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Procedia Manufacturing 35 (2019) 236–241

Procedia MANUFACTURING

www.elsevier.com/locate/procedia

2nd International Conference on Sustainable Materials Processing and Manufacturing

(SMPM 2019)

Constitutive Model to Analyse the effect of Variation in Temperature Distribution on the Rate of Heat Transfer around a Spur Gear Tooth

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Abstract

The study employed the finite element approach for the solution of 1-dimensional steady state heat transfer to determine the temperature distributions on a spur gear tooth. The gear tooth was subjected to temperature variations both at the top and root as well as variations in the respective coordinate of the material particles. The developed model was further used to evaluate the rate of heat transfer through the gear material. The result showed that the heat flux decreases with increasing values of Z which represent the particle coordinate. However, change in Z coordinate also constitute increased random motion of the particles in the gear causing steady degradation even at decreasing heat flux. Therefore, the study has revealed that temperature variation has negative effect on the wear mode of the gear material and at higher temperature, failure of the gear tooth would set in.

Peer-review under responsibility of the organizing committee of SMPM 2019.

Keywords: Failure, Production Technology, Gears, Reliability

Nomenclature

t_1 temperature at the top surface of the gear tooth

t_2 temperature at the top surface of the gear tooth

Z_1 coordinate of the particle at the top of gear tooth

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Peer-review under responsibility of the organizing committee of SMPM 2019.
10.1016/j.promfg.2019.05.033

Z_2 coordinate of the particle at the root of the gear tooth

K thermal conductivity of mild steel given as 54 W/m. K

Q_z heat transfer

A cross sectional area

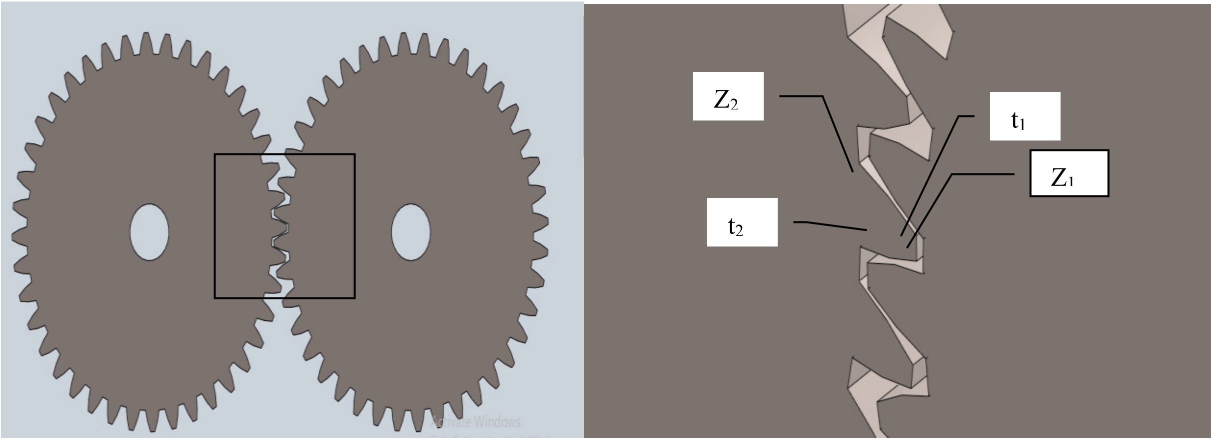
1. Introduction

The efficiency of spur gear tooth is a function of the load and temperature which the tooth is subjected to [1]. Variation in load and temperature resulted in thermal stress on the gear tooth [2, 3]. Assessment of temperature variation of gear in an unsteady state condition revealed flash temperature differences which is attributed to the dynamic loading [4]. Salawu *et al.* [5] reported that critical design of the gear tooth involute would give efficient gear transmission ratio and reduction in gear failure that could have resulted from increased temperature variations. Various model has been developed to address failures associated with gears. For instance, Shi *et al.* [6] developed a model to analyse the temperature variation on the gear tooth of a traction gear in the direction vertical to the gear tooth. However, there is limitation in the efficiency and applicability of the model as a result of the fact that tooth contact temperature, friction at the surface and mesh stiffness are factors which usually result to variations in temperature [7,8]. More so, Mertens and Senthilvelan [9] reported that polymer gears exhibit increased temperature variations when meshed with mild steel gears under increased load. This was attributed to area of contact and friction at the surface of the gear tooth in pair. In addition to this, it was possible to predict the temperature variation at the tooth contact in gears using finite element approach. This involved modelling of the rolling and sliding processes that gear tooth is subjected to [10]. Consequently, rise in temperature at the point of mesh can be predicted using coupled thermos-elastic finite element model. However, this also is limited by coefficient of friction which can only be determined experimentally [11]. On the same hand, Fernandez [12] suggested that finite element model could be effective in predicting the rise in temperature of gears if the load can be determined. Similarly, temperature variation in spur gear tooth is also attributed to both speed of operation and the environmental condition. In fact, vibration is a function of increased temperature [13,14]. These parameters have great effect on the tribological performance of the gear, especially the polymer gears [15]. Based on these uncertainties associated with the existing model for temperature variation on gear tooth, this study focused on developing a model to predict the temperature at the top and root of a single tooth spur gear under certain assumptions using 1-dimension approach of Fourier heat conduction. The study further evaluated the numerical values of temperature both at the top and the root of the spur gear tooth. The focus was based on the need to establish the reason for efficiency enhancement of the gear tooth during design.

2. Methodology

1.1 Problem Formulation

The study considered a spur gear in mesh as shown in Figs. 1a and b. Fig. 1a, showed the cross section of the gear in mesh during operation while Fig. 1b describes a hypothetical situation of gear tooth in service. The study assumed finite points Z_1 and Z_2 at the tip and root of a tooth, while t_1 and t_2 represent the temperature at the tip and root of the gear tooth. As the temperature increases, linear expansion occurs within the tooth signifying the material displacement of point Z from the original position to a new position and possibly induces thermal stress build up and eventual failure. Therefore, the focus of this study was to determine, theoretically by way of modelling, the effect of temperature variation on the gear tooth.



a b

Fig.1: Diagrammatic representation of a Spur gear tooth

2.2 Model Development

Applying the Fourier law of heat conduction in 1-dimension to model the heat distribution on the gear tooth gives:

$$\frac{dt}{dz} = \frac{Q_{z_g}}{K_g A_g} \text{ [16]:} \qquad (1)$$

$$A = \frac{\pi d^2}{4} = \frac{\pi (bz)^2}{4} = \frac{\pi b^2 z^2}{4} \qquad (2)$$

Putting equation (2) into equation (1) gives:

$$4Q_{z_g dz} = -K_g \pi b^2 z^2 \qquad (3)$$

From which,

$$\frac{4Q_{z_g} dz}{\pi b^2 z^2} = -K_g dt \qquad (4)$$

Integrating both sides of equation (4) at the interval of z_1 to z and t_1 to t along the wall of the gear tooth gives:

$$\int_{z_1}^{z} \frac{4Q_{z_g} dz}{\pi b^2 z^2} = \int_{t_1}^{t} -K_g dt \qquad (5)$$

$$\frac{4Q_{z_g}}{\pi b^2} \int_{z_1}^{z} \frac{dz}{z^2} = -K_g \int_{t_1}^{t} dt \qquad (6)$$

$$\frac{4Q_{z_g}}{\pi b^2} \left[-\frac{1}{z}\right]_{z_1}^{z} = -K_g [t]_{t_1}^{t} \qquad (7)$$

$$\frac{4Q_{z_g}}{\pi b^2} \left[\frac{1}{z} + \frac{1}{z_1}\right] = -K_g \left[t - t_1\right] \qquad (8)$$

Equ (8) gives;

$$-\frac{4Q_{z_g}}{\pi b^2 K_g} \left[\frac{1}{z_1} - \frac{1}{z}\right] = t - t_1 \qquad (9)$$

$$t = -\frac{4Q_{z_g}}{\pi b^2 K_g} \left[-\frac{1}{z} + \frac{1}{z_1}\right] + t_1 \qquad (10)$$

$$t(z) = t_1 - \frac{4Q_{z_g}}{\pi b^2 K_g}\left[\frac{1}{z_1} - \frac{1}{z}\right] \tag{11}$$

To find the heat distribution on the gear tooth, it is worthwhile to note that Q_z is constant and independent of z

Thus, assume that $z = z_2$ and $t(z_2) = t_2$

$$t_2 = t_1 - \frac{4Q_{z_g}}{\pi b^2 K_g}\left[\frac{1}{z_1} - \frac{1}{z_2}\right] \tag{12}$$

Solving gives:

$$t_2 \pi b^2 K_g = \pi b^2 K_g t_1 - 4Q_{z_g}\left(\frac{1}{z_1} + \frac{1}{z_2}\right) \tag{13}$$

$$\frac{\pi b^2 K_g (t_2 - t_1)}{4\left(\frac{1}{z_1} - \frac{1}{z_2}\right)} = -Q_{z_g} \tag{14}$$

$$Q_{z_g} = -\frac{\pi b^2 K_g (t_2 - t_1)}{4\left(\frac{1}{z_1} - \frac{1}{z_2}\right)} \tag{15}$$

Determination of fractional change in length

This is given by:

$$\frac{\Delta L}{L_0} = \alpha \Delta T \tag{16}$$

Knowing that the coefficient of linear temperature expansion for all steels is between (11 and 12.5) $\times 10^{-6}$m/(m k). The fractional change in length falls between (1.1 and 1.25) $\times 10^{-3}$

Equation (15) therefore gives the variation in the heat distribution along the path of travel of the finite points z_1 and z_2. The relationship between the heat transfer rate and temperature variation as well as the temperature along the path of travel are displayed in Figs. 2 to 6.

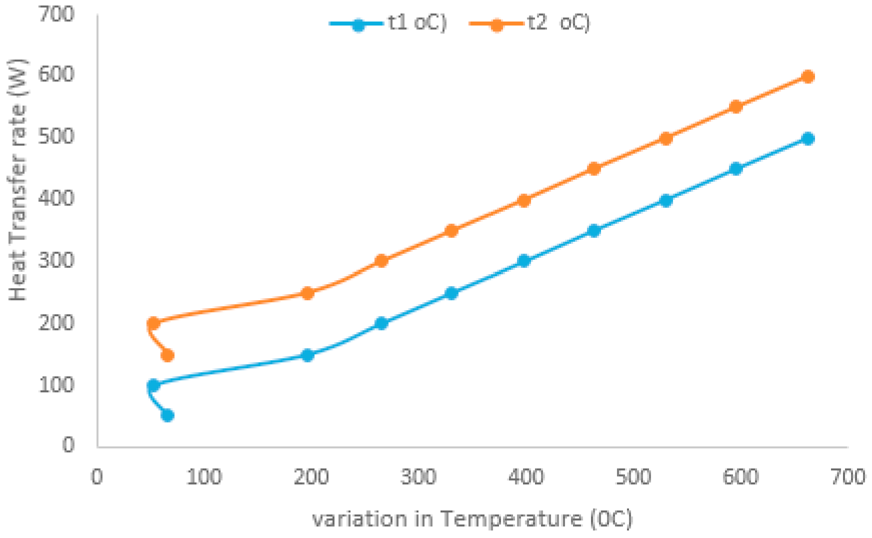


Fig.2: Variation of Temperature with Heat transfer rate

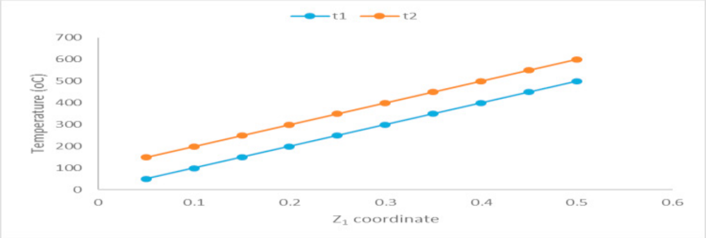


Fig.3: Variation in Temperature with Z_1 co-ordinate

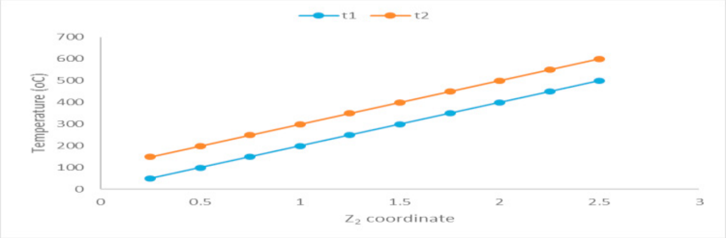


Fig.4: Variation in Temperature with Z_2 co-ordinate

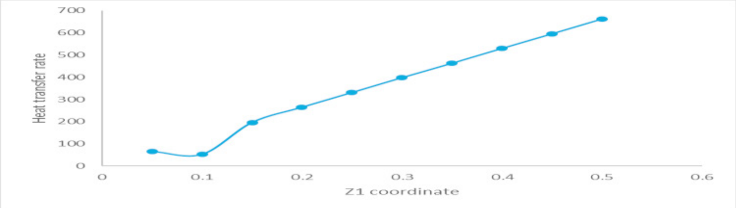


Fig.5: Variation of Heat Transfer rate with Z_1 Co-ordinate

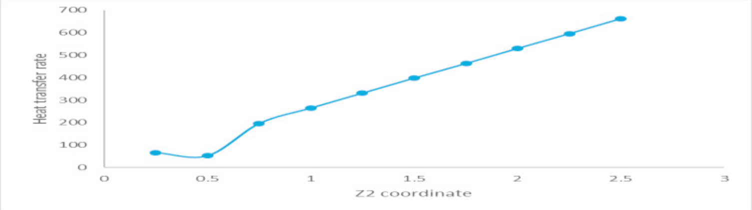


Fig.6: Variation of Heat Transfer rate with Z_2 Co-ordinate

3. Results and Discussion

Fig. 2 presents the result of temperature variation with rate of heat transfer. From the result, at temperatures of 50^0C and 150^0C, the magnitude of heat transfer rate at both the tip and the root of gear tooth was 66.3 W. Further increase in temperature resulted in sharp increase in heat transfer rate. This represent the phenomenon of the initial state of the gear tooth in operation. Also, at constant temperature increase, the rate of heat transfer increased. Going by the value of the fractional change in length (equation (16)) and the result thereby, the deformation of the gear tooth would not be obvious at this stage due to the steady state conduction through the material. Although, fatigue would be inevitable when the material is subjected to operation under such condition at longer hours.

Similarly, Figs. 3 - 4 are the variation of temperature with the Z co-ordinates. t_1 displayed linearity in temperature variation while there was non-linearity in the variation in t_2 at 150°C and 200°C respectively. This was due to slight variation in temperature. However, linearity in temperature variation was later maintained by t_2 just as it was in t_1. More so, the z coordinate increases with corresponding temperature variations. Thus, it is worthy of note that increase in temperature will cause increase in displacement of the particles from their initial positions. In the same vein, Figs. 5 to 6 depict the variation in z coordinate with the rate of heat transfer. The figures show a linear relaionship between heat transfer rate and the Z co-ordinates. However, there was a noticable dip at onset of the heat transfer afterwhich the heat transfer rate increased with corresponding increase along the path of travel of the finite points (i.e. Z co-ordinates).

Conclusion

The study employed the finite element approach for the solution of 1-dimensional steady state heat transfer to determine the temperature distributions on a spur gear tooth. The gear tooth was subjected to temperature variations both at the top and root as well as variations in the respective coordinate of the material particles. The developed model was further used to evaluate the rate of heat transfer through the gear material. The result showed that the heat flux decreases with increasing values of Z which represent the material position. However, change in position Z also constitute increased random motion of the particles in the gear, causing steady degradation even at decreasing heat flux. Therefore, the study has revealed that temperature variation has adverse effect on the wear mode of the gear material.

Acknowledgement

The authors wished to appreciate the management of Covenant University for the contribution toward the success of this study.

References

[1] Fernandes, C. M., Rocha, D. M., Martins, R. C., Magalhães, L., & Seabra, J. H. (2018). Finite element method model to predict bulk and flash temperatures on polymer gears. *Tribology International*, *120*, 255-268.

[2] Salawu, E. Y., Ajayi, O. O., & Olatunji, O. O. (2015). Theoretical Modelling Of Thermal-Hoop Stress Around The Tooth Of A Spur Gear In A Filler Machine. *Journal of Multidisciplinary Engineering Science and Technology (JMEST)*, *2*(2), 1635-1640.

[3] Salawu, E. Y., Okokpujie, I. P., Ajayi, O. O., & Agarana, M. C. (2018). Analytical Technique for the Determination of Hoop Stress and Radial Stress on the Tooth Spur Gear under Vertical Loading in a Food Packaging Machine.

[4] Li, W., & Tian, J. (2017). Unsteady-state temperature field and sensitivity analysis of gear transmission. *Tribology International*, *116*, 229-243.

[5] Salawu, E. Y., Okokpujie, I. P., Ajayi, O. O., Afolalu, S. A., & Agarana, M. C. (2018). Numerical Modeling and Evaluation of Involute Curve Length of a Spur Gear Tooth to Maintain Constant Velocity Ratio While in Motion.

[6] Shi, Y., Yao, Y. P., & Fei, J. Y. (2016). Analysis of bulk temperature field and flash temperature for locomotive traction gear. *Applied Thermal Engineering*, *99*, 528-536.

[7] Gou, X., Zhu, L., & Qi, C. (2017). Nonlinear dynamic model of a gear-rotor-bearing system considering the flash temperature. *Journal of Sound and Vibration*, *410*, 187-208.

[8] Touret, T., Changenet, C., Ville, F., Lalmi, M., & Becquerelle, S. (2018). On the use of temperature for online condition monitoring of geared systems–A review. *Mechanical Systems and Signal Processing*, *101*, 197-210.

[9] Mertens, A. J., & Senthilvelan, S. (2015). Effect of mating metal gear surface texture on the polymer gear surface temperature. *Materials Today: Proceedings*, *2*(4-5), 1763-1769.

[10] Evans, S. M., & Keogh, P. S. (2016). Efficiency and running temperature of a polymer–steel spur gear pair from slip/roll ratio fundamentals. *Tribology International*, *97*, 379-389.

[11] Zhang, J. G., Liu, S. J., & Fang, T. (2017). Determination of surface temperature rise with the coupled thermo-elasto-hydrodynamic analysis of spiral bevel gears. *Applied Thermal Engineering*, *124*, 494-503.

[12] Fernandes, C. M., Rocha, D. M., Martins, R. C., Magalhães, L., & Seabra, J. H. (2018). Finite element method model to predict bulk and flash temperatures on polymer gears. *Tribology International*, *120*, 255-268.

[13] Li, S., & Anisetti, A. (2016). On the flash temperature of gear contacts under the tribo-dynamic condition. *Tribology International*, *97*, 6-13.

[14] Stark, S., Beutner, M., Lorenz, F., Uhlmann, S., Karpuschewski, B., & Halle, T. (2013). Heat flux and temperature distribution in gear hobbing operations. *Procedia Cirp*, *8*, 456-461.

[15] Kalin, M., & Kupec, A. (2017). The dominant effect of temperature on the fatigue behaviour of polymer gears. *Wear*, *376*, 1339-1346.

[16] Bergman, T. L., Incropera, F. P., DeWitt, D. P., & Lavine, A. S. (2011). *Fundamentals of heat and mass transfer*. John Wiley & Sons.