]. These classical solutions consider Drucker-Prager yield criteria and the Ulm-Gathier yield function is made asymptotically to this function, establishing that $\eta = 1$, ie, there is no porosity. Simulations using incremental plasticity were performed with CodeBright code for the slopes with 60^o and 80^o. After validation, some results considering soil porosity are shown.

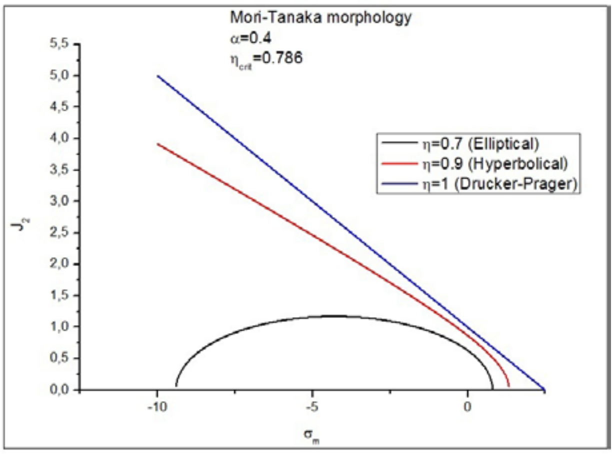

Figure 2: Yield function regimes: elliptical below η_{crit}, hyperbolical above this value and limited by Drucker-Prager when $\eta = 1$.

The continuum form of the limit analysis problem is discretized into 2-D mixed finite elements. Triangular elements are used, with quadratic interpolation for velocity and linear interpolation for stresses fields. An adaptive mesh refinement is used and the goal of this approach is to achieve a mesh-adaptive strategy accounting for mesh size refinement, as well as redefinition of the element stretching. More details about adaptive approach are in [2] and [1].

Figure 3 shows a refined finite element mesh.

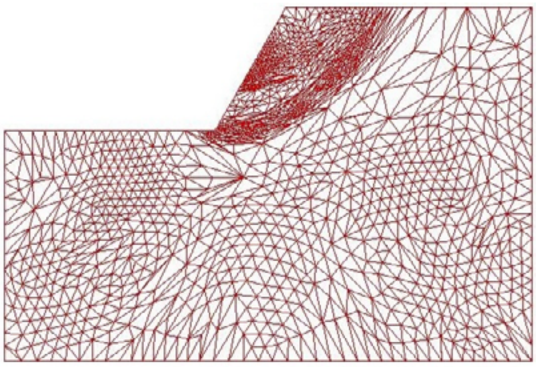

Figure 3: Finite element refined mesh for a slope.

5.1 Validation

The called Culmann solution considers a straight failure surface and the stability number is calculated from forces equilibrium requirements. The called Log-spiral solution considers limit analysis and a log-spiral failure surface. These classical solutions are compared to limit analysis solution and incremental plasticity.

The slope angles were made to vary: 40^o, 60^o and 80^o. For each slope angle, the soil friction angle was made to vary as well. Figures 4, 5 and 6 shows the stability numbers for each friction and slope angles.

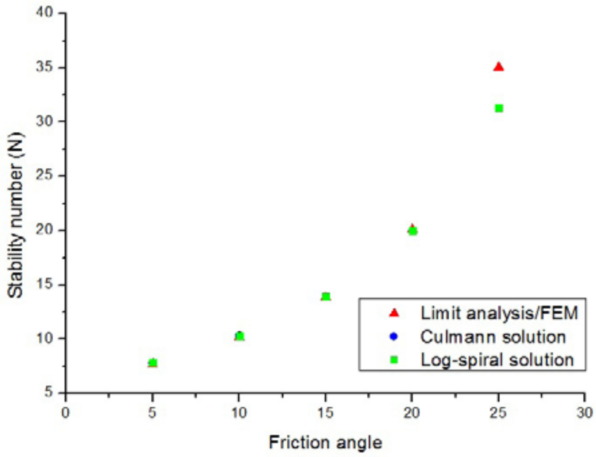

Figure 4: Stability number for slope angle=40^o.

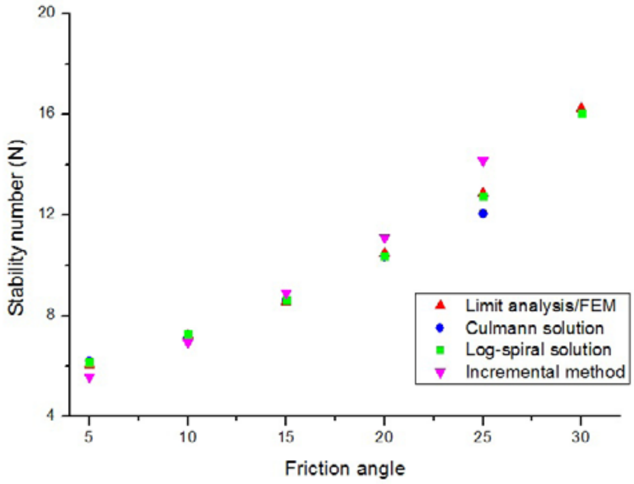

Figure 5: Stability number for slope angle=60^o.

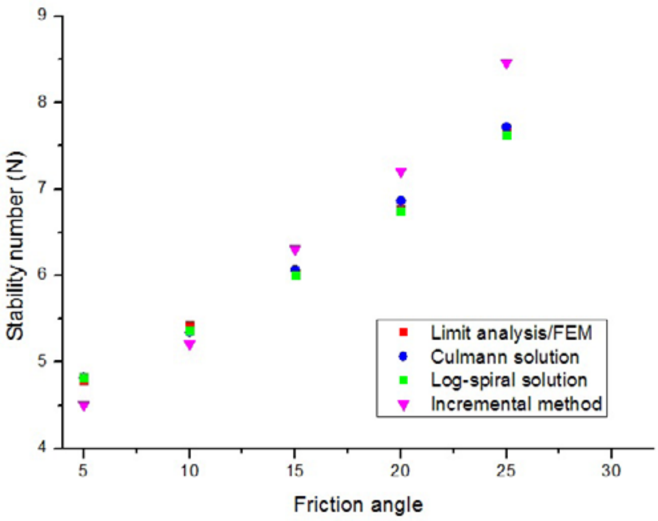

Figure 6: Stability number for slope angle=80^o.

Figures 7 and 8 show the velocity field and the slip line for a slope angle 40^o and soil friction angle 20^o. The slip line in Figure 8 represents the slope failure surface.

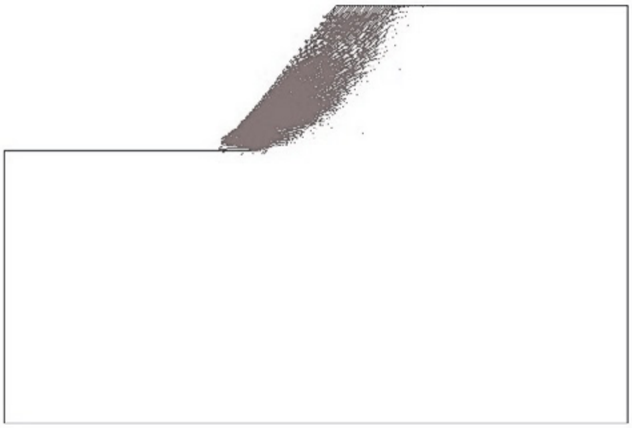

Figure 7: Velocity field for a slope angle=40^o and friction angle=20^o.

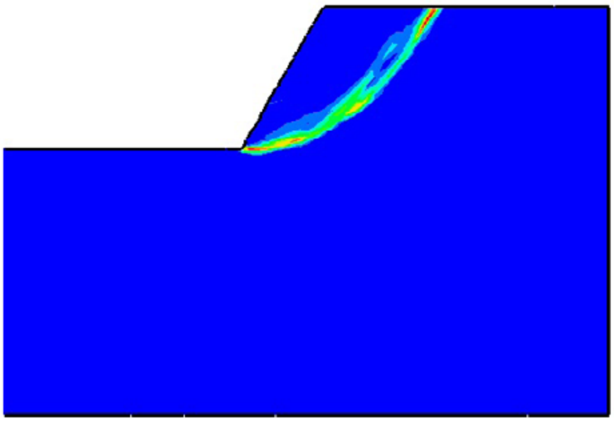

Figure 8: Slip line: slope failure surface for a slope angle=40^o and friction angle=20^o.

In the incremental method, the finite element mesh used for the 60 degrees slope is illustrated in Figure 9.

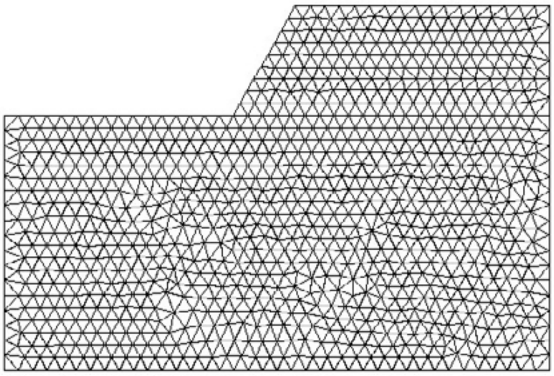

Figure 9: Finite element mesh for CodeBright.

The velocity fields and slip lines for a 60^o and 80^o slope angle and friction angle equal to 20^o are shown in Figures 10 and 11.

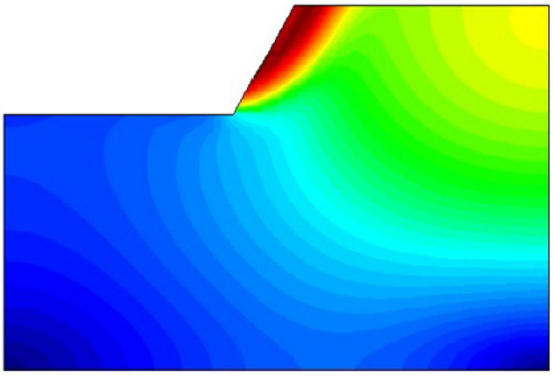

Figure 10: Velocity field modulus for a slope angle=60^o and friction angle=20^o.

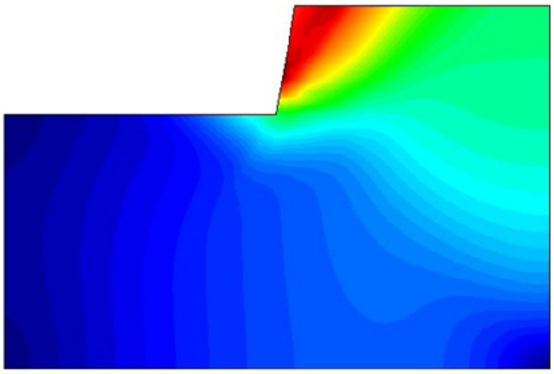

Figure 11: Velocity field modulus for a slope angle=60^o and friction angle=20^o.

5.2 Stability Number for a Porous Material

Since the model validation was made, the porosity was made to vary in order to study the stability number for a 30^o friction angle soil and slope angle equal to 60^o.

Figure 12 shows the variation of stability number with soil porosity.

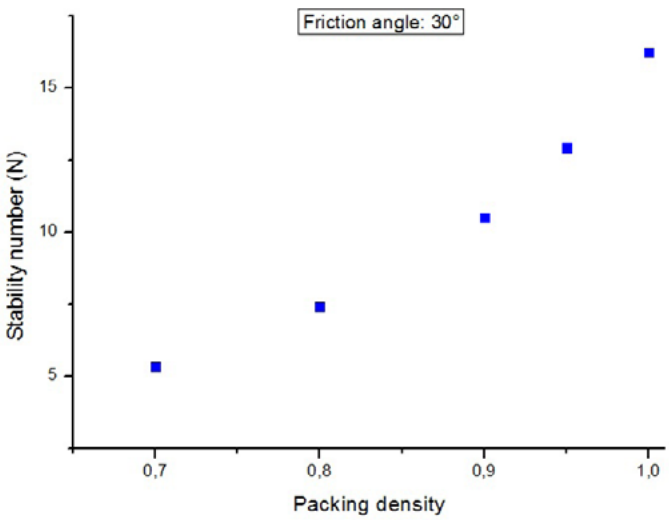

Figure 12: Stability number for a porous material.

The stability number along friction angle can also be plotted by several curves, showing variation of this number with porosity, as in Figure 13:

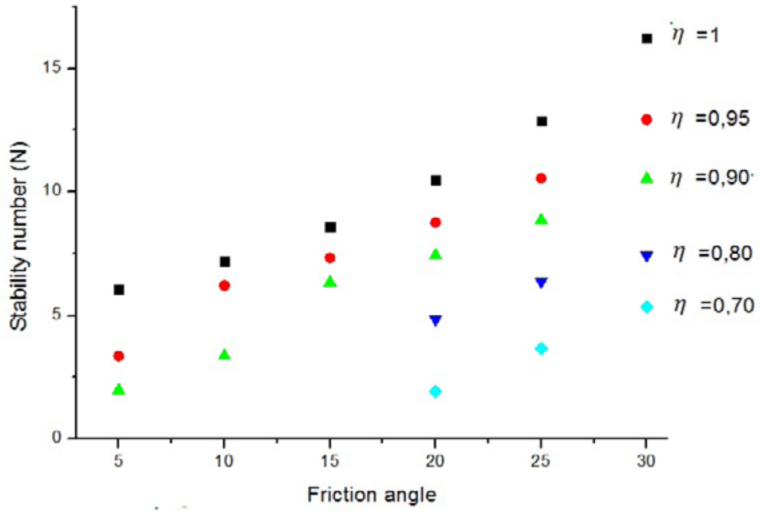

Figure 13: Stability number for some packing densities.

6 CONCLUSION

The study of stability of slopes is a geotechnical problem and its aim is to characterize the potential failure surface and the slope safety factor for a given geometry and loading conditions.

Many solutions have been developed based on a proposed failure surface and forces equilibrium requirements. Limit analysis principles is also applied, according to [13]. Classical yield functions such as Drucker-Prager and Mohr-Coulomb are also applied on the development of these solutions. However, some issues must be observed: it is not possible to know the shape of the failure surface *a priori* and the classical yield surfaces do not take into account the porosity effect as a parameter. In the proposed limit analysis/FEM method it is possible to determine the failure surface. Moreover, the Ulm-Gathier yield function is applied in the model and it considers, among friction angle and cohesion, the soil porosity effect.

In this work, the stability of slope problem is developed under limit analysis theory and finite element method. This approach considers a 2-D finite element and under strain plane hypothesis, a mixed formulation is applied. The stress, velocity and strain rate fields are determined, besides the collapse factor and the plastic multiplier, which characterizes the slip lines and the failure surface. The incremental method is also applied and some studies are carried out by the software CODEBRIGHT. The collapse factor is calculated and the stability number is a suitable parameter since it is a non-dimensional number.

At first, the model validation was executed and the limit analysis/FEM results were compared to classical solutions and incremental solution. Some studies were released considering slope angles equals to 40, 60 and 80 degrees and some soil friction angles. The stability numbers from limit analysis/FEM were compared to classical solutions and incremental method and one can observe a close behavior with those solutions.

On the next step, the soil porosity was considered. The graphic in Figure 12 shows the stability number variation with density packing. When $\eta = 1$, the Drucker-Prager criterion is established. When density packing decreases and some porosity is introduced, the stability number decreases, as expected. The next graphic in Figure 13 shows some curves for some porosity values: the top curve represents a curve without porosity. When porosity is introduced, the stability numbers for the other porosity are below each other, since porosity diminishes the soil resistance.

7 ACKNOWLEDGEMENTS

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