Treball de Fi de Grau

Grau en Enginyeria en Tecnologies Industrials

Study of the Effects of Fluid-Structure Interaction on Submerged Bodies

REPORT

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Abstract

In this project the modes of vibration of a circular plate surrounded by a mixture of air and water have been studied. This is a scenario that a lot of solids may encounter, for example a machine part submerged in water that may be mixed with some air. How this air concentration will affect the solid modes of vibrations wants to be studied.

A preliminary bibliographic survey was conducted, explaining concepts as mass effect and void ratio as well as looking into other papers with relevant previous results. The numerical model used was that of a fluid-solid interaction. A harmonic analysis type was chosen for the plate modes and a modal type for the acoustic modes. The geometry studied consists of a circular plate surrounded by a two-phase fluid, a mixture of air and water. This mixture was defined by its void-ratio parameter, which is dependent on the density and the sound speed. Ansys simulations were conducted in order to find the wanted modes of vibration for the plate and the acoustic modes for the surrounding fluid.

The acoustic modes of vibration (considering all solid eliminated) were analysed, studying its frequency's dependence on void ratio, density and sound speed and comparing its mode shapes. Fluid-structure coupled modes of vibration for the plate were also studied, to be more specific those with an axisymmetric circular shape. Finally, the coupling effect between the acoustic and the structural modes was studied.

The results confirm that a dependence between the plate modes of vibration and the surrounding mixture exists, and it's ruled by the mixture density. A big coupling effect between some acoustic modes and plate modes was also found, which made the further analysis of the plate modes of vibration difficult.

Resum

En aquest projecte han sigut estudiats els modes vibratoris d'un plat circular submergit en una barreja d'aigua i aire. Aquest és un cas que es pot donar per molts sòlids, per exemple si una peça d'una màquina està envoltada d'aigua i aquesta es barreja amb aire. Com la concentració d'aire en aquesta mescla afectarà els modes vibratoris de la peça és el que es vol estudiar en aquest projecte.

Un estudi previ bibliogràfic ha sigut realitzat, explicant conceptes com l'efecte de massa afegida i el ràtio d'aire, i alhora buscant informació rellevant d'altres projectes. El model numèric utilitzat és el d'una interacció fluid-estructura. Un anàlisis harmònic va ser escollit pels modes del plat i un anàlisi modal pels modes acústics. La geometria estudiada consisteix en un plat circular envoltat per un fluid de dues fases, aire i aigua. Aquest mescla va ser definida pel paràmetre ràtio d'aire, que depèn de la densitat i la velocitat del so. Es van realitzar simulacions amb el programa Ansys per trobar el modes de vibració del plat i del fluid evolvent.

Els modes acústics de vibració (eliminant tots el sòlids) van ser analitzats, estudiant la dependència de la seva freqüència amb el ràtio d'aire, la densitat i la velocitat del so, així com observant la seva forma modal. Els modes acoblats del fluid i l'estructura presents en el plat també van ser estudiants, concretament aquells amb una forma circular axisimètrica. Finalment, l'efecte d'acoblament entre ambdós modes (l'acústic i el del plat) va ser estudiat.

Els resultats obtinguts confirmen una dependència entre els modes vibratoris del plat i la mescla que l'envolta, la qual és dominada per la densitat. També es va trobar un gran efecte d'acoblament entre els modes acústics i els del plat que causa una impossibilitat d'estudiar els modes del plat en més profunditat.

Summary

GLOSSARY

x [m]: mass displacement

m [kg]: system mass

m_a *[kg] added mass*

b [kg/s]: linear damping coefficient

k [N/m]: spring coefficient

f(t) [N]: force

ω [Hz]: frequency

c [m/s]: sound speed

ρ $[kg/m^3]$*: density*

μ [Pa s]: dynamic viscosity

p [Pa]: acoustic pressure

t [s]: time

α [-]: void ratio

r [m] : distance between the plates axis and the points of zero vertical displacement

a [m]: plate's diameter

$[M_F]$*: mass matrix*

$[C_F]$*: acoustic fluid damping matrix*

$[K_F]$*: acoustic fluid stiffness matrix*

[R]: fluid acoustic boundary matrix

{f}: acoustic fluid load vector

1. Preface

1.1. Project's origins

This project is born from a desire to expand the research on fluid-structure interactions. This is a field that has been explored as of lately, and still has a long way to go. Within this wide subject, the effect of the fluid different parameters was chosen as desired topic. To be more specific, the question of what happens if some air is mixed with water, and how this could affect a solid submerged on it was arisen.

This knowledge could be applied in different fields. One good example is in the design of a machine part that is going to be submerged in the mixture while vibrating. The existence of this fluid and its influence upon the piece are taken into consideration as for lately, but the appearance of air in the mixture has not been taken into account yet while designing the piece. This project wants to study these cases more deeply to understand if the filtration of some air may have a big impact into the piece and its proper operation.

To do so, the paper A*xisymmetric vibrations of a circular Chladni plate in air and fully submerged in water [1]* by Xavier Escaler and Oscar De La Torre was studied and taken as a starting point. In said paper a plate is submerged in both air and water and its frequency modes are studied. The current project wants to study what happens if the fluid surrounding the plate is a mixture of water and air.

1.2. Motivation

Since this is my final thesis project, I was eager to find an interesting subject where I could apply the knowledge I had gained during my degree. Since the classes that I enjoyed the most were those related to fluid mechanics, especially those involving simulations with the programme Ansys, I started to search for a relevant topic in this area.

When this project was presented to me, I found it especially interesting since I could see all the possible applications that it could have. Any piece in contact with a fluid can be affected by the change of the air concentration in said mixture, and this is an aspect that is not usually taken into account while designing, and it may have notable consequences latter on. It was in this spirit that I started this project.

2. Introduction

2.1. Aim

The aim of this project is to study the influence that a change in the air concentration of a mixture of air and water has on a circular plate submerged in it. Some vibration modes are going to be studied, changing the properties of the surrounding mixture, and its effects on the modal frequency and modal shape are going to be analysed. The possible interaction between the acoustic modes and the plate modes is also going to be studied.

The specific objectives of this project are:

- Find the acoustic modes of the fluid once all solids are removed
- Analyse the acoustic mode frequency and its behaviour in front of the sound speed of the mixture
- Analyse the acoustic mode shape for different mixtures
- Find the first 6 fluid-structure coupled modes of vibration for the 11 different mixtures
- Analyse the fluid-structure coupled mode frequency and its behaviour in front of the void ratio, density and sound speed of the mixture
- Study the fluid-structure coupled mode shape changes
- Study the possible interaction between the acoustic modes and the plate modes.

2.2. Scope

This project is going to study a circular plate free at its edges and excited at its centre, submerged in different mixtures of air and water. For each mixture, the first 11 mode shapes with an axisymmetric circular displacement are going to be studied. Each fluid is going to be defined using the void ratio, which defines its density and its sound speed. The simulations are going to be done using the programme Ansys.

3. Background

3.1. Added mass effect

For a body submerged in a fluid we must consider the additional force that results from the fluid acting on the structure, forcing the body to move an external fluid mass. This added effect is called added mass.

Structures can be modelled by a system equation similar to equation 1, where x is the mass displacement, m is the system mass, b is the linear damping coefficient, k is the spring coefficient and f(t) is the force acting on the mass. The frequency ω can be then calculated with equation 2. If the added mass effect is now taken into account as the weight added to the system, equation 1 is rewritten as shown in equation 3, were m_a is the added mass. The new natural frequency is then calculated with equation 4. It is observed that the addition of this mass effect implies a reduction in the frequency ω, which means that according to this equation a lower frequency is expected to be found in water (where the mass effect takes place) than in air.

$$m\ddot{x} + b\dot{x} + kx = f(x) \tag{1}$$

$$\omega = \sqrt{\frac{k}{m}} \tag{2}$$

$$(m + m_a)\ddot{x} + b\dot{x} + kx = f(x) \tag{3}$$

$$\omega = \sqrt{\frac{k}{m+m_a}} \tag{4}$$

This effect needs to be calculated in all directions, since added mass can arise in one direction due to motion in a different direction. It is usually calculated through a mass matrix.

3.2. Governing equations

In a fluid-structural interaction (FSI) problem as the one presented in this project, the structural dynamics equation must be considered along with the Navier-Stokes equation of fluid momentum and the flow continuity equation. The following assumptions need to be made: the fluid is compressible (density does change due to pressure variations) and there is no mean flow of the fluid. This results on the lossy wave equation (5) (from [5]), where *c* refers to the sound speed through the fluid, ρ_0 to its density, μ to its dynamic viscosity, p is the acoustic pressure (which is a function of x,y,z and t), Q is the mass source in the continuity equation and t refers to time.

$$\nabla\left(\frac{1}{\rho_0}\nabla p\right)-\frac{1}{\rho_0 c^2}\frac{\partial^2 p}{\partial t^2}+\nabla\left[\frac{4\mu}{3\rho_0}\nabla\left(\frac{1}{\rho_0 c^2}\frac{\partial p}{\partial t}\right)\right]=\frac{-\partial}{\partial t}\left(\frac{Q}{\rho_0}\right)+\nabla\left[\frac{4\mu}{3\rho_0}\nabla\left(\frac{Q}{\rho_0}\right)\right] \tag{5}$$

This equation is then reduced to the Helmholtz equation which includes the harmonically varying pressure and depends on the frequency of oscillation of this pressure. Then, using the Galerkin procedure, the equation is discretized to obtain the so called wave equation *(6)*.

The wave equation can be expressed in matrix notation (6) where $[M_F]$ is the acoustic fluid mass matrix, $[C_F]$ is the acoustic fluid damping matrix, $[K_F]$ is the acoustic fluid stiffness matrix, [R] the fluid acoustic boundary matrix and {f} is the acoustic fluid load vector. The acoustic fluid mass density, which is considered constant, is shown as ρ_0. The fluid pressure is expressed by p_e and its velocity is represented by $\ddot{u}$.

$$[M_F]\{\ddot{p_e}\}+[C_F]\{\dot{p_e}\}+[K_F]\{p_e\}+\overline{\rho_0}[R]^T\{\ddot{u}_{F,e}\}=\{f_F\} \tag{6}$$

The matrix equation that needs to be solved in a fluid-solid interaction problem can be expressed in function of the frequency ω as well, as shown in equation 7 (from [5]). This is the equation that is going to be solved by Ansys.

$$\left(-\omega^2\begin{bmatrix} M_s & 0 \\ \rho_0 R^T & M_f \end{bmatrix} + j\omega\begin{bmatrix} c_s & 0 \\ 0 & c_s \end{bmatrix} + \begin{bmatrix} K_s & -R \\ 0 & K_f \end{bmatrix}\right)\begin{Bmatrix} u \\ p \end{Bmatrix} = 0 \tag{7}$$

3.3. State of the art

3.3.1. Air and water data

The current project wants to be an extension of the research on fluid-structure interactions, to be more specific it wants to study the variations on the modes of vibration of a circular plate submerged in a mixture of air and water, when the two-phase fluid proportion changes. For this purpose the results from the paper [1] by Escaler and De La Torre are taken as a starting point for further analysis.

Said paper aims to study the dynamic responses of bodies submerged in two-phase flows, to be more specifics to prove the small but relevant differences between the mode shapes in air and water.

For this porpoise an experimental setup was made, consisting of a Chladni plate inside a square tank filled by either air or water, free at its edge an excited at its centre through a cylindrical bar. This is the same model used in the present paper, which is further explained in point 4.2.

The Chladni technique used consists of exciting a horizontal plate at a single vibration mode and then waiting for sand grains scattered on the top surface to collect along the nodal lines.

Ansys harmonic simulations were also used to verify the Chladni experiments, proving them to be accurate. The Ansys model used is the same as in the present paper, which is further explained in point 4.

The modal frequencies found are shown in Table 1 for the first six modes (S1 to S6) and its shapes are shown in Figure 1. The current project aims to be an extension of this one, wanting to compare the modes of vibration in the mixture of water an air.

Table 1: mode frequency in air and water for the modes S1 to S6, from [1]

	S1	**S2**	**S3**	**S4**	**S5**	**S6**
Air [Hz]	70	305	846	1666	2756	4130
Water [Hz]	26	116	281	563	996	1599

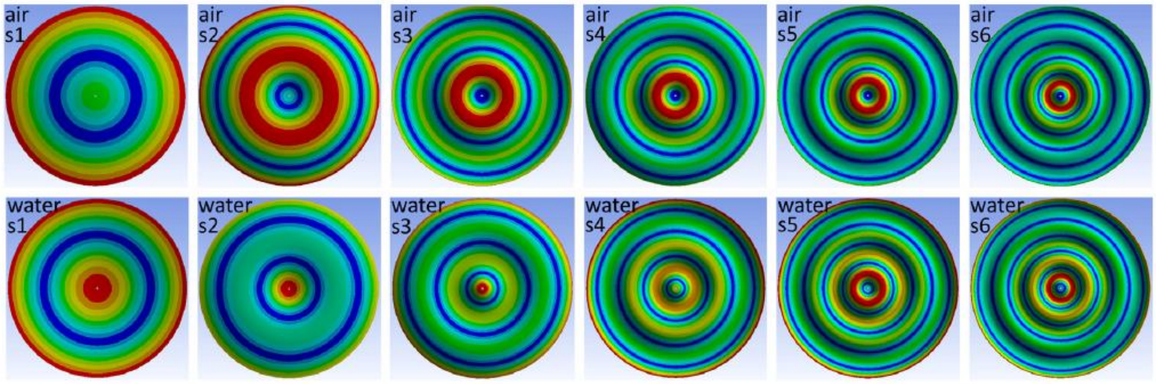

Figure 1: top view of the simulated mode shapes in air (top row) and water (bottom row) from [1]

The relative nodal radius is defined as r/a, being *r* the distance between the plate axis and the points of zero vertical displacement and *a* the plate radius. It was found in water (Table 2) and compared with that of air (Table 3), finding a significant increase in the innermost nodal radius and a slight decrease of all the rest of outer radii compared to the mode shapes in air.

Table 2: relative nodal radius for the first six modes in air (S1 to S6), from [1]

AIR	**Relative Nodal Radius [-]**					
S1	0.457					
S2	0.801	0.126				
S3	0.876	0.523	0.06			
S4	0.907	0.656	0.378	0.046		
S5	0.934	0.735	0.51	0.298	0.033	
S6	0.94	0.781	0.603	0.424	0.245	0.033

Table 3: relative nodal radius for the first six modes in water (S1 to S6) from [1]

WATER	**Relative Nodal Radius [-]**					
S1	0.603					
S2	0.774	0.323				
S3	0.848	0.510	0.172			
S4	0.881	0.641	0.364	0.099		
S5	0.907	0.709	0.497	0.285	0.006	
S6	0.921	0.762	0.589	0.417	0.232	0.046

3.3.2. Fluid and plate mode interaction

In the paper [2] by Escaler et. al. information about the interaction of the surrounding acoustic modes and a fluid-structure coupled modes can be found. This paper aims to study the influence of the acoustic modes on the mode shapes and natural frequencies of a 2D hydrofoil placed in a high-speed water tunnel.

Both experimental results and simulated data were obtained and compared. The first seven acoustic modes of the cavity domains were calculated and plotted in figure 2 for both a short cavity (continuous line) and a long cavity (dotted line). The acoustic values were normalized using the hydrofoil chord length. It can be observed that all frequencies decrease linearly as the speed of sound decreases. The order of the mode shapes is different in each cavity.

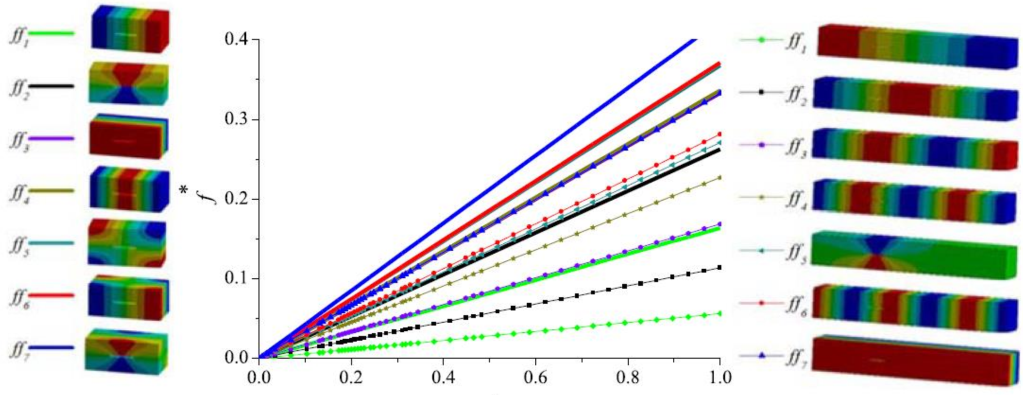

Figure 2: natural frequencies and mode shapes of the seven first acoustic modes of the water cavity as a function of the speed sound for a long domain (right-hand side plots and dotted lines) and for a short domain (left-hand side plots and continuous lines), from [2].

3.4. Circular plate modes

A circular plate with free edges and excited by its centre presents different mode shapes. The first 6 shapes can be seen in figure 3, with no fluid-solid interaction present. It can be observed that diametric and circular modes are found and also combined, resulting in multiple possible mode shapes. In this project, the shape mode studied is the 5th one, consisting of a pure circular mode with an axisymmetric radius of displacement. The first 6 modes presenting this shape are going to be studied, which are shown in figure 4.

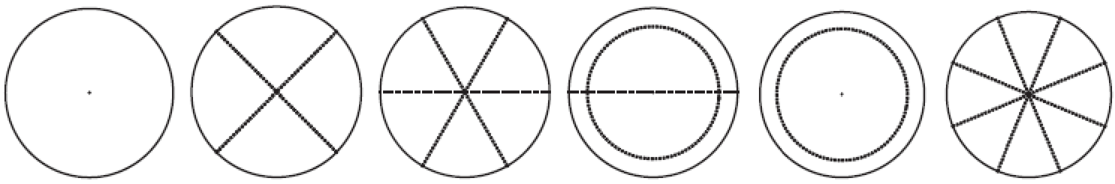

Figure 3: first 6 mode shapes for a circular plate free at its edges and clamped at its centre, from [3].

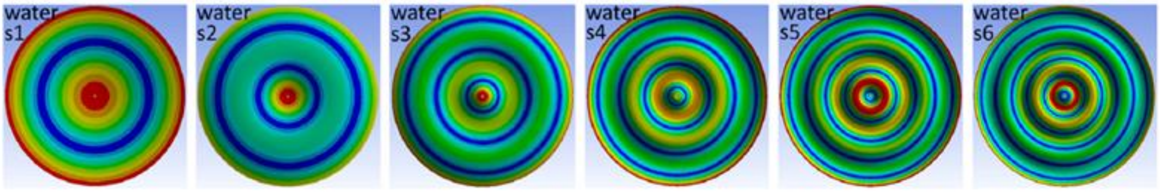

Figure 4: mode shapes S1 to S6 studied in this project, from [1].

4. Ansys

4.1. Model

The first decision to make is which type of analysis is going to be used. There are four different types of analysis that can be performed: modal analysis, harmonic response analysis, transient analysis and response spectrum.

The modal analysis aims to determine frequencies and wave patterns within a structure. Given a frequency range, the programme will give all modes of vibration within it. This analysis was used in the fluid simulations, to find the acoustic modes of the fluid alone.

The harmonic analysis objective is to calculate the response of the system as a function of frequency. This allows us to determinate the mode frequencies by looking at both its amplitude and phase graphs, where a resonance with a natural frequency is going to be shown as a maximum amplitude and change of phase point (see figure 5). This analysis was chosen for the plate simulations, since the modes of vibration may not be clearly defined for the disc so a personal inspection of the graphs was preferred.

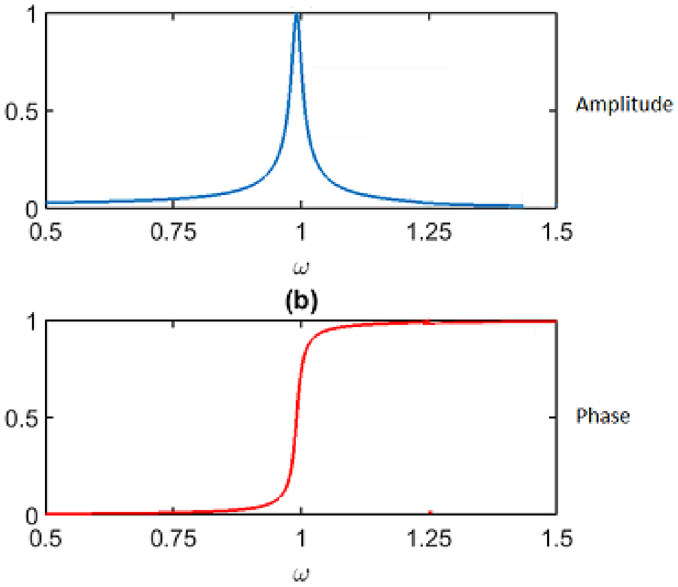

Figure 5: amplitude and phase in front of frequency, used to identify a mode, from [6].

4.2. Ansys model

The Ansys model used is the same as in the paper [1] by Escaler and De La Torre. For more precise information on its settings, consult said paper.

The model is comprised by one fluid domain, whose properties are going to be changed during the experiment to simulate different proportions of a mix of water and air, and two structural solid domains representing a Plexiglas tank and the aluminium plate that is going to be studied.

The plate has a 0.12 m outer radius and a thickness of $0.8 \cdot 10^{-3}$ m. It is made of T6 temper 6061 aluminium alloy. It has a small hole at its centre of $4{\cdot}10^{-3}$m in diameter. This plate is placed inside a Plexiglas container, of inner dimensions 0.48x0.48x0.49 m^3. The wall thickness is 0.01m. A mass of 0.08 kg representing the bar subjecting the plate and the frequency exciter (which will make the plate vibrate) together is used in the Ansys model. This setup can be seen in figure 6.

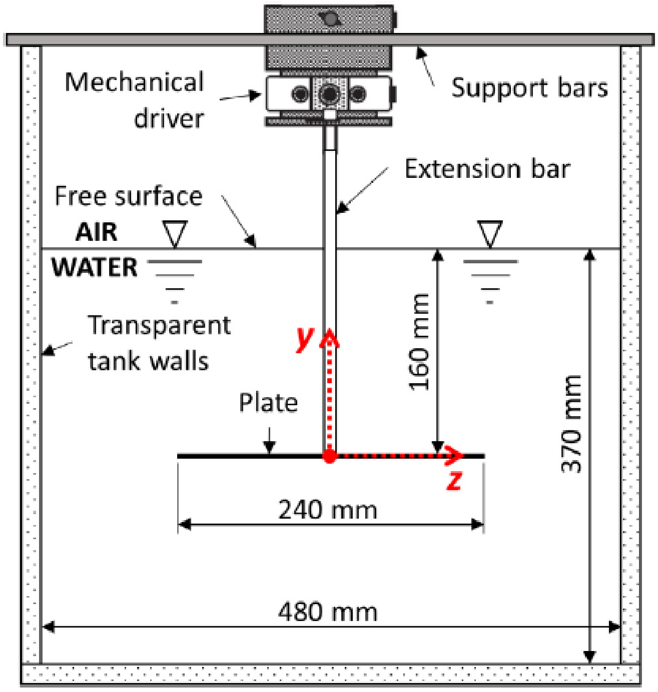

Figure 6: Schematic of the setup with the plate submerged in a water tank, from [1]

A total of 231 bodies was used in order to create a good axisymmetric mesh. It is composed by a high mesh density at the plate and surrounding fluid and low mesh density further away from the plate, since the area of study and the fluid-solid interaction are prioritized. The number of plate divisions in the radial, angular and axial directions are 50, 48 and 2 respectively. The mesh is proven valid in paper [1], and it can be seen in figure 7.

The fixed boundary conditions are zero displacement of the bottom of the tank, since it is supposed to be in contact with the ground, and zero pressure for the free surface elements.

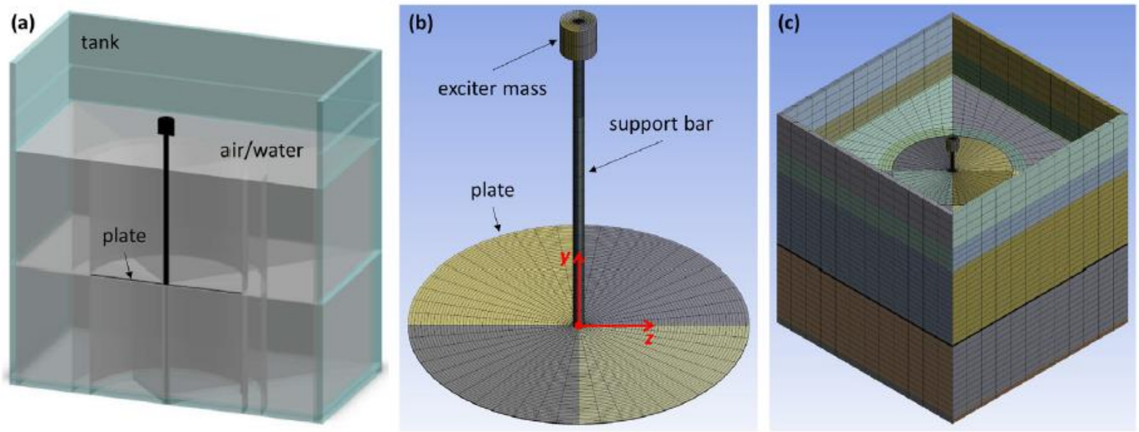

Figure 7: section of the model showing the fluid and structural domains (a), plate mesh system (b) and the mesh of the complete model, from [1]

4.3. Fluid

4.3.1. Void ratio (alpha)

The same simulation is going to be repeated just changing one parameter: the mixture surrounding the plate. This mixture is water and air with different concentrations of each fluid. The parameter used is the void ratio (alpha), calculated as the volume of air in relation to the total volume, as expressed in equation (8).

$$\alpha = \frac{volume\ air}{total\ volume} \tag{8}$$

This way, if α=0 all the fluid is considered water and if α=1 it is considered all air.

For every change in the parameter alpha, the values of density and sound speed of the resulting fluid mixture will change. These two properties will then define the fluid used in each simulation.

4.3.2. Density

The density (ρ) of the resulting mixture of water and air is a lineal property. Considering the density of water 998kg/m^3 and the density of air 1.2041 kg/m^3, equation (9) can be used to obtain the density in several mixes of the two fluids. In Table 4 the density values used in the simulations in function of alpha can be found.

$$\rho = 1.2041\alpha + 998(\alpha - 1) \tag{9}$$

Table 4: Density of the fluid in function of alpha

α [-]	**0**	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**	**1**
Density [kg/m^3]	998	898.32	798.64	696.96	599.28	499.6	399.9	300.24	200.6	100.9	1.2

4.3.3. Sound speed

For homogeneous two-phase mixtures as the one used in these simulations, it is proven that the speed of sound of the mixture can reach lower values that those of the sound speed of either of the pure phases. Assumptions such as thermodynamic and mechanical equilibrium between the two phases need to be made. Then, equation 10 (from [4]) can be used to calculate the speed of sound *c* in the mixture, being ρ_v and ρ_l the density of the vapour and liquid, and α the void ratio.

$$c^2 = \left[(\alpha\rho_v + (1-\alpha)\rho_l) \left(\frac{\alpha}{\rho_v \alpha^2} + \frac{1-\alpha}{\rho_l \alpha^2} \right) \right]^{-1} \tag{10}$$

In Figure 8, the sound speed of a mixture air-water is shown, calculated by Eq. 10. As mentioned before, the sound speed when α=0.5 is the lowest, below both pure phases. For this kind of mixture the drop in sound speed is even more drastic, since the vapour density is lower than the air by two orders of magnitude.

The sound speed values used in the simulations can be found in Table 5.

Table 5: Sound speed of the fluid in function of alpha

α [-]	**0**	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**	**1**
c_{so} [m/s]	1481.6	39.727	29.79	26	24.33	23.83	24.3	25.98	29.7	39.53	343.24

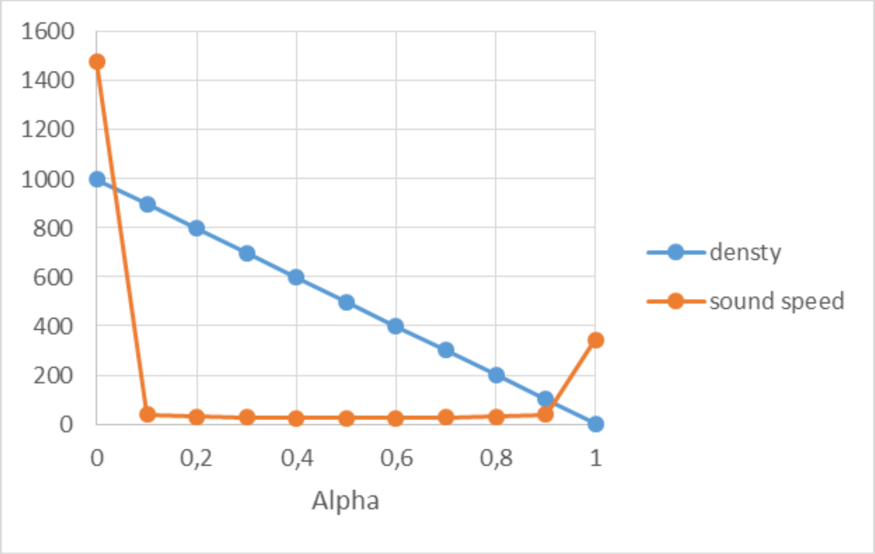

Figure 8: sound speed and density of the fluid in function of alpha

4.3.4. Frequency reduction ratio

The frequency reduction ratio FRR is used to compare frequencies from different mixtures, since it is has been explained in point 3.1 that air frequencies are going to be higher than water ones. It is calculated using equation 11, were ω_{air} is the frequency for air (the highest of frequencies in the different mixtures) and ω is the frequency for each point of study. This ratio is going to be used later on this paper to compare frequencies from different modes.

$$FRR = 100\frac{\omega_{air} - \omega}{\omega_{air}} \tag{11}$$

5. Simulations

5.1. Acoustic modes

The modal frequency of the fluid without the plate was simulated. For this purpose, the analysis was chosen as modal analysis and all solid geometries were eliminated. The simulation was made from α=0 to α=1 with an interval of 0.1, consisting of 11 different mixtures. For each mixture, the first 100 modes of vibration were found. This made a total of 11 simulations, of about 10 minutes each. The total time of simulation was of 2 hours.

As for the order of simulation, the pure fluids were simulated first (water with α=0 and air with α=1). Then, starting from α=0 the fluid quantity of air was increased, changing α 0.1 each time until reaching pure air again.

5.2. Fluid-structure coupled modes

In order to find the different modes of vibration a total of 54 simulations were needed. The first 6 modes were searched for 9 different fluids, with alpha between 0.1 and 0.9, with a gap of 0.1. For each surrounding fluid, the parameters Mass Density and Sound Speed were changed. The results found on paper [1] were added to the ones found in this paper, getting a total of 11 different mixtures with alpha between 0 and 10, with a gap of 0.1.

The simulation order was decided as the following: for each mode, the fluid was started as water (α=0) and then moved towards air incrementing α 0.1 each time, until reaching a pure air fluid (α=1). The first model simulated was s1, followed by s2 and so on.

The range of simulation was chosen as follows: for α=0.1, each simulation was started at the last known frequency mode, and increased until the next wanted mode of vibration was found. For the other mixtures, it was assumed that a similar frequency profile in front of alpha to mixture α=0.1 was followed. Probable frequency modes were found this way, and it was around these values that the real modes were looked for.

Some troubles were encountered during the simulation process, so the simulation plan was slightly changed. When the mode 3 simulations started, the mode of vibration for 0.1 was not found within the expected range. It was decided to skip mode 3 and look at mode 6 first, since the modes are more recognizable at higher frequencies. When the correct mode was still not found in mode 6, a different strategy was taken. Going back to mode 3 the parameter alpha was reduced in order to see exactly how the mode was moving because of the fluid change, simulating mode 3 for alphas 0.05 and 0.07. The 3rd mode shape for alpha 0.1 was still not found.

The average time of a simulation is 3 hours. It should also be mentioned that not all modes of vibration were found at the first simulation, making the total of simulations made a lot higher than the 54 simulations needed, up to 64 simulations, and still not all modes were found. This makes the total time of simulation 192 hours.

6. Acoustic modes

These simulations consist of extracting all solid elements from the model and analysing only the remaining fluid and its modes of vibration. The first 100 modes were found for each mixture, some of which are symmetrical mode shapes at the same frequency.

If the different mode frequencies are plotted for a same mixture, as it can be seen in figure 9 for α=0.5, it is observed that it follows an exponential curve. This means that at higher frequencies the number of acoustic modes in a range is higher than at low frequencies. If we compare this curve for the pure fluids and a mixture (figure 10), it is shown that the mixture has a more horizontal curve than pure fluids. This means an increase in the number of possible mode interactions using a mixture of air and water.

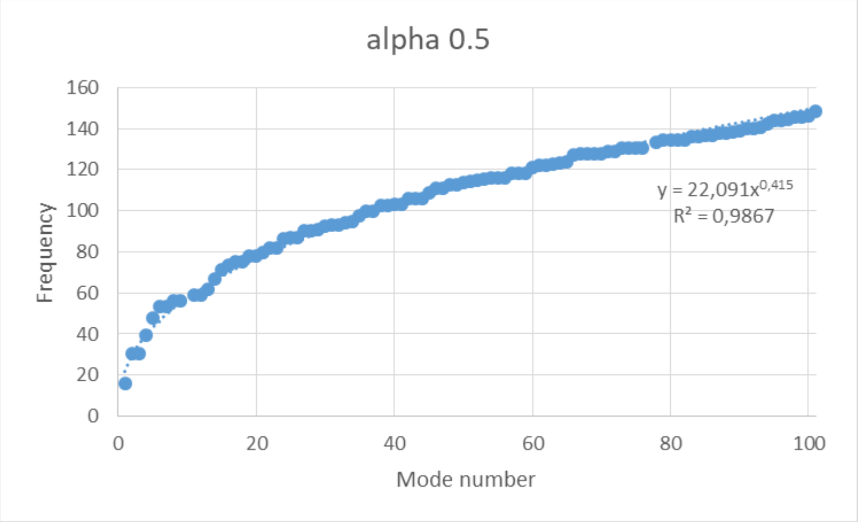

Figure 9: Mode frequency in front of mode number for alpha 0.5

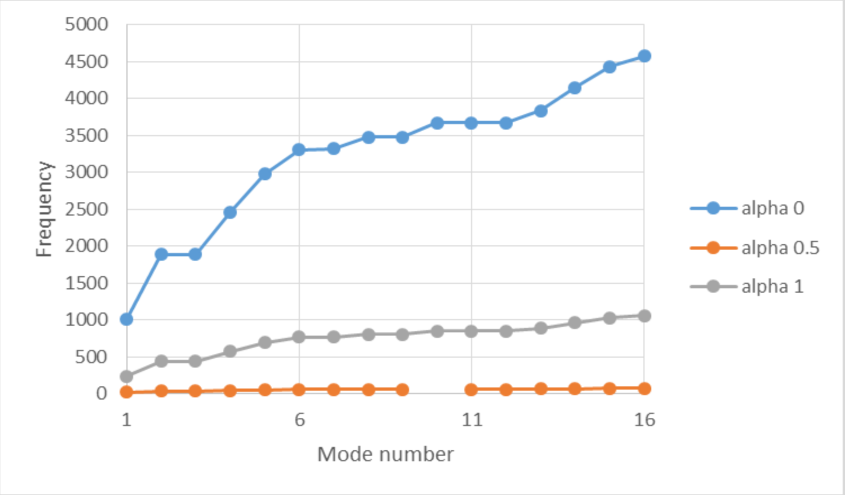

Figure 10: Mode frequency in front of mode number for alpha 0, 0.5 and 1

If the frequencies for each mode are plotted in front of alpha, it is observed that whilst they are quite distant in the pure fluids (water and air), they became instantly together at the minimum appearance of the other. This can be seen in figure 11, where the first 25 modes are plotted for the different mixtures. This behaviour can be explained since sound speed is dominant in the control of the acoustic modes, so the frequency graph in front of alpha resembles that of the sound speed in front of alpha. This sudden aggrupation of the acoustic modes means that the number of possible interactions with the plate in a mixture is going to be higher than in the pure phases.

A zoom of the lower frequencies can be seen on figure 12, where the mode frequency changes can be observed more accurately. It can also be seen that at higher frequencies the modes of vibration become closer to one another. This means that for a mixture fluid at higher frequencies (higher plate mode number), the number of possible interaction modes is going to be higher.

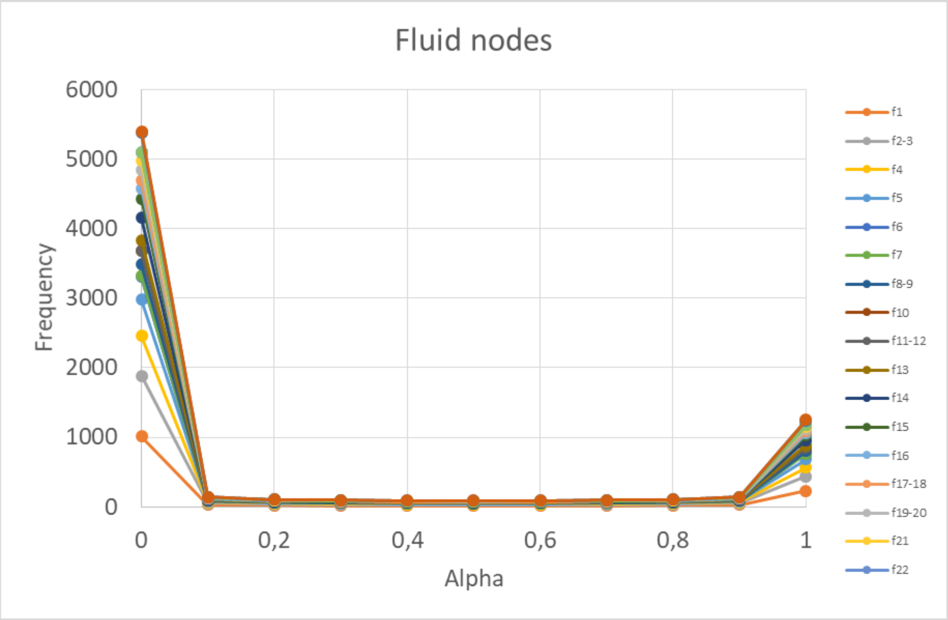

Figure 11: first 25 fluid nodes frequency in front of alpha.

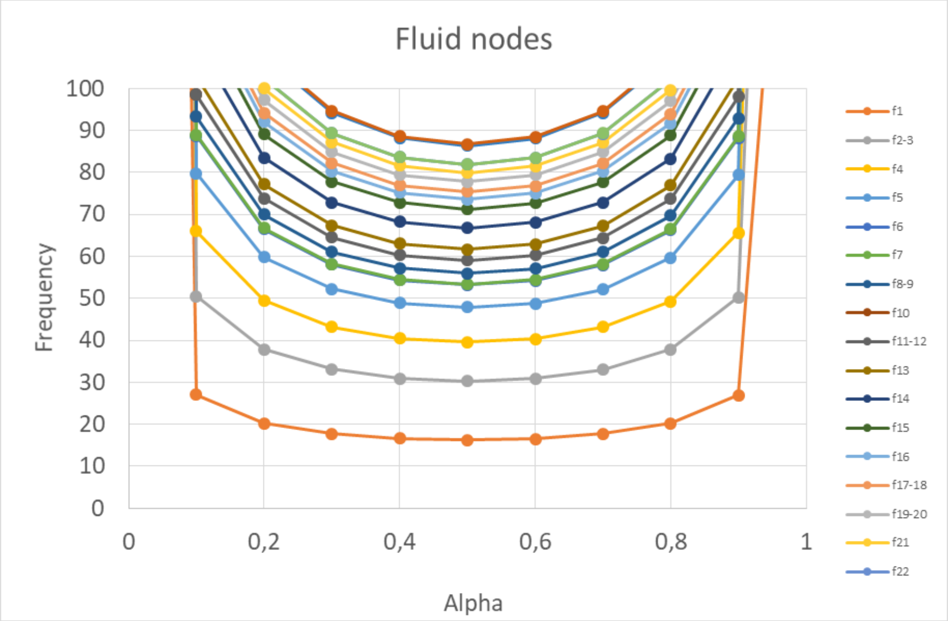

Figure 12: Zoom of the lower frequencies of figure 11.

If the *acoustic* modes are now plotted in front of the sound speed, we can see (figure 13) as has been proved in previous studies (mentioned in point 3.3.2) a linear behaviour. The frequency follows a straight line that starts at zero Hz and has a higher slope for higher mode number. In the current project the mixture sound speed does not follow a linear variation, as explained in point 4.3.3, so this linear behaviour is not relevant to the current simulations.

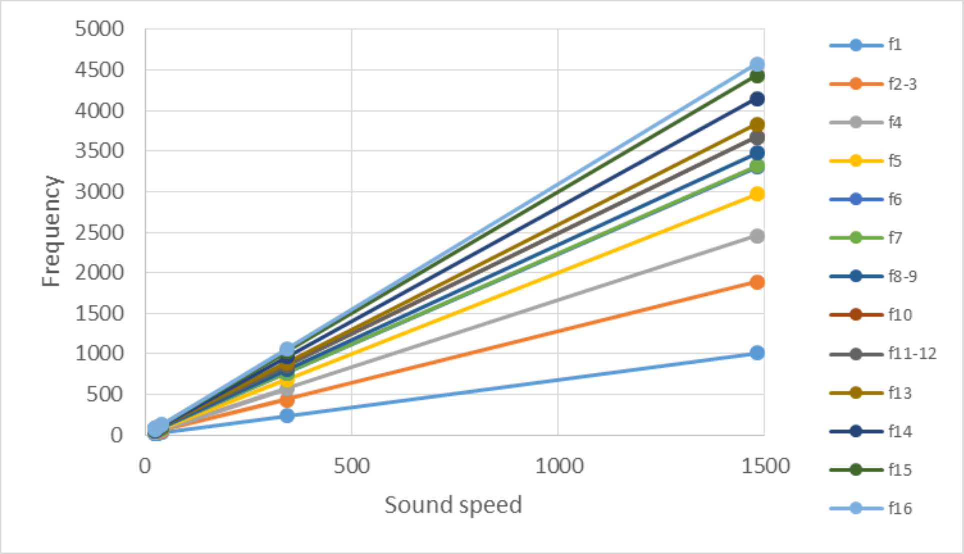

Figure 13: Mode frequency in front of sound speed for the first 16 acoustic modes

The 12 first mode shapes for water are shown in figure 14. In modes like F1 where the acoustic mode shape is not similar to the plate mode shape, even if the frequencies at which they appear are similar there will be no coupling effect. For modes similar to the plate mode shape, a coupled effect may occur where the fluid and the plate mode interact and the acoustic mode may be dominant, making the finding of the plate mode impossible. The actual interaction between fluid and plate modes is studied in point 8.

It should be mentioned that the acoustic mode shape number 10 was only found for the pure fluids and the mixture with alpha 0.1, and it is located at the exact same frequency as mode shape 8-9. For the rest of mixtures, the mode shape 11 was found after the 9th. In all of the following figures, the mode shape 10 has been left out.

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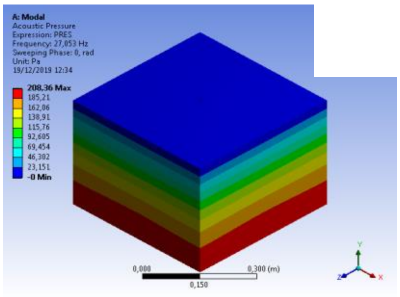

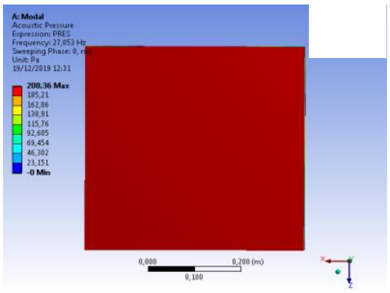

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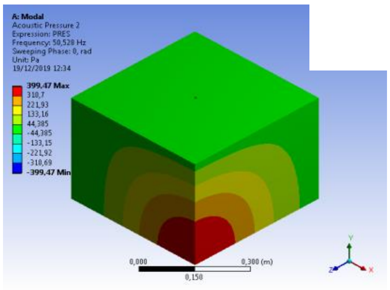

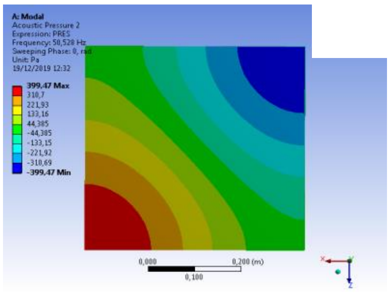

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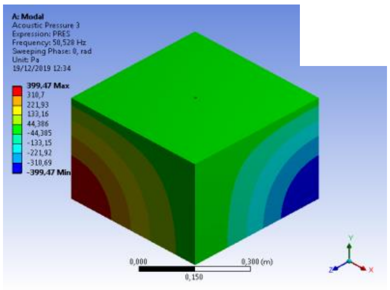

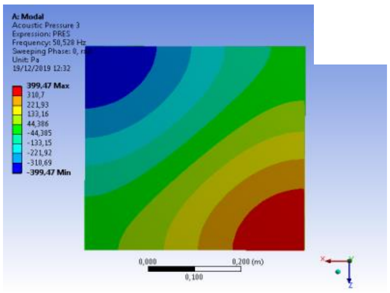

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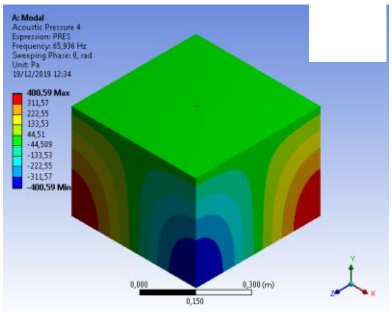

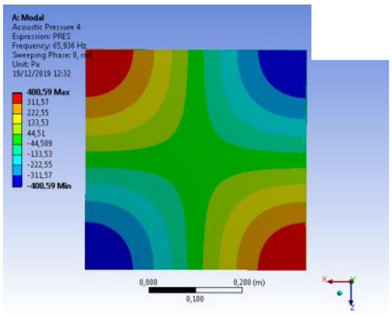

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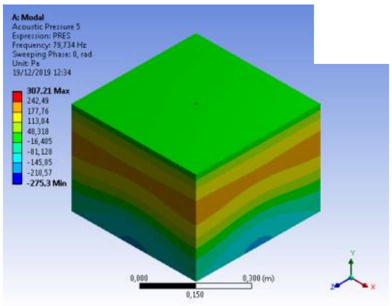

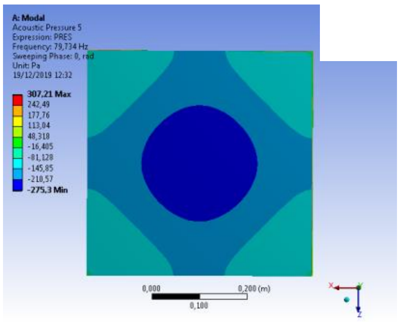

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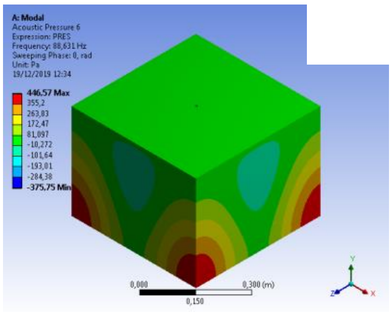

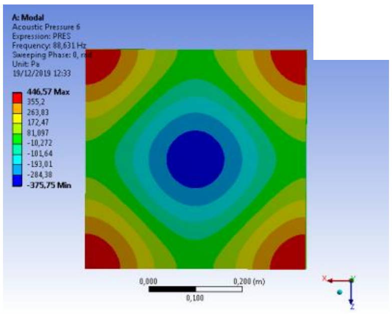

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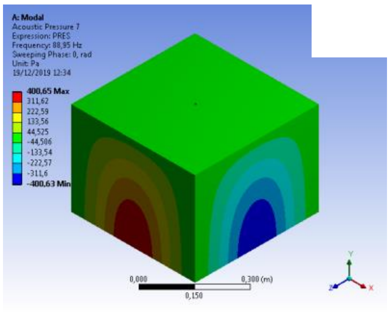

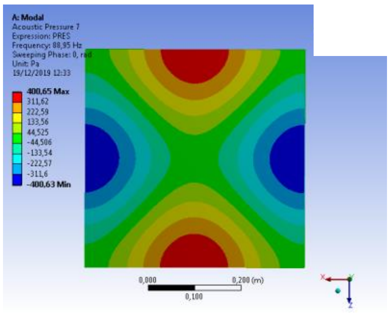

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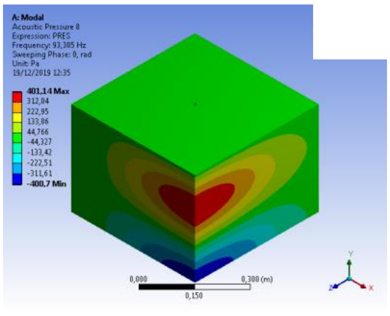

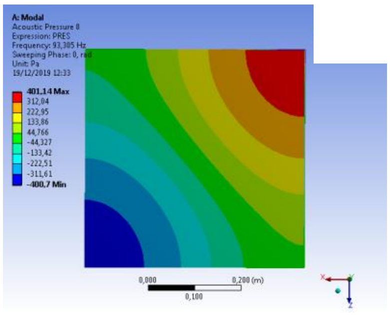

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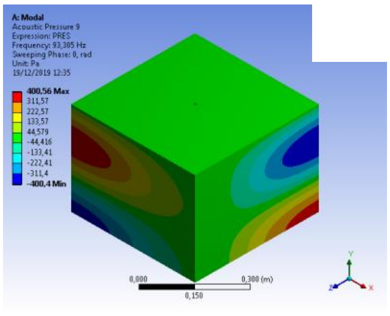

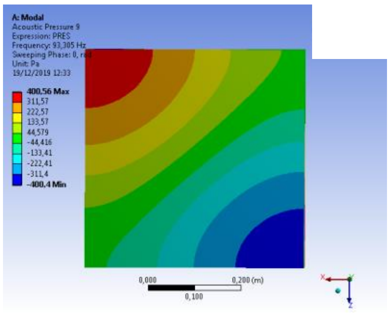

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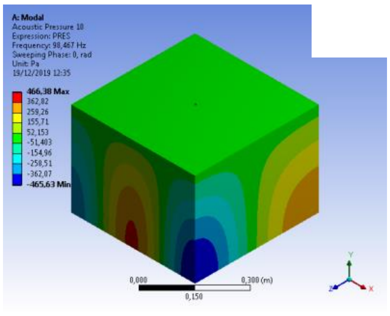

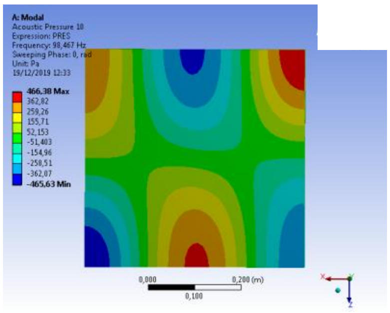

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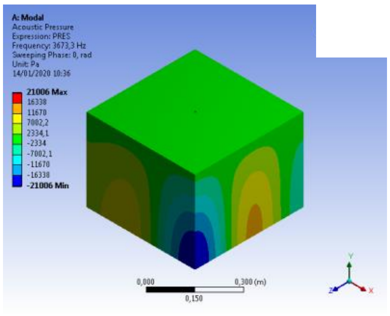

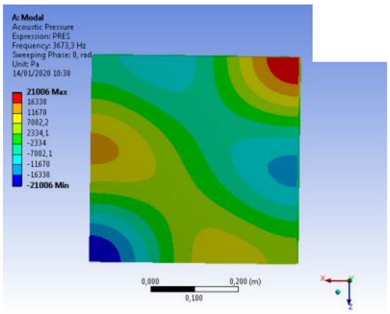

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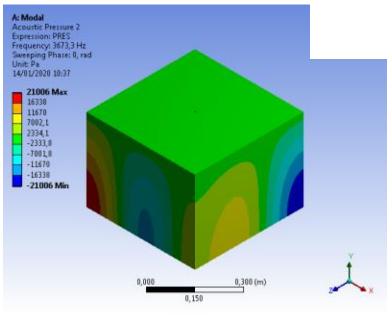

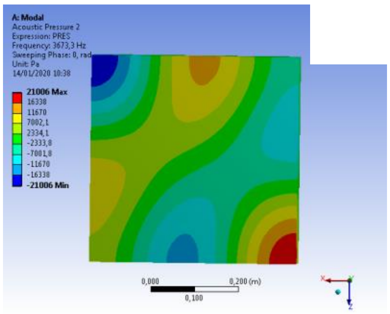

Figure 14: isometric and bottom view of the first 12 acoustic mode shapes for water

The acoustic mode shapes were also studied in different mixtures to observe if its properties affect the mode shape. For the first 5 acoustic modes, the mode shapes for $\alpha=0$, $\alpha=0.5$ and $\alpha=1$ can be seen in figure 15. The acoustic mode 25 was also studied, to see if higher frequencies cause a higher mode shape difference. It was observed that the mode shape did not depend on the fluid, since the acoustic mode shape doesn’t change when the mixture is changed.

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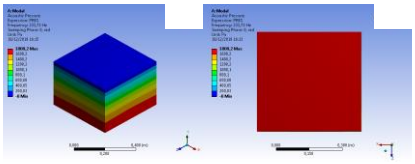
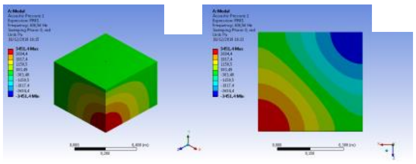
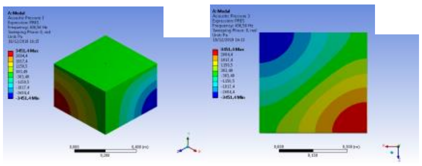
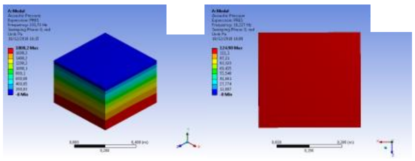
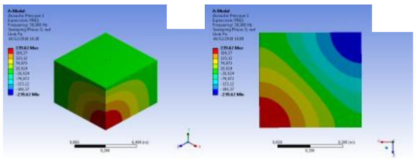
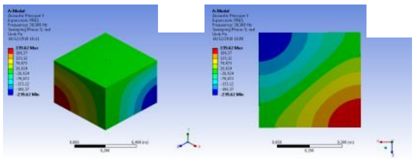
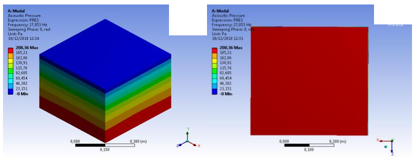
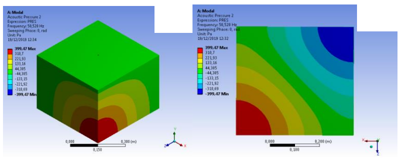
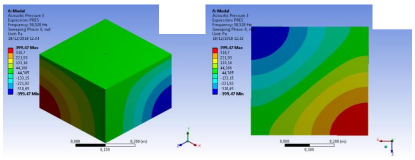
0			
0.5			
1			

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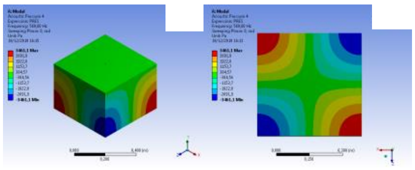
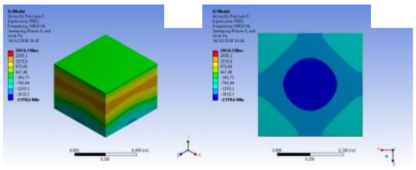
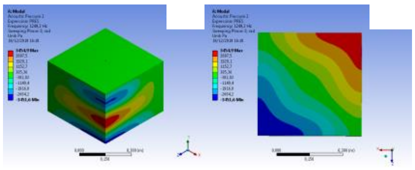
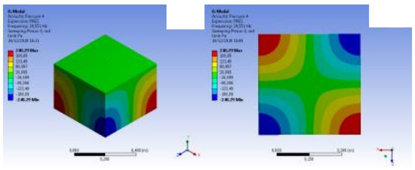
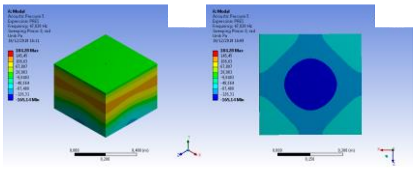
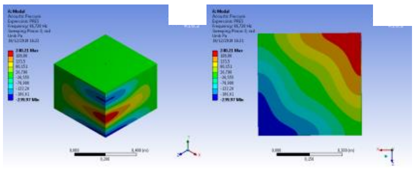
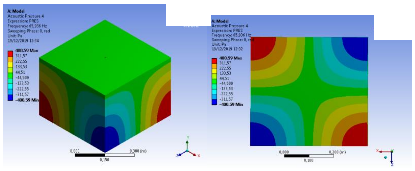
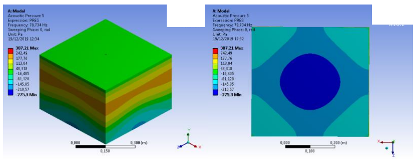
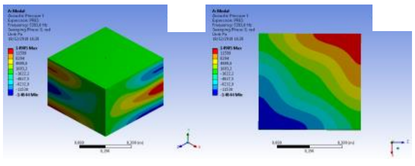
0			
0.5			
1			

Figure 15: evolution of acoustic mode shapes for different alphas.

7. Fluid-structure coupled modes

As mentioned in point 5, the plate modes of vibration were only found for the first 2 modes in all 11 fluids. When the third mode simulations were started, the axisimetric radial displacement mode shape was not found anymore. An explanation can be found on point 8.

The modal frequencies were identified by a change of sign in the phase diagram at the same time as a maximum value was found in the amplitude graph, as can be seen in figures 16, 17, 19 and 20 (first and second mode for α=0.1). In the first mode these points were more clearly identified than in the second one. It was also checked that the modal shape was the one searched, consisting of a circular axisymmetric deformation, as is shown in figure 18 and 21 (first and second mode for α=0.1).

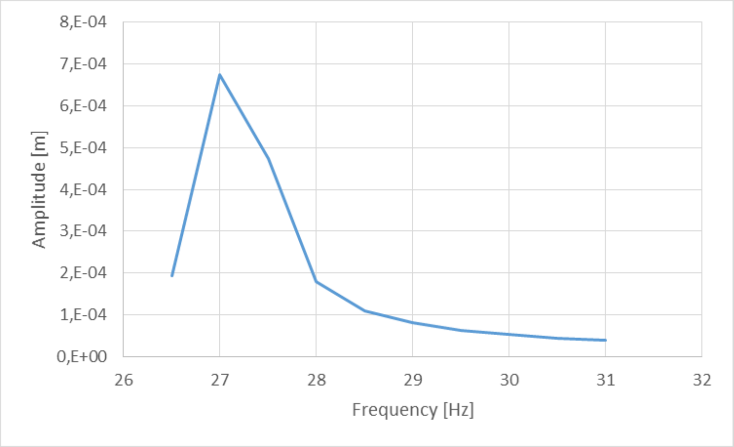


Figure 16: graph amplitude in front of frequency, used to recognize the mode shape 1 for α=0.1

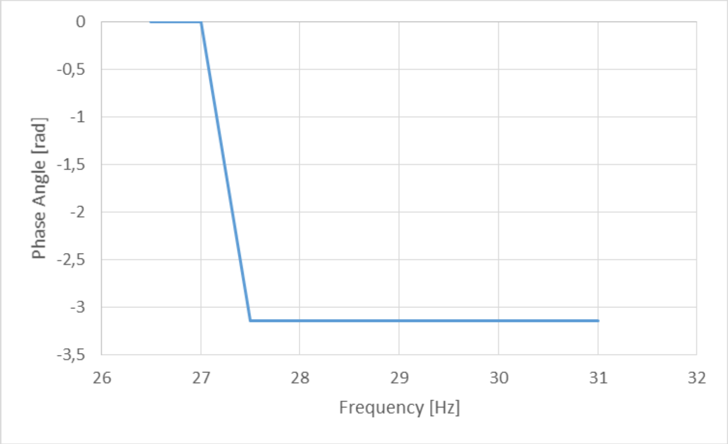


Figure 17: graph phase angle in front of frequency used to recognize the mode shape 1 for α=0.1

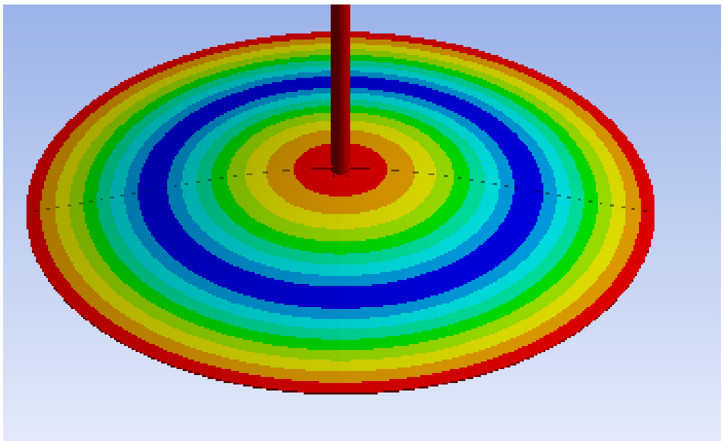


Figure 18: shape for mode 1 and α=0.1

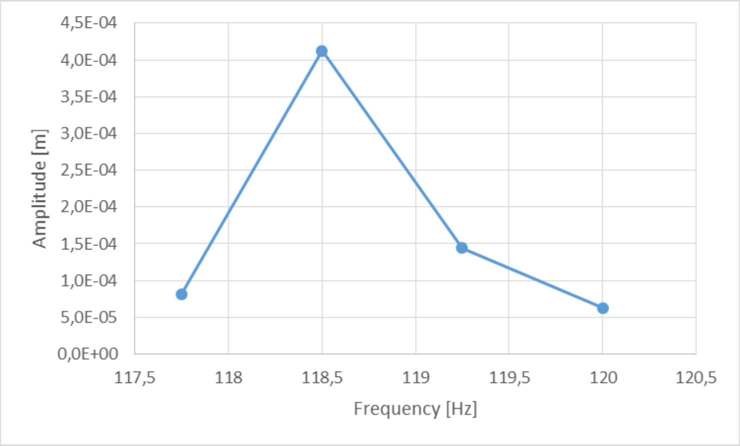


Figure 19: graph amplitude in front of frequency used to recognize the mode shape 2 for α=0.1

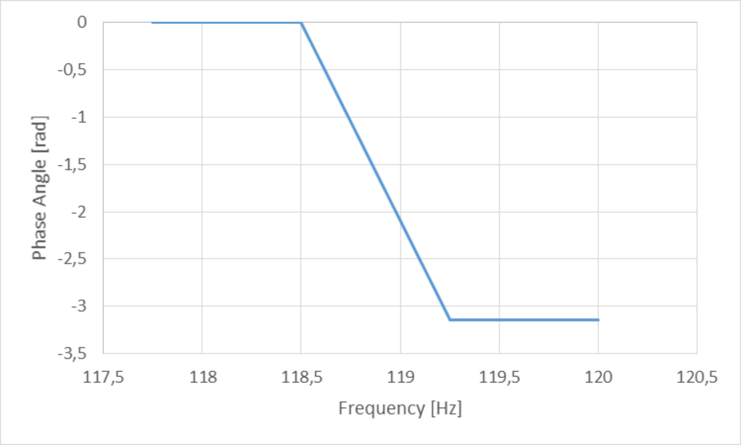


Figure 20: graph phase angle in front of frequency used to recognize the mode shape 2 for α=0.1

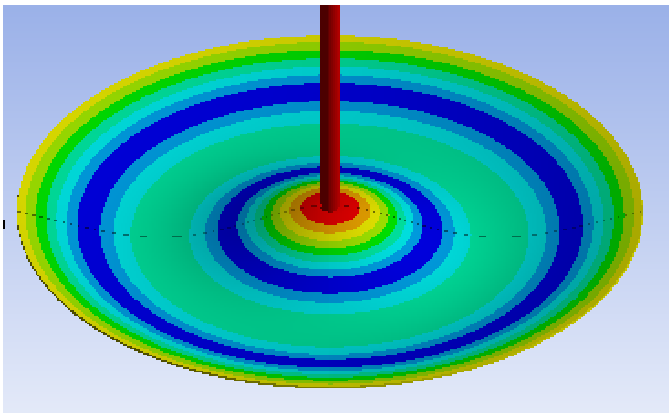


Figure 21: shape for mode 2 and α=0.1

The mode frequency for modes 1 and 2 can be found in table 6, depending on parameter alpha (which defines a specific sound speed and density). The second mode of vibration for alpha 0.9 was not found.

Table 6: mode frequencies for the first 2 modes, in function of alpha

Alpha	**Sound speed**	**Density**	**Mode 1**	**Mode 2**
1	343.24	1.204	70	305
0.9	39.53	100.9	52.667	-
0.8	29.7	200.6	45.25	163
0.7	25.98	300.24	40.2	155.67
0.6	24.3	399.9	36.75	144
0.5	23.83	499.6	34.2	138.6
0.4	24.33	599.28	31.714	132
0.3	26	696.96	29.75	126.17
0.2	29.79	798.64	28.333	122.25
0.1	39.727	898.32	27	118.5
0	1481.58	998	26	116

If we compare the frequencies in front of alpha for mode 1 (figure 22) and mode 2 (figure 23), it can be seen that the first one has a higher frequency interval between two consecutive points, whilst mode 2 is more linear with a wider gap at the last point. It should also be taken into account that the point for alpha 0.9 on mode 2 is missing. Both modes follow a parabolic equation.

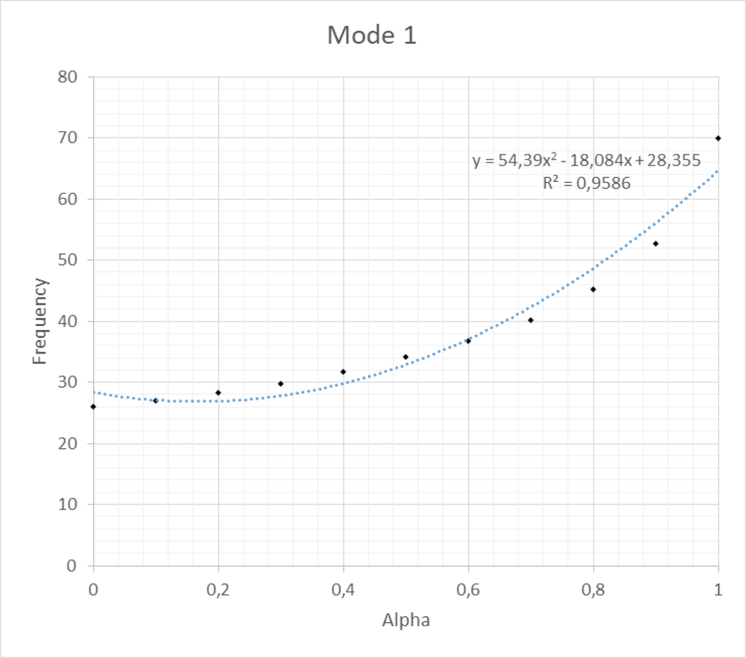


Figure 22: mode 1 frequency in front of alpha

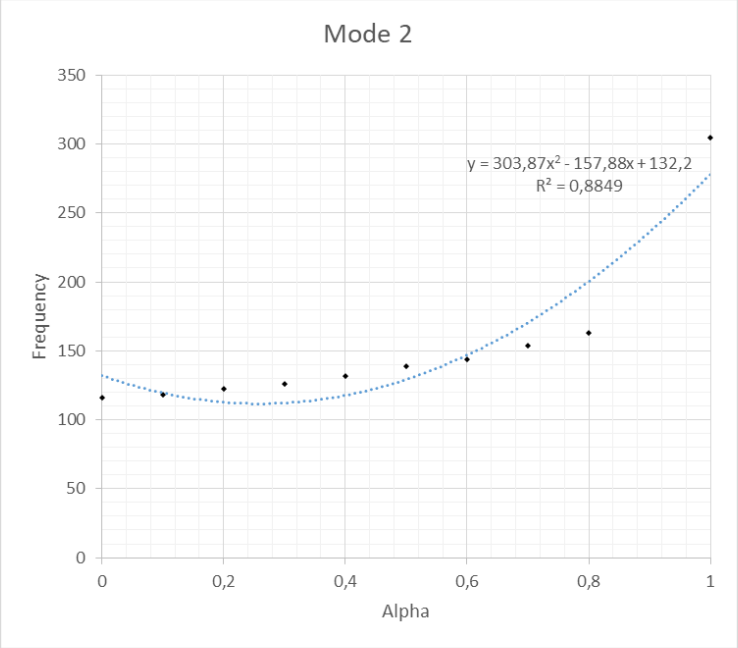


Figure 23: mode 2 frequency reduction ratio in front of alpha

If the mode frequency is plotted in front of the density for both modes (figure 24 and 25), it can be seen that there is a clear influence between these parameters. Especially in mode 1 an exponential relation is found between the fluid density and the mode frequency. In mode 2 the relation in not as good, but the missing point may be the problem.

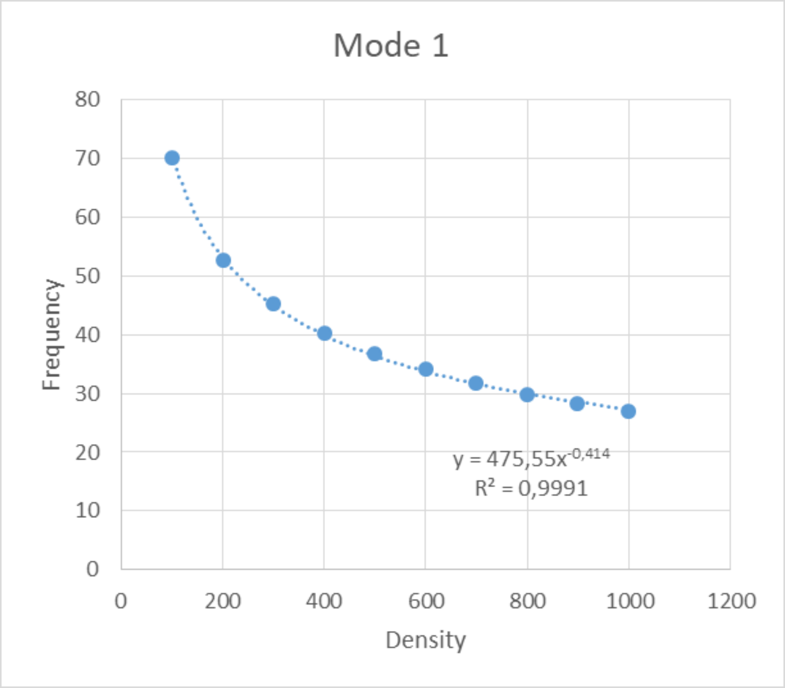


Figure 24: Frequency in front of density for mode 1

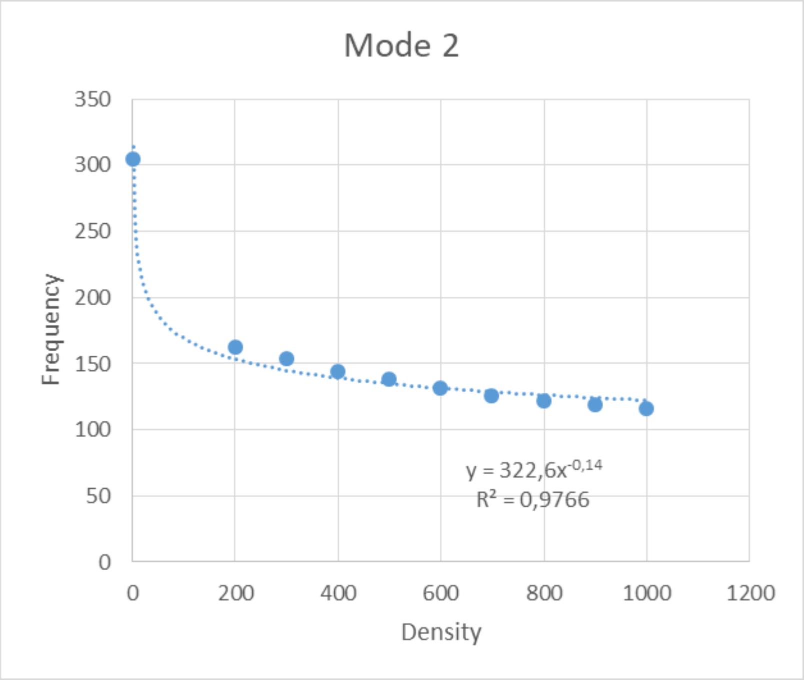


Figure 25: Frequency in front of density for mode 2

If the mode frequency for both modes is plotted in front of its sound speed (figure 26 and 27), a clear relation cannot be found between both parameters. Its form may resemble that of the sound speed in front of alpha graph (figure 9) but turned sideways.

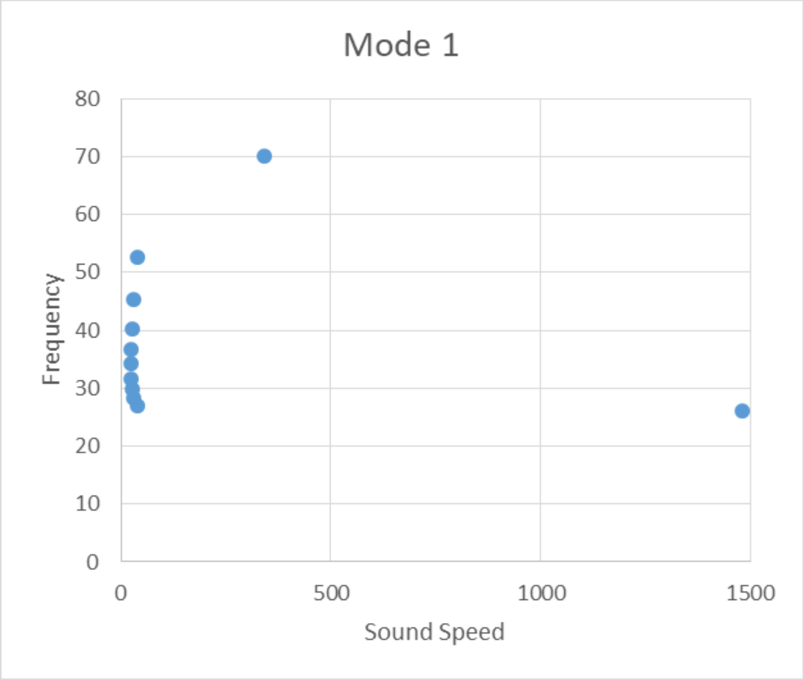


Figure 26: mode 1 frequency in front of fluids sound speed

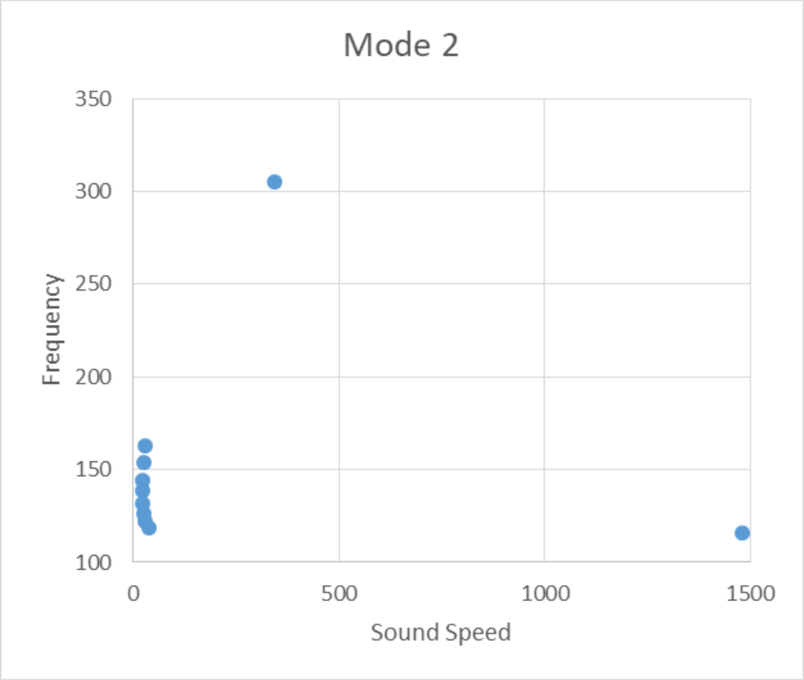


Figure 27: mode 2 frequency in front of sound speed

The frequency reduction ratio FRR (explained in point 4.3.4) is now used to compare different modes of vibration. In figure 28 the first two modes are plotted in front of alpha. It can be observed that FRR for mode 2 are slightly above those of mode 1. It is then expected that mode 3 FRR should be even higher. This can also be seen in figure 29 where both modes are plotted in front of density. Finally, both modes are plotted in front of sound speed in figure 30.

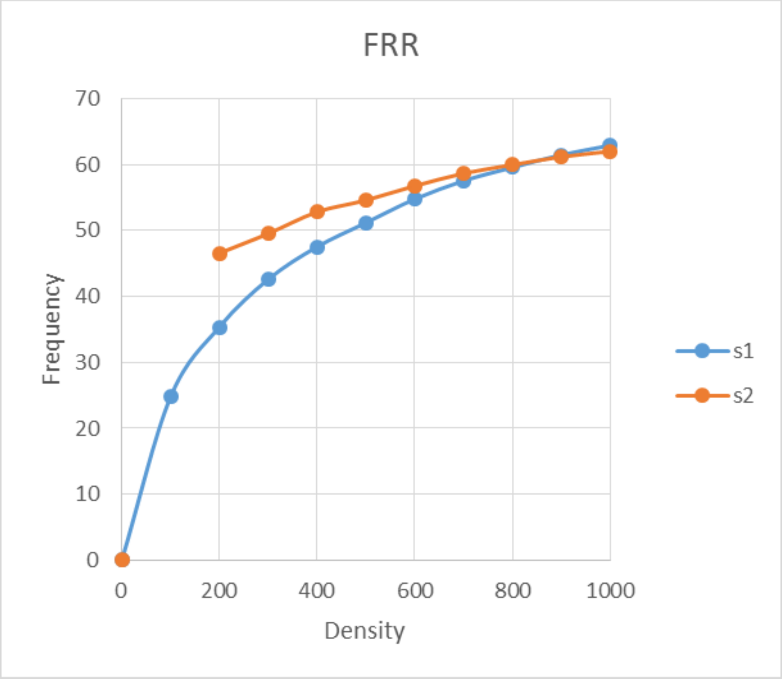


Figure 28: mode 1 (s1) and 2 (s2) frequency reduction ratio in front of alpha

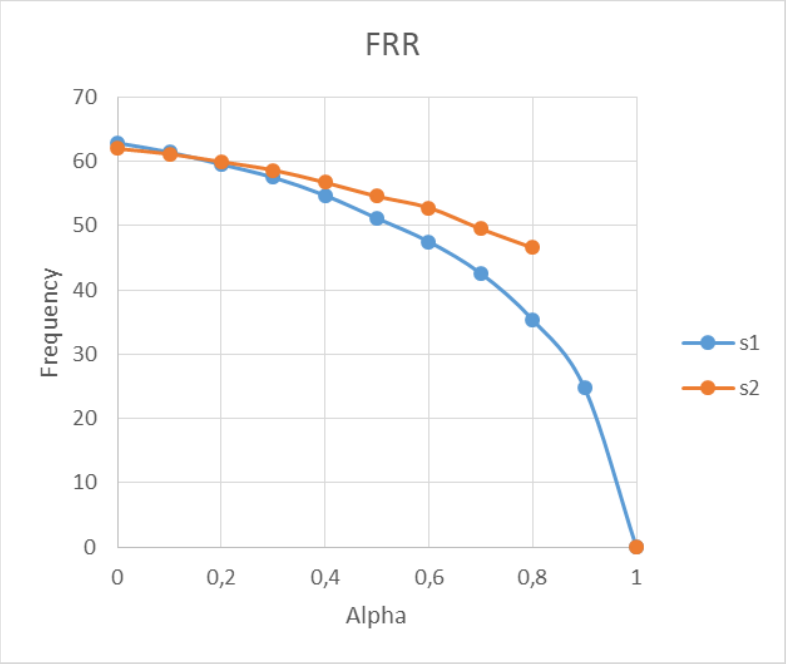


Figure 29: mode 1 (s1) and 2 (s2) frequency reduction ratio in front of density

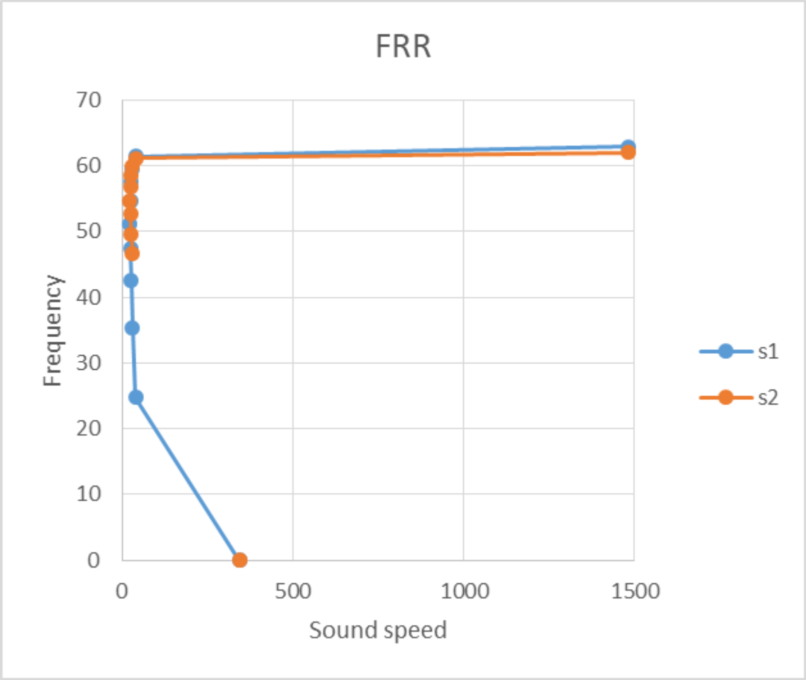
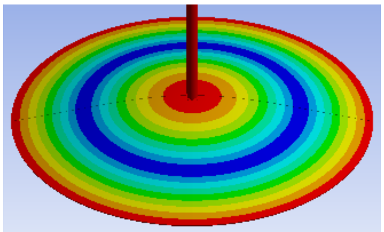
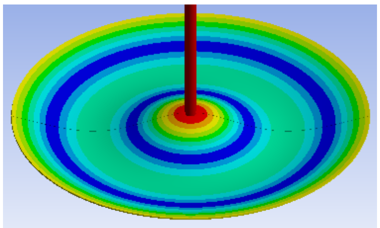
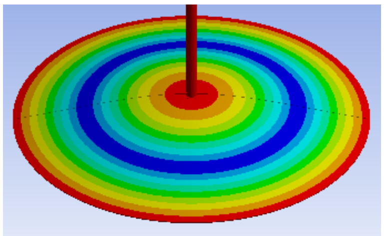
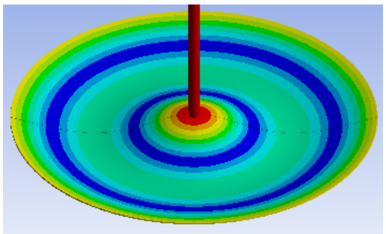
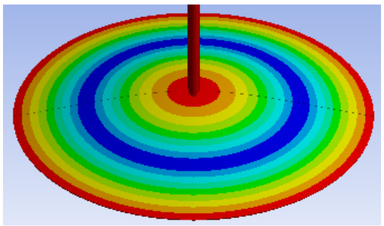
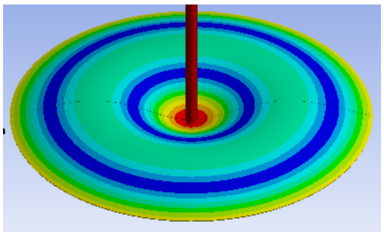
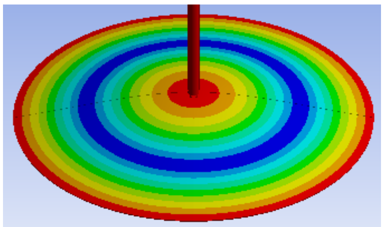
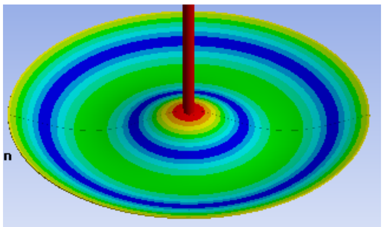
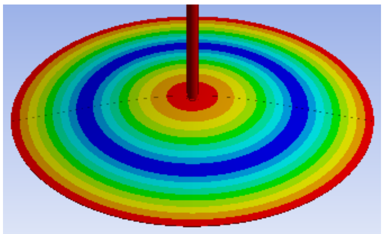
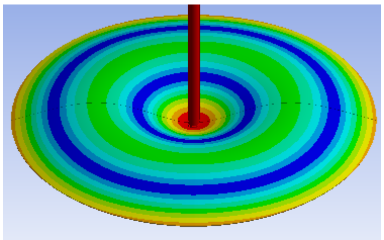


Figure 30: mode 1 (s1) and 2 (s2) frequency reduction ratio in front of sound speed

The different mode shapes found for both modes are shown in figure 31. It is observed that for mode 2 some shapes are slightly distorted. This is further explained in point 8.

α	Mode 1	Mode 2
0		
0.1		
0.2		
0.3		
0.4		

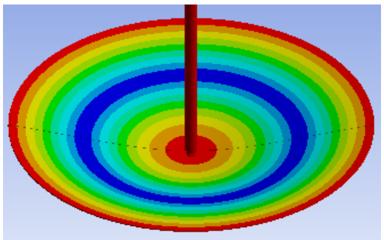
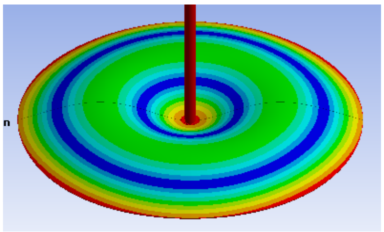
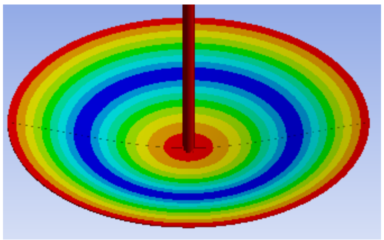
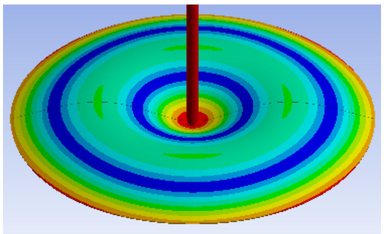
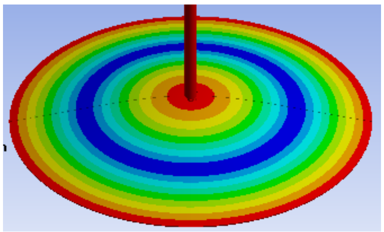
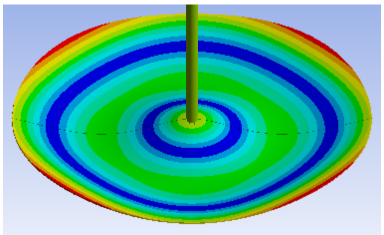
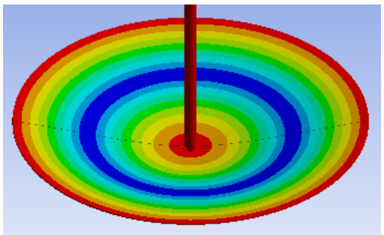
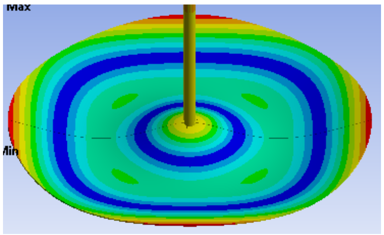
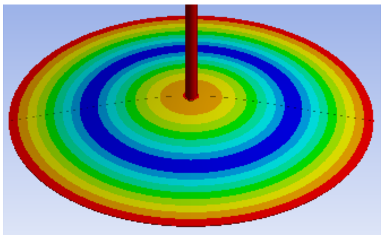
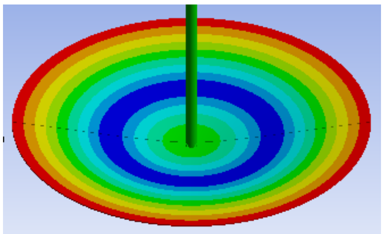
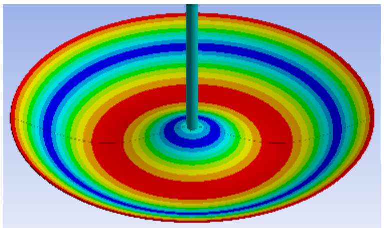
0.5		
0.6		
0.7		
0.8		
0.9		-
1		

Figure 31: plate mode shapes for the different fluids

7.1. Fluid-structure coupled mode shape changes

Being *r* the radial distance between the plate axis and the points of zero vertical displacement (see figure 32), and being *a* the outer plate radius that equals 0.12m, the parameter r/a is plotted for the first 2 plate modes. The results are shown in table 7.

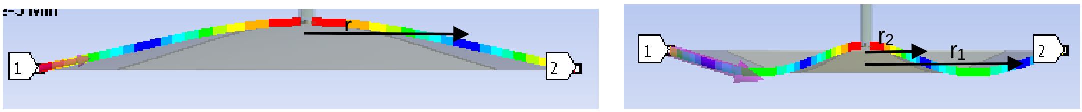


Figure 32: parameter r for mode 1 on the left and mode 2 on the right

Table 7: Parameter r/a in meters for mode 1 and 2, for each different alpha.

Alpha	**r/a [-]**		
	Mode 1	Mode 2	
1	0.453	0.801	0.126
0.9	0.563	-	-
0.8	0.576	0.775	0.245
0.7	0.589	0.735	0.298
0.6	0.589	0.748	0.285
0.5	0.589	0.761	0.285
0.4	0.589	0.788	0.311
0.3	0.589	0.801	0.298
0.2	0.603	0.775	0.325
0.1	0.603	0.775	0.324
0	0.603	0.775	0.325

If r/a displacement is plotted in front of alpha, it can be observed in figure 33 how the outer circle of mode 2 (s2 1) increases while the inner circle (s2 2) suffers a radius decrease. It should be mentioned that small errors in the mode 2 radius may be caused by the coupling of this mode with the acoustic modes, as mentioned in point 8. This makes the behaviour of r/a in the mixture not consistent, so further conclusions regarding the radius displacement may not be extracted from this experiments.

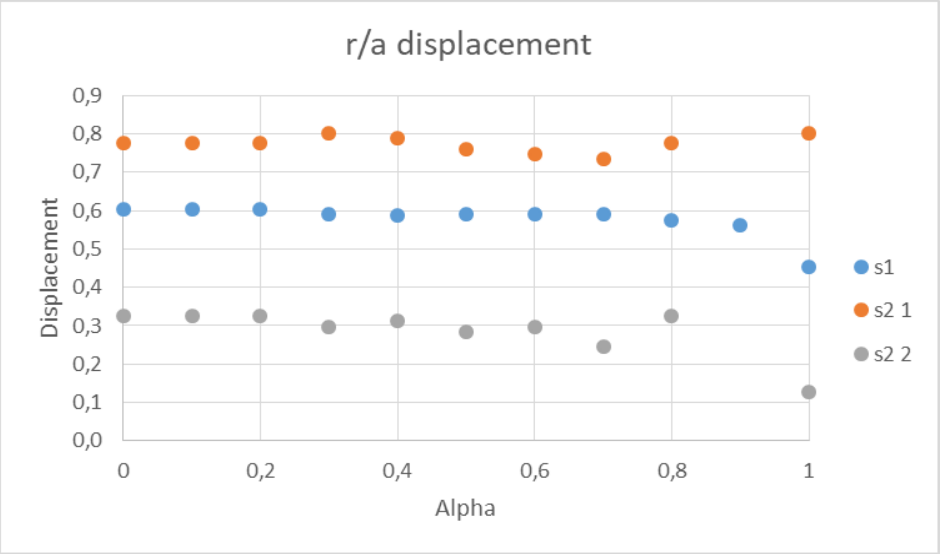


Figure 33: plate mode raius for the first two modes in funtion of alpha

If the radius displacement is plotted in front of the fluids density and its sound speed (figures 34 and 35 respectively), it can be observed that the behaviour is mostly ruled by the parameter density, whilst it presents a rare behaviour in front of sound speed.

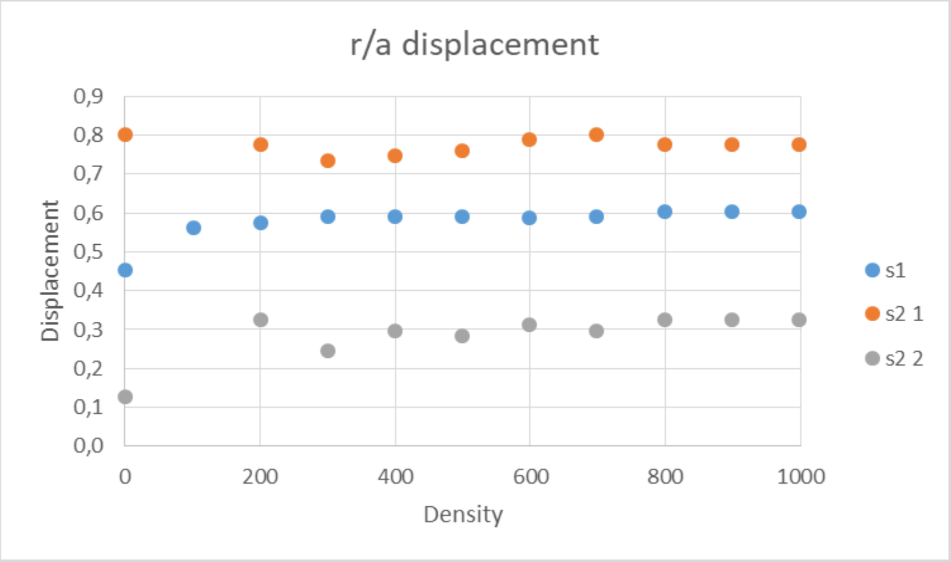
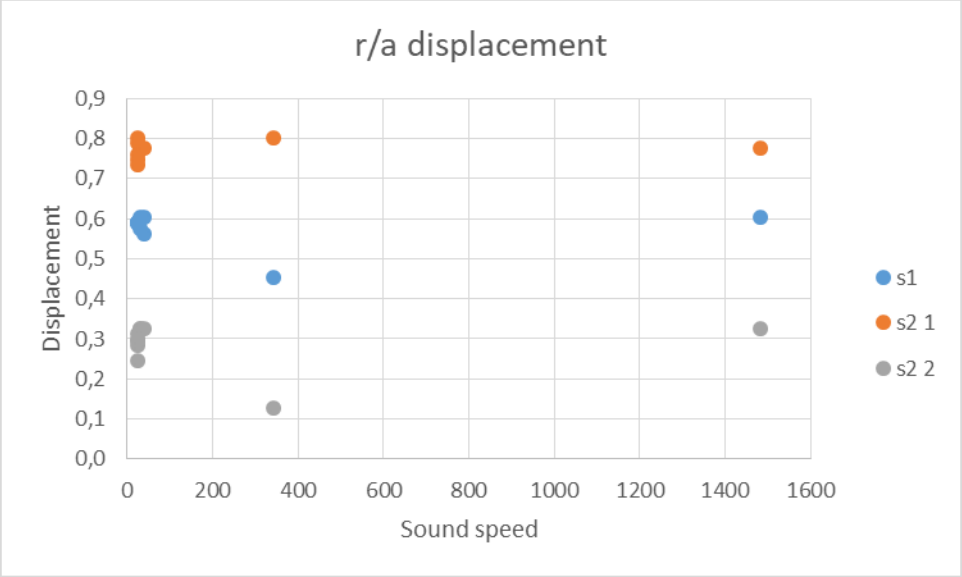


Figure 34: plate mode raius for the first two modes in funtion of fluids density



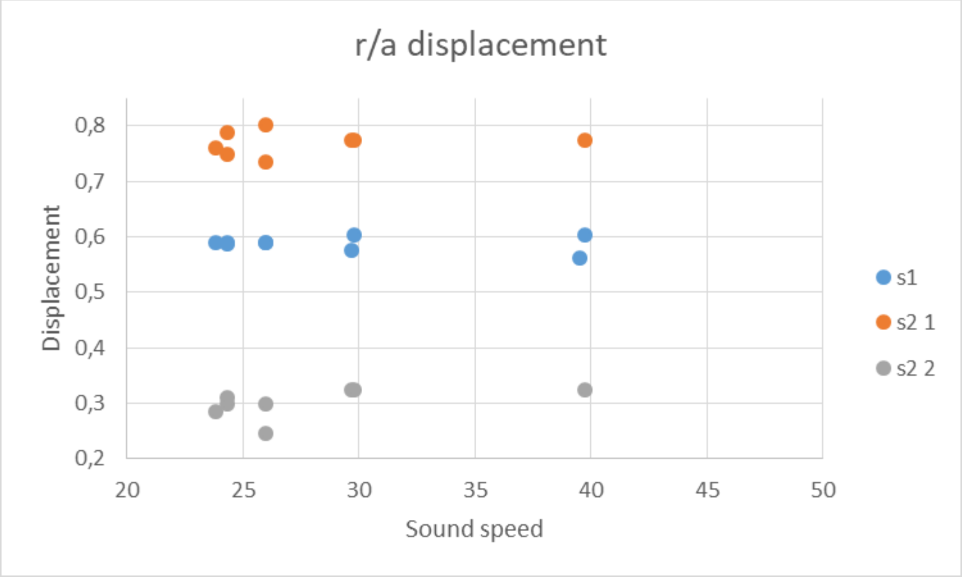


Figure 35: plate mode raius for the first two modes in funtion of fluids sound speed. On the bottom, a zoom for lower sound speed

8. Plate and acoustic mode interaction

The interaction between the two first plate modes and the acoustic modes is going to be studied now.

The first plate mode found in part 7 is plotted with the first 4 acoustic modes, which are the ones it may interact with. As it can be observed in figure 36, the possible interaction occurs in alpha 0.1, where the plate mode is close to the acoustic mode f1 and in alpha 0.4 where the plate mode is close to the acoustic mode f2-3.

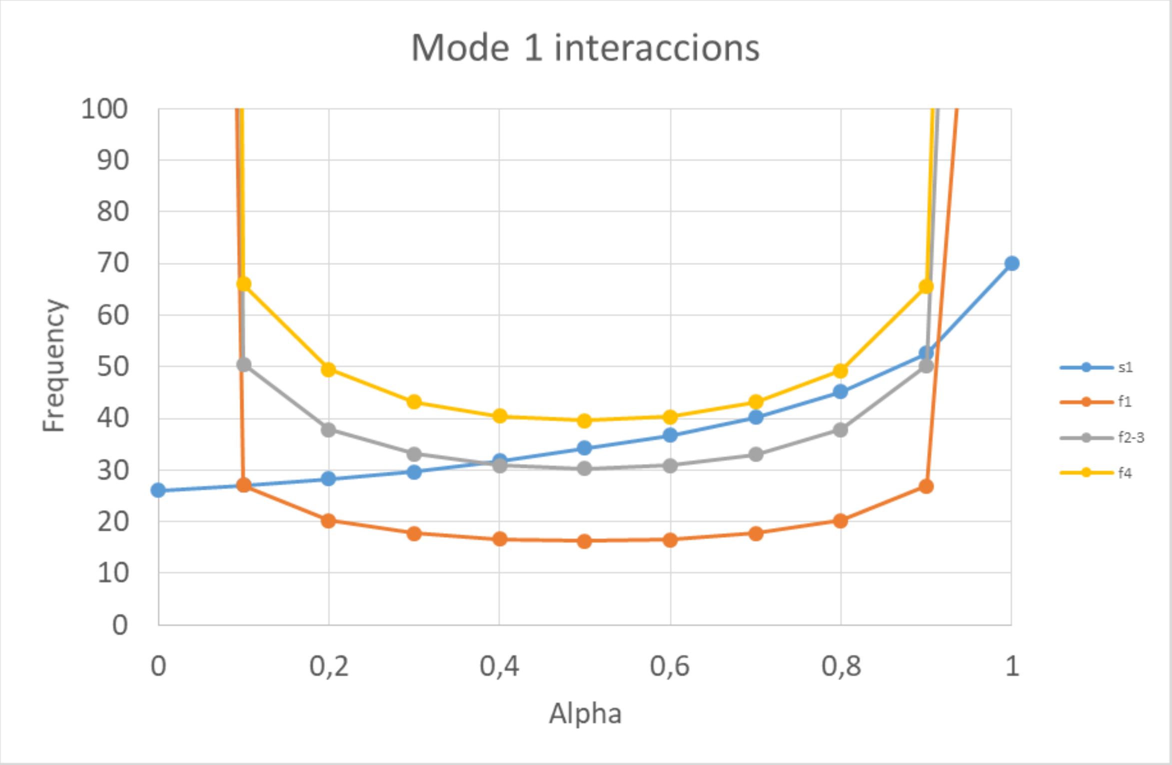


Figure 36: 1st plate mode frequency (s1 in blue) and first 4 fluid nodes frequency in front of alpha (f1-f4)

<table>
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f1</th><th>α=0.4
f2-3</th></tr>
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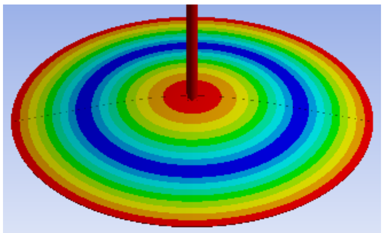
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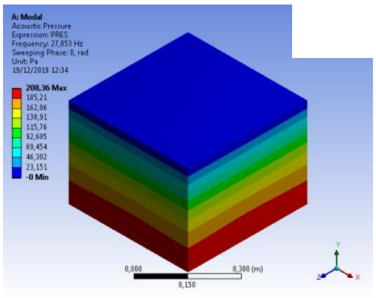

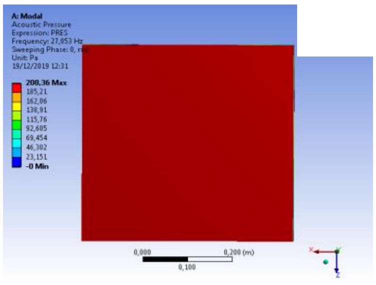

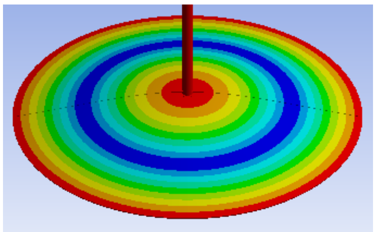
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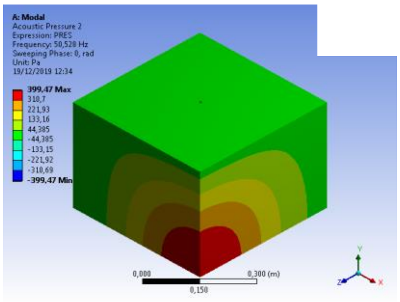
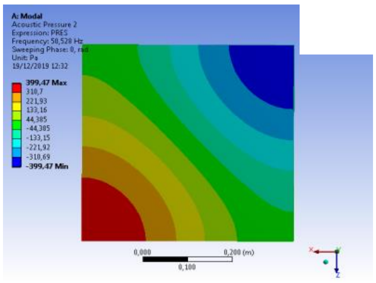




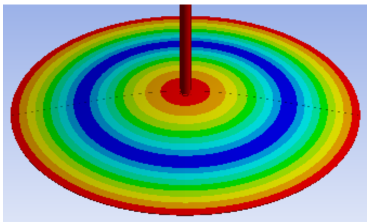

</td>
</tr>
</table>

Figure 37: acoustic mode shape of the possible interactions with the 1st plate mode

In figure 37 the acoustic mode shapes that can interact with the plate mode are studied. Since the shapes found are not similar to the axisymmetric shape of the plate mode, it can be concluded that there will be no coupling effect and the plate mode is not going to be affected.

The second plate mode found in point 7 is plotted in figures 38-40 together with the acoustic modes that may cause coupling. It is observed that the possible interactions are: α=0.1 with f15, α=0.2 with f35, α=0.3 with f53, α=0.4 with f71-72, α=0.5 with f89-90, α=0.6 with f93, α=0.7 with f92, f93 or f94 and α=0.8 with f73, f74, f75 or f76.

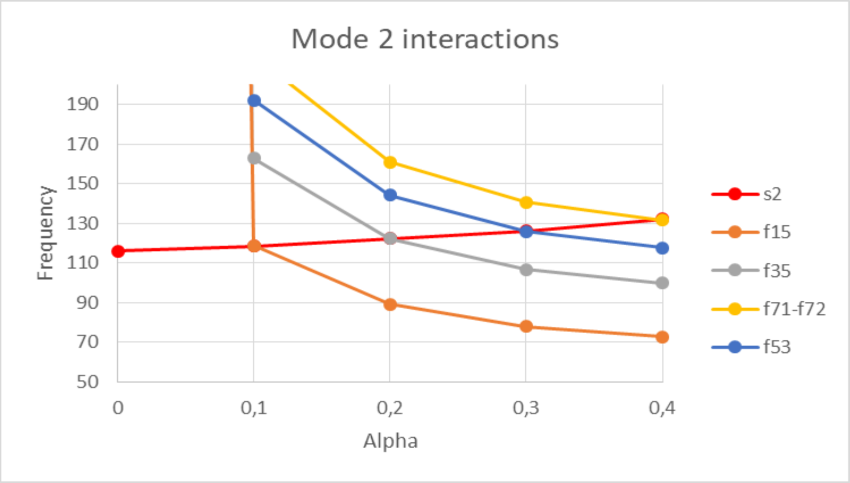


Figure 38: 2nd plate mode frequency (in red) and its possible acoustic mode interactions for alphas from 0 to 0.4

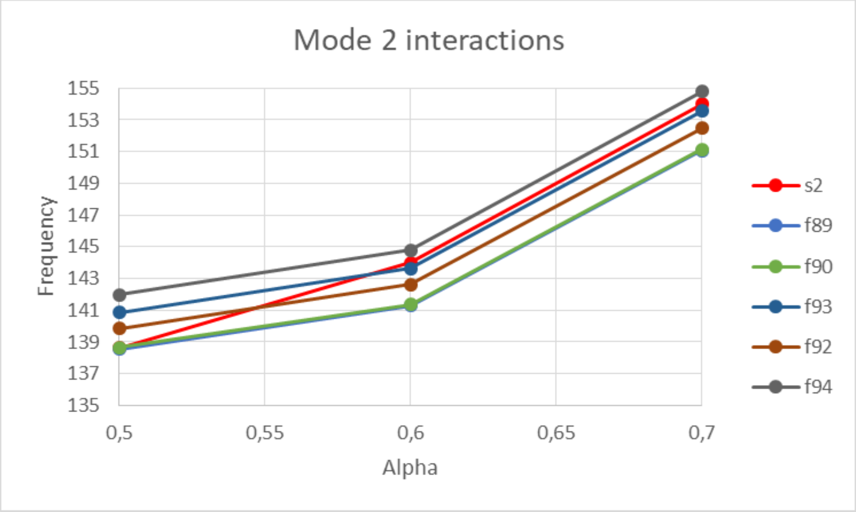


Figure 39: 2nd plate mode frequency (in red) and its possible acoustic mode interactions for alphas from 0.5 to 0.7

Figure 40: 2nd plate mode frequency (in red) and its possible acoustic mode interactions for alpha 0.8

The mode shapes of the possible interactions are studied, as shown in figures 41 to 46, to look for possible coupling effects. At alpha 0.4 (figure 42) these effects start to show. It can be seen that the plate mode is slightly rectangular, showing some orange colour deformation at some points of its outer radius but not in all of it. The same effects are shown in alpha 0.5 (figure 43) and alpha 0.6 (figure 44). For alpha 0.7 (figure 45) the coupling is more clearly shown, deforming the plate mode more obviously. This is also the case of alpha 0.8 (figure 46), where a clear rectangular mode shape is shown for the plate. This different coupling levels are due to the different acoustic mode shapes that interact with the plate, since some are more easily going to affect its shape than others.

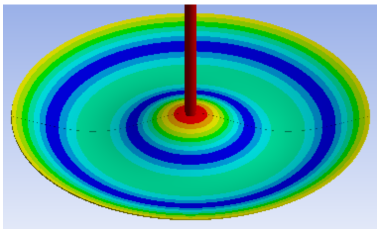
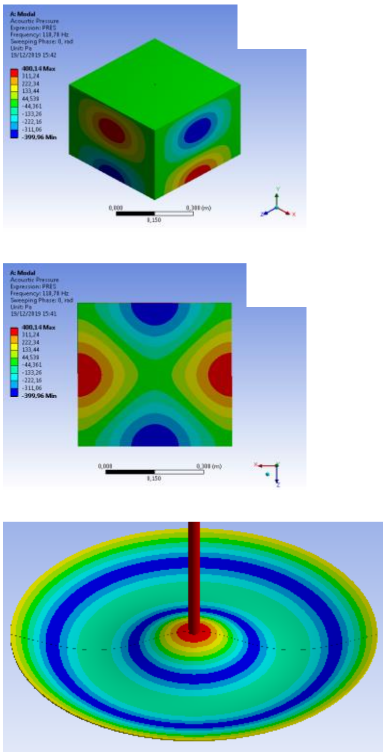
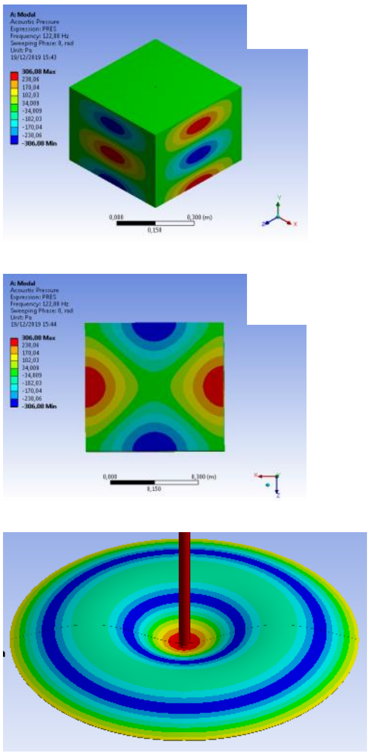
α=0	α=0.1 f15	α=0.2 f35
		

Figure 41: possible interactions for alpha 0.1 and 0.2. Isometric and bottom view of the acoustic mode. Plate mode at the bottom. On the left, plate mode for water (where there is no interaction)

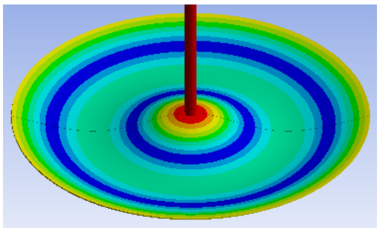
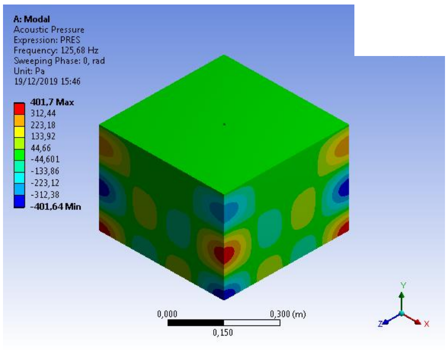
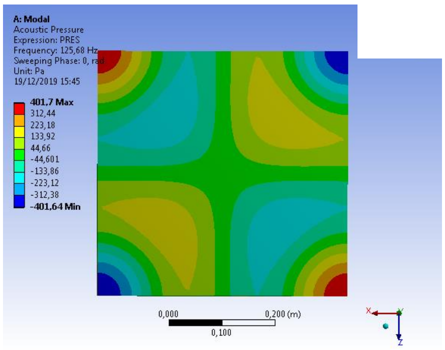
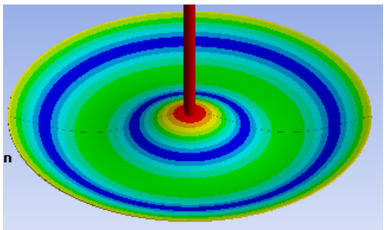
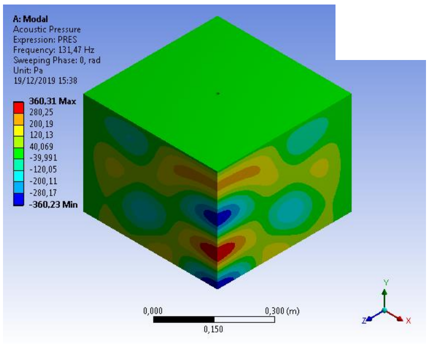
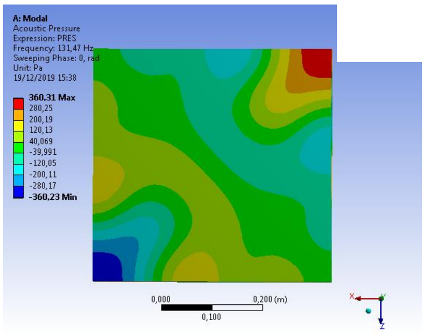
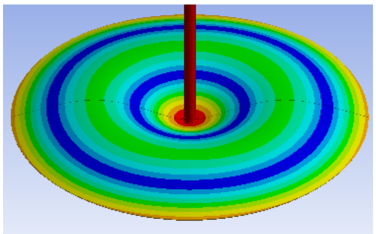
α=0	α=0.3 f53	α=0.4 f71-72
	  	  

Figure 42: possible interactions for alpha 0.3 and 0.4. Isometric and bottom view of the acoustic mode. Plate mode at the bottom. On the left, plate mode for water (where there is no interaction)

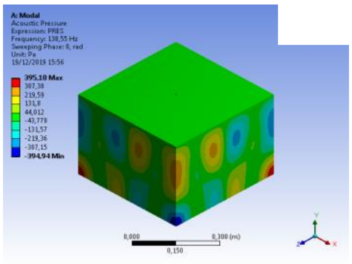
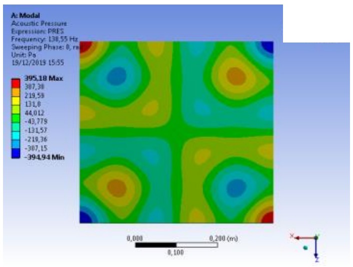
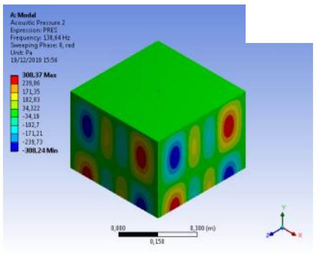
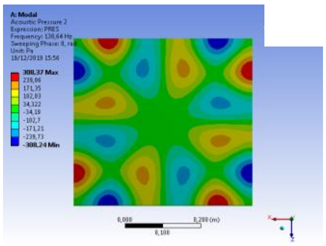
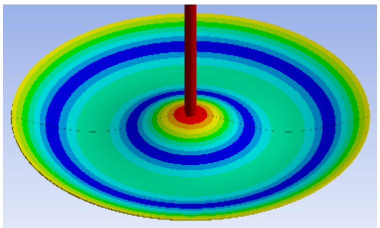
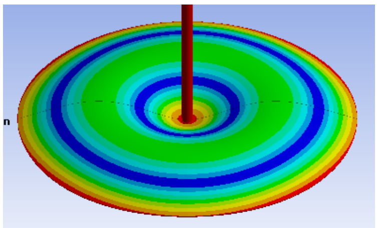
α=0	α=0.5	
	f89	f90
	 	 
		

Figure 43: possible interactions for alpha 0.5. Isometric and bottom view of the acoustic modes. Plate mode at the bottom. On the left, plate mode for water (where there is no interaction)

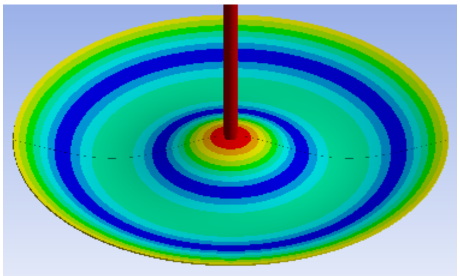
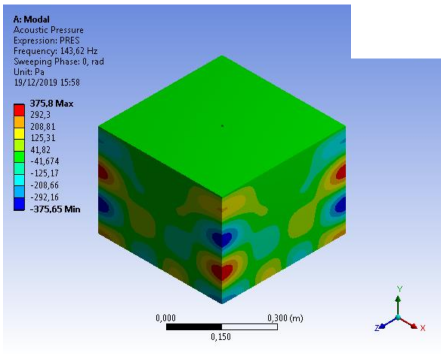
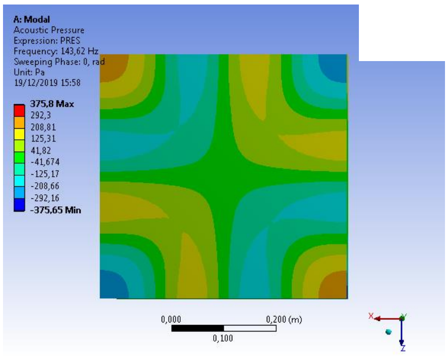
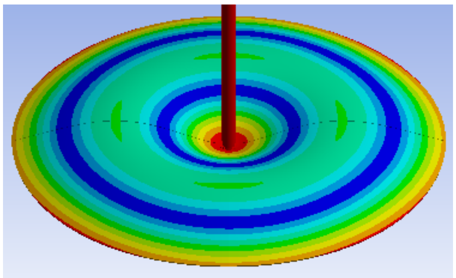
α=0	α=0.6 f93
	  

Figure 44: possible interactions for alpha 0.6. Isometric and bottom view of the acoustic mode. Plate mode at the bottom. On the left, plate mode for water (where there is no interaction)

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f92, f93, f94</td></tr>
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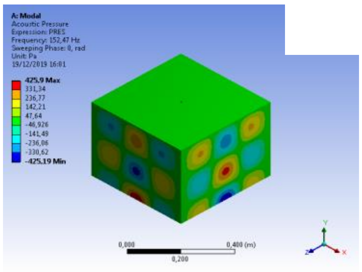

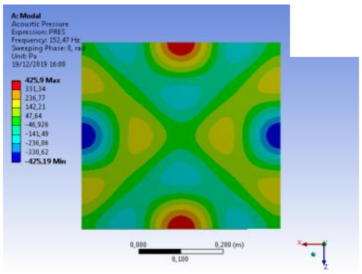

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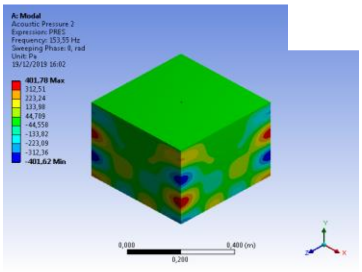

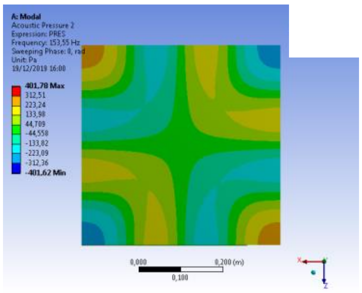

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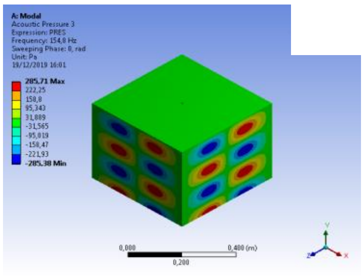

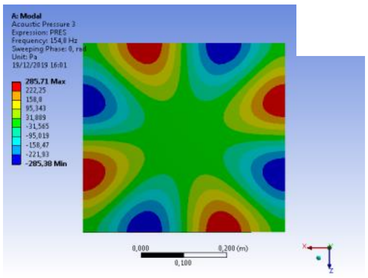

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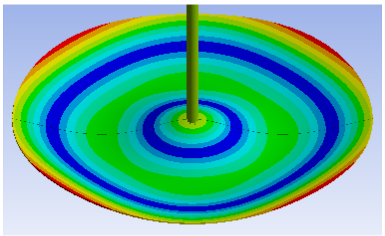
<tr><td colspan="3"></td></tr>
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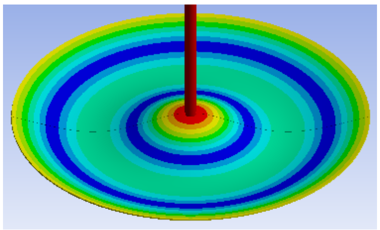
<tr><td>α=0</td><td></td></tr>
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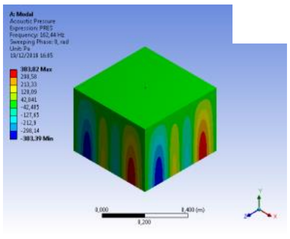
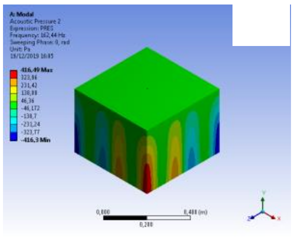
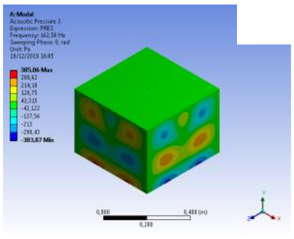
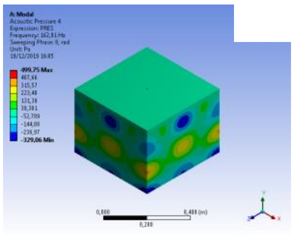
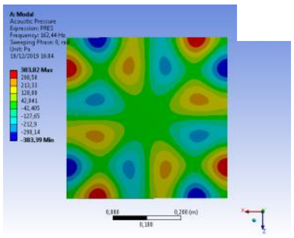
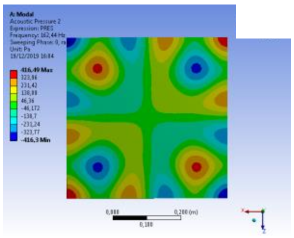
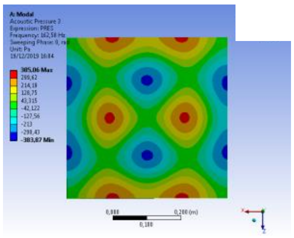
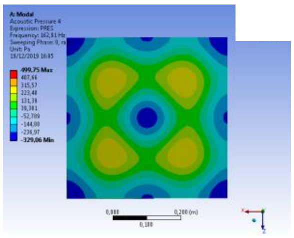
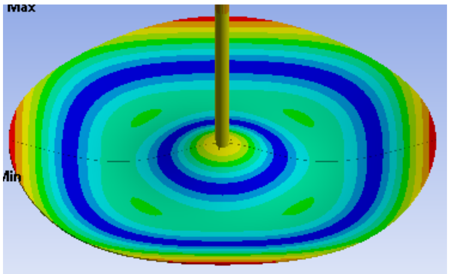
Figure 45: possible interactions for alpha 0.7. Isometric and bottom view of the acoustic mode. Plate mode at the bottom. On the bottom figure, plate mode for water (where there is no interaction)

α=0.8 f73, f74, f75, f76			
			
			
			

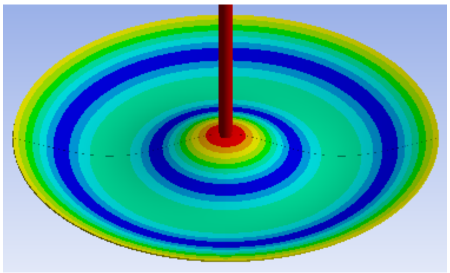
α=0	
	

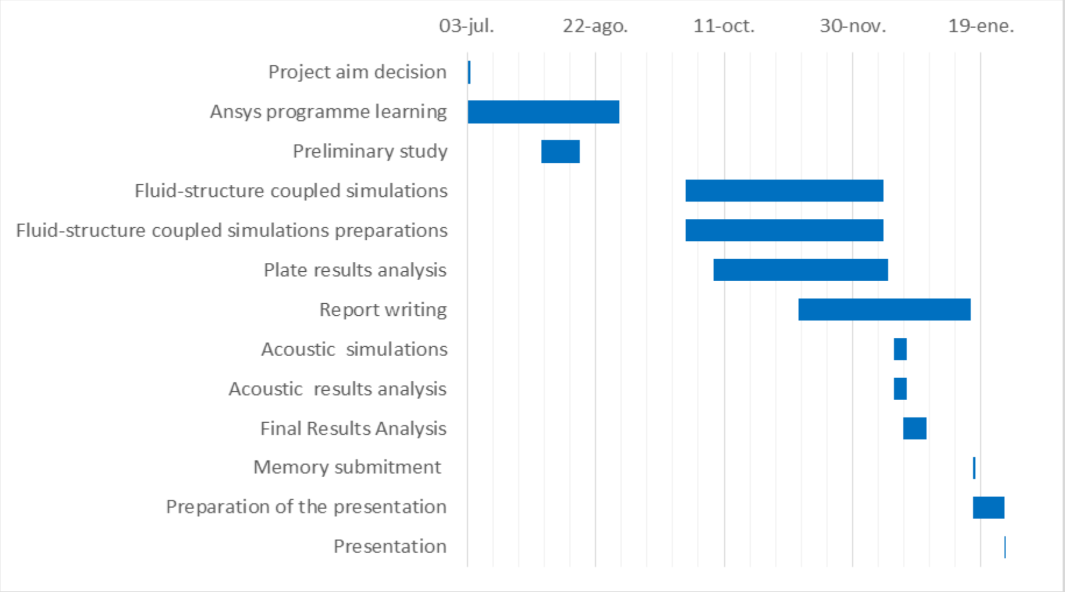
Figure 46: possible interactions for alpha 0.8. Isometric and bottom view of the acoustic mode. Plate mode at the bottom. On the bottom figure, plate mode for water (where there is no interaction)

It is observed that the number of possible interactions on the second mode of vibration is a lot higher than in the first one. This is because the acoustic modes become closer with a higher frequency, and the second structural mode is found at a higher frequency than the first. If the third mode was studied, the number of possible interactions would be even higher. This may be the reason why the plate mode 3 frequencies were not found, since the mode is so affected by an acoustic mode that it could not be recognise. It may be even said that the mode itself doesn’t exist anymore.

9. Planning

This project planning is made using a Gantt chart in order to calculate the overall time of its making as well as the different activities that need to be achieved. The project is divided into 13 tasks, with a total of 350 hours.

The project begins on July 2019, with the decision of its toping and a wide idea about its contents. During the months of July and August a background study is made, learning how to properly use the needed programme Ansys. Some previous papers about the subject to study are also analysed, what leads to the decision of the specific objective for this project. It is during September when the plate frequency simulations as such will begin. During the next 4 months, the simulations are going to be realized. This task is divided between simulation preparations (which include the changing of the parameters and planning of the simulation order) and simulations time as such. As soon as some results are obtained, they will be analysed in order to improve the accuracy of the following simulations. During the month of November the report will be started, which writing will conclude in January. It is mid-December when the acoustic mode frequency simulations are going to be started. This simulations together with their analysis will last for a week. When all the simulations are over, at December 20, the final results analysis is going to be done, which will take 1 week. The 16th of January the project will be submitted, and the presentation preparations will begin. Finally, the 28th of January the project will be presented.



Task	**Start Date**	**Duration [days]**	**Finish Date**	**Hours**
Project aim decision	3 Jul.	1	3 Jul.	3
Ansys programme learning	3 Jul.	59	31 Aug.	30
Preliminary study	1 Aug.	15	15 Aug.	10
Fluid-structure coupled simulations preparations	26 Sept.	77	13 Dec.	34
Fluid-structure coupled simulations	26 Sept.	77	13 Dec.	192
Fluid-structure coupled results analysis	7 Oct.	68	14 Dec.	20
Report writing	9 Nov.	67	16 Jan.	30
Acoustic simulations	16 Dec.	5	20 Dec.	3
Acoustic results analysis	16 Dec.	5	20 Dec.	8
Final Results Analysis	20 Dec.	9	29 Dec.	8
Memory submitment	16 Jan.	1	16 Jan.	1
Preparation of the presentation	16 Jan.	12	28 Jan.	10
Presentation	28 Jan.	1	28 Jan.	1
Total				350

10. Budget

The total budget for this project can be divided in two parts: that concerning human resources expenses and that concerning technological and infrastructure expenses.

Human resources expenses:

It is considered human resources expenses all those costs related to staff, all investigation hours from engineers, time used by learning the programs needed (Ansys, SolidWorks) and time writing the project report. All these expenses are expressed in the following table.

Concept	**Price [€/h]**	**Time consumed [h]**	**Cost[€]**
Junior engineer hour	20	350	7000
Senior engineer hour (professor)	60	70	4200
Total cost HR			11,200

Technological resources expenses

It is included in these expenses all the costs related to software license, as well as those concerning the material used and the place of work needed for this project. Some of the most important item costs can be found in the following table. In order to obtain the project budget, the amortization of these items is taken into account in the same table.

Concept	**Price [€]**	**Amortization time [years]**	**Cost [€]**
Computer	1500	5	100
Ansys	4000	1	1315
Microsoft Office	0 [student license]		0
Office	1920	1	640
Total			2,055

The total budged used in this project is 17,245.7€, calculated as follows.

Human resources expense	**11,200€**
Technological resources expenses	2,055€
Total costs	13,255€
Overhead (7% of subtotal)	997.7€
Subtotal	14,252.7€
VAT (21%)	2,993€
Project budget	17,245.7€

11. Environmental impact

This project has a low environmental impact, since little material is used. There was no need to buy a new computer, since an old one was available. The computer used was also shared with other researchers, and at the end of this project it was left for others to use. It is for these reasons that the environmental impact of manufacturing the computer and its latter residual disposal are not considered as an environmental impact of the current project.

All of the environmental impact generated in the making of this project, then, is due to the electrical energy consumed by the computer. A desktop computer uses around 170kW per hour. If the used computer is running 10 hours per day, this will mean 1700kW generated per day. According to the planning of this project, the simulations made with the computer will take 82 days, making it a total of 1394,000 kWh generated. Knowing that 1kWh is equivalent to 0.283kg CO_2 emitted to the atmosphere, we can easily calculate the total CO_2 emitted during this project, which equals 39,450.2kg of CO_2.

Conclusions

In this project a fluid-structure interaction setup was studied, consisting of a circular plate surrounded by a two-phase fluid, a mixture of air and water. The simulations were conducted with the programme Ansys. As for the mathematical approach, a harmonic analysis was chosen for the plate modes of vibrations and a modal analysis for the acoustic modes of the surrounding fluid. Each different mixture was defined by the void ratio, a parameter that is dependent on the density (in a linear way) and the sound speed. The sound speed of the two-phase fluid follows an unusual curve, reaching lower values in the mixture than in the pure phases. This is reflected in some results.

The plate modes of vibration were studied. Its circular mode shapes were searched, and only the first two modes were found due to coupling effects with the acoustic modes of the surrounding fluid. Its mode frequencies were analysed respect the mixture’s void ratio, density and sound speed. It was found that the mode frequency for each mode of vibration is dependent of its density, and it follows a parabolic tendency in front of its void ratio. The fluid-structure coupled mode shape changes were also analysed, and a dependence with the mixture density was found.

The surrounding fluid was studied, eliminating all solids and analysing its acoustic modes. It was found that the different mode frequencies were quite distant at pure fluids (air and water), but suffered a sudden frequency decrease and became all close to each other when the mixture of both fluids was conducted. This implies a higher number of possible acoustic and plate mode interactions once the mixture is done. It was also observed that at higher frequencies the number of acoustic modes within a range increased, making the determination of structural modes at higher frequencies even more complicated. These are the reasons why higher modes of vibrations for the plate were not found. As for the acoustic mode shapes no difference was found between mixtures.

The possible interactions between the plate modes and the acoustic modes were found and analysed, finding as expected a higher number of interaction in the second mode than in the first. It was found that the coupled effect occurs for certain acoustic mode shapes that influence the plate mode shapes strongly. This is probably because of the geometry difference between a circular plate and a quadratic recipient.

A further study following this project is recommended, changing the experiment design. A circular fluid recipient should be used or its dimensions increased so the coupling effect is suppressed. This way, the fluid-structure modes could be found clearly and further analysed.

Acknowledgements

First of all I want to express my gratitude to Xavier Escaler both for being my teacher and my project director, because it is thanks to his interest, patience and dedication that I am finishing my studies with a sense of fulfilment. I want to thank him for his guidance in this project, for all the help that he has given me and also for showing me an interesting branch of science that otherwise I may have not gotten to know this deeply.

I would also like to thank my dear family who supported me during these four years, pushing me forwards when I needed it the most and always staying by my side. A special mention to my mother, Anna Graell, who showed me how to do what I love and helped me discover my passion in my studies. I would also like to mention my friends who have studied and struggled beside me, who helped me keep going when think seemed to be going in the wrong direction. It is thanks to these people that I was able to finish this project, so it is for all of you that I am thankful.

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