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H-ADAPTIVE FINITE ELEMENT ANALYSIS OF CONSOLIDATION PROBLEMS IN GEOMECHANICS

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Abstract. In this paper, a computational framework based on the h-adaptive finite element strategy is presented for the solution of consolidation problems in geomechanics. The efficiency and performance of alternative error estimation techniques is demonstrated via the analysis of a slope stability problem.

1 INTRODUCTION

In the finite element analysis of geotechnical problems, displacements are often coupled with pore water pressures, resulting in a highly nonlinear system of equations, particularly in cases where the material response is nonlinear, drainage of excess poor pressures is rapid or where large changes in the geometry occur [11]. The solution of such problems by the finite element method requires discretisation of the domain into a finite number of elements. While the accuracy of the analysis is usually improved by increasing the number of elements, the efficiency of the analysis, in terms of computational time and required memory storage, significantly decreases as denser meshes are employed. However, in modelling consolidation, the behaviour of soil is influenced by drainage of excess pore pressures. As excess pore water gradients, and hence the rates of pore water dissipation may vary significantly between loading and their complete dissipation, the optimal finite element mesh at the start of the analysis will unlikely be economical for the final stages of the finite element computations.

The adaptive finite element methods allow the discretisation of the physical domain to change based on a prescribed criterion and as they may modify and adapt the mesh during the course of an analysis, are suitable to deal with the time-dependent consolidation of soil. In the literature, the use of the r-adaptive method, such as the Arbitrary Lagrangian-Eulerian (ALE) strategy, has proved effective in solving large strain consolidation problems [8]. However, as the number of elements and the number of nodes do not change in the ALE method, the accuracy of the analysis strongly depends on the quality of initial mesh. Considering this fact, it seems that the h-adaptive finite element method, in which the topology may continuously change throughout the analysis, can be a more efficient strategy to deal with complex problems of consolidation in geomechanics. By using an h-adaptive technique, neither prior assumption nor judgment is required while discretising the region.

The h-adaptive finite element method has been developed to increase the efficiency of analysis by continuously subdividing elements into smaller areas in the regions of higher nonlinearity based on a measure of computational error. In addition, a robust h-adaptive method can eliminate the issue of mesh distortion in problems involving relatively large deformations. Recently, the h-adaptive strategy has been widely used in problems of solid mechanics as well as in single-phase problems of geomechanics [5], [7], and [9]. However, the application of this robust technique in tackling the coupled analysis of consolidation has rarely been considered in the literature, mainly due to uncertainties in using an appropriate and efficient error estimator. El-Hamalawi and Bolton [4] introduced an error estimator for calculating the error in finite element domain. This error estimator was based on the maximum value of the L_2 norm of displacements and the L_2 norm of pore water pressures at nodal points.

In this study, we present a general framework for the h-adaptive analysis of geotechnical problems, and consider three alternative error estimators based on the energy norm, Green-Lagrange strain tensor, and the plastic dissipation. The efficiency as well as applicability of these error estimators is studied by analysing the long term stability of a vertical slope.

2 H-ADAPTIVE FINITE ELEMENT METHOD

In this paper, the h-adaptive method developed in [6] is extended to the consolidation analysis of geotechnical problems involving large deformation. The consolidation analysis by this method includes three main steps. In the first step, the Updated Lagrangian (UL) method is employed to solve the global governing equations to achieve equilibrium. Secondly, a new finite element mesh is generated based on the new sizes of the elements, usually obtained by an error estimator that determines which areas should be subdivided into smaller elements by measuring the error in each element. In the third step, all state variables at integration points as well as nodal points are transformed from the old mesh to the new generated mesh. These steps are briefly explained in the following.

In analysis of coupled deformation and fluid flow for soils, the governing equations are usually derived from mechanical equilibrium of the soil skeleton and mass balance of the pore fluid [8]. The discretised governing equations can be written as

$$\begin{bmatrix} \mathbf{K}_{\mathrm{ep}} & \mathbf{L} \\ \mathbf{L}^{\mathrm{T}} & 0 \end{bmatrix} \begin{Bmatrix} \dot{\mathbf{U}} \\ \dot{\mathbf{P}} \end{Bmatrix} + \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \dot{\mathbf{H}} \end{bmatrix} \begin{Bmatrix} \mathbf{U} \\ \mathbf{P} \end{Bmatrix} = \begin{Bmatrix} \dot{\mathbf{F}}^{\mathrm{ext}} \\ \dot{\mathbf{Q}}^{\mathrm{ext}} \end{Bmatrix} \tag{1}$$

where $\mathbf{K}_{\mathrm{ep}}$, $\mathbf{H}$ and $\mathbf{L}$ are the stiffness, flow and coupling matrices, respectively, $\mathbf{U}$ is the nodal displacement vector, $\mathbf{P}$ represents the nodal pore pressure vector, $\mathbf{Q}$ is the fluid supply vector, and $\mathbf{F}^{\mathrm{ext}}$ and $\mathbf{F}^{\mathrm{int}}$ represent the external and the internal force vectors, respectively.

After solving Equation (1), the coordinates of all nodal points are updated according to the incremental displacements. Then, an automatic mesh generation technique is employed which refines the mesh in the critical areas of discretisation based on the error η defined by

$$\eta = \frac{\sqrt{\sum_{i=1}^{N_{el}} \left\| e_{el}^{*} \right\|_{i}}}{E} \tag{2}$$

in which $\left\| e_{el}^{*} \right\|$ is the error in each element, E represents the error in the finite element domain, and N_{el} is the total number of elements. If this error exceeds a prescribed accuracy ($\bar{\eta}$), a new element area, A_{new}, will be defined based on the old element area, A_{old}, and an equal distribution of the finite element error over the elements in the domain, according to

$$A_{new} = \left(\frac{\bar{\eta} E}{\sqrt{N_{el}} \left\| e_{el}^{*} \right\|_{i}} \right)^{\frac{1}{p}} \times A_{old} \tag{3}$$

where p represents the polynomial order of displacement shape functions.

To estimate the error, three alternative error estimators are considered in this study. The first error estimator is based on the energy norm. According to this estimator, the error in each element, $\left\| e_{el}^{*} \right\|$, and the error in finite element domain, E, are obtained by:

$$\begin{aligned} \left\| e_{el}^{*} \right\| &= \sum_{i=1}^{N_{gp}} w_i \left(\sigma_i^{*} - \hat{\sigma}_i \right) \left(\Delta\varepsilon_i^{*} - \Delta\hat{\varepsilon}_i \right) \\ E &= \left(\sum_{i=1}^{N_{el}} \sum_{j=1}^{N_{gp}} w_j \sigma_{ij}^{*T} \Delta\varepsilon_{ij}^{*} \right)^{\frac{1}{2}} \end{aligned} \tag{4}$$

where N_{gp} and w_i are the number of Gauss points and the standard Gauss quadrature weights, σ^{*} and $\Delta\varepsilon^{*}$ represent the recovered stresses and incremental strains, respectively, and $\hat{\sigma}$ and $\Delta\hat{\varepsilon}$ are the corresponding finite element approximations. The second error estimator is based on the Green-Lagrange strain tensor [2], E_G, and the errors in each element and the finite element are calculated by

$$\begin{aligned} \left\| e_{el}^{*} \right\| &= \left(\int \left| \left(E_G^{*} - \hat{E}_G \right)^T \left(E_G^{*} - \hat{E}_G \right) \right| dV_{el} \right)^{\frac{1}{2}} \\ E &= \left(\sum_{i=1}^{N_{el}} \sum_{j=1}^{N_{gp}} w_j E_{Gij}^{*T} E_{Gij}^{*} \right)^{\frac{1}{2}} \end{aligned} \tag{5}$$

where V_{el} is the volume of an element. The third error measurement is based on the plastic dissipation [10] and is defined by

$$\begin{aligned} \left\| e_{el}^{*} \right\| &= \left(\int \left| \left(\sigma^{*} - \hat{\sigma} \right)^T \left(\Delta\varepsilon^{p*} - \Delta\hat{\varepsilon}^{p} \right) \right| dV_{el} \right)^{\frac{1}{2}} \\ E &= \left(\sum_{i=1}^{N_{el}} \sum_{j=1}^{N_{gp}} w_j \sigma_{ij}^{*T} \Delta\varepsilon_{ij}^{p*} \right)^{\frac{1}{2}} \end{aligned} \tag{6}$$

in which ε^{p} is the plastic strain.

After estimating the error and refining the mesh, all the history-dependent variables, displacements, and pore water pressures need to be transferred from the previous mesh to the new mesh. This process, which is known as remapping, must guarantee the plasticity consistency as well as the equilibrium in the new mesh. The nodal variables such as displacements can be remapped by a direct interpolation. However, for the values defined in Gauss points the Supper Convergent Patch (SPR) technique is used. For more details refer to [6] and [8].

3 NUMERICAL EXAMPLE

The h-adaptive finite element method described in the previous section is implemented into SNAC, the in-house finite element code developed by the Geotechnical research group at the University of Newcastle, Australia [1]. SNAC was used to analyse the numerical example in

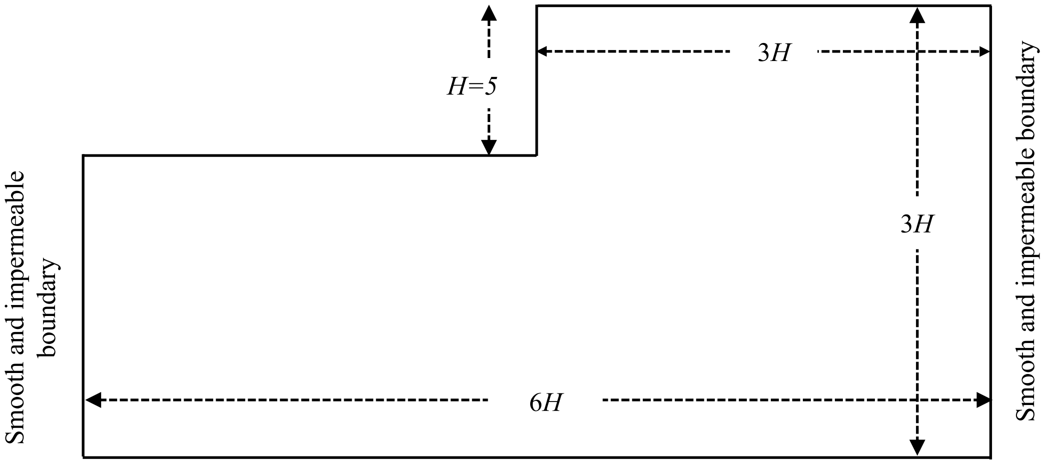

Figure 1: A vertical slope

this section. To study the performance of the three error estimator techniques as well as the efficiency of the h-adaptive technique presented here we study the long term stability of a 5m high vertical slope. The material properties describing the soil behaviour are shown in Table 1. Also, the problem domain, its dimensions, and boundary conditions are shown in Figure 1.

Table 1: The material properties describing the soil behaviour

Property	Value
drained Young's modulus, E'	1000 kPa
drained Poisson's ratio, ν'	0.3
drained cohesion, c'	20 kPa
drained friction angle, φ'	20°
dilation angle, ψ'	0°
coefficients of permeability in x and y directions, $k_x = k_y$	10^{-4} m/day
unit weight of water, γ_w	10 kN/m^3

Based on Culmann's method of slope stability analysis, the critical height of a vertical slope for which the failure occurs, H_{cr}, is obtained by [3]

$$H_{cr} = \frac{4c'}{\gamma}\tan\left(\frac{\pi}{4}+\frac{\varphi'}{2}\right) \tag{7}$$

where γ is the unit weight of soil. To investigate the stability of a slope by the finite element method, a common practice is to gradually increase the unit weight of soil until failure occurs. Assuming $H_{cr} = 5$ m, the unit weight of the soil at failure, according to Equation (7), is 22.85 kN/m^3. In order to investigate this value by the finite element method, the soil is assumed to be weightless at the beginning of the analysis. Then, the unit weight of the soil is increased during a long period of time to ensure that any excess pore pressure dissipates, viz., the total analysis time exceeds 10^7 years.

In all analyses, 6-noded quadratic triangular elements with 6 integration points were used. The area of elements in the initial finite element mesh is assumed to be ~$0.04H^2$, and the minimum area of elements during refinement is limited to $0.0004H^2$. The finite element mesh

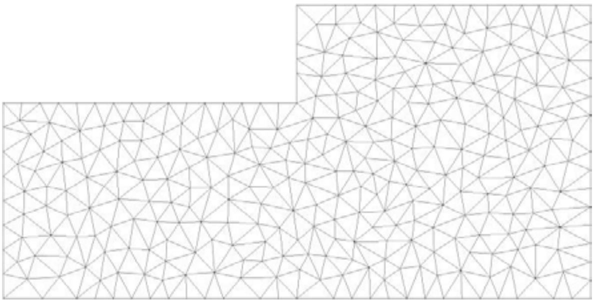

Figure 2: The initial finite element mesh of slope including 589 elements and 1264 nodal points

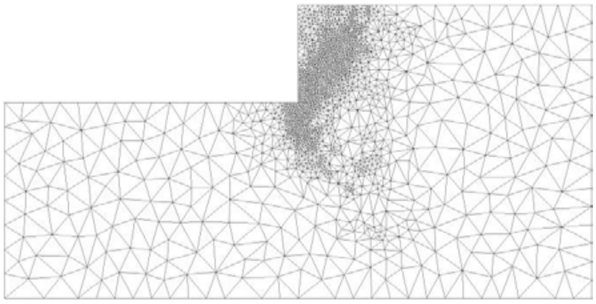

Figure 3: Finite element mesh at the end of analysis based on the plastic dissipation error estimator and small strain assumption

at the beginning of the analysis includes 589 elements and 1264 nodal points as shown in Figure 2. The soil is modelled as a Mohr-Coulomb material with a non-associated flow rule. The unit weight of soil at the failure, estimated by the error estimators considered in this study, and assuming small strain as well as large strain conditions, are summarised in Table 2.

Table 2: Finite element results for the unit weight of soil at failure

Error estimator technique	Small strain analysis		Large strain analysis	
	γ_w (kN/m^3)	Error in γ_w (%)	γ_w (kN/m^3)	Error in γ_w (%)
Energy norm	20.7	9.41	21.03	7.96
Green-Lagrange strain	26.76	17.11	24.84	8.71
Plastic dissipation	21.12	7.57	21.24	7.05

According to Table 2, the error estimator based on the plastic dissipation outperforms the error estimators based on the Green-Lagrange strain tensor and the energy norm. In addition, the results predicted by the large deformation analyses are more realistic than the results obtained by assumption of small strains. As a typical visualisation, the finite element mesh at the end of analysis obtained by the plastic dissipation error estimator is depicted in Figure 3, which includes 3672 elements and 7489 nodal points.

4 CONCLUSIONS

An h-adaptive finite element procedure for modelling consolidation problems, in which the displacements are coupled with pore fluid pressure, was presented. Three alternative error estimators for controlling the refinement of the finite element mesh, based upon the energy norm, Green-Lagrange strain tensor, and plastic dissipation respectively, were considered. The performance and accuracy of the error estimators was studied by analysing long term stability of a vertical slope, assuming small strain as well as large deformation conditions. For the problem considered in this study, the numerical results indicated that the error estimator based on the plastic dissipation is more accurate that the other error estimators. Nonetheless, the performance and efficiency of these error estimators for coupled analysis of geomechanics problems require further investigation through solving a wider range of applications such as analysis of short term as well as long term bearing capacity of soil under a footing, consolidation settlement of soil under embankments, and analysis of short term stability of slopes.

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