Chapter 7
Grey Number Based Methodology for Non-homogeneous Preference Elicitation in Fuzzy Risk Analysis Management

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7.1 Introduction

The incomplete and vagueness of real-world information has triggered the emergence of grey system in human decision making environment. The grey system serves as an alternative methodology that plays the role in complementing the uncertainty in systems with partial information [1–4]. Similarly as fuzzy sets [5] and rough sets [6–8], grey sets characterised the uncertainties in the form of grey

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numbers as the basic concept in grey systems [9, 10]. A grey number is defined as a number with an unknown position within clear lower and upper boundaries [3, 9]. The main aim of introducing grey numbers in the literature is to define the membership or characteristic function value that is unclear in traditional crisp sets and fuzzy sets [3, 9].

Membership or characteristic function values are often used in decision making process as the preferences elicited by decision makers. However, determination of a suitable preference elicitation for a situation is not an easy task as different decision makers may have different types of perception. For instance, in real risk analysis world scenarios, it is a big challenge for risk analysts to make a proper and comprehensive decision when coping with the risks. This is because different risk analysts may mitigate the level of harm of the same risk differently. Another major concern in many practical risk analysis problems is they do not have flexibility with regards to knowledge elicitation and disagreements in the group. This is due to the non-homogeneous nature of risk analysts' preferences that lead to inconsistent agreements in the process of group decision making. Thus, the element of non-homogeneous in the membership or characteristic function values is important to be addressed in order to complement the non-homogeneous nature of risk analysts' preferences.

In the literature, there are many established concepts that are also concerned with the study on membership or characteristic function value such as rough sets, type-2 fuzzy sets and interval-valued fuzzy sets [11, 12]. Nonetheless, all of them have weaknesses from one to another. Rough sets have successfully expressed this situation by representing the probability of an element being a member of the set using rough membership function [6–8]. However, the representation is incomplete when some well-defined values that belong to the decision making situations are missing. Type-2 fuzzy sets [13] on the other hand, define the membership value using another fuzzy set which includes the Footprint of Uncertainty [14, 15]. Nevertheless, it is difficult to clarify one fuzzy set with another fuzzy set [10] due to the fact that the uncertain membership value needs a representation that can express both possible values of type-2 fuzzy sets.

More importantly, the value is a single value as defined in fuzzy sets. Interval-valued fuzzy sets conceptually solve this issue in the case of fuzzy sets when grey sets are considered to be the same as interval-valued fuzzy sets. This is due to grey numbers and intervals shared some common aspects [16]. Nonetheless, this understanding is a misconception, as grey numbers have special features in which intervals do not have. In addition, this concept is inconsistent with respect to the epistemic uncertainty of an interval representation. Furthermore, grey sets provide better coverage when dealing with partial information than interval-valued fuzzy sets [10].

As grey numbers [3, 9] are capable to efficiently describe non-homogeneous membership or characteristic function values [10], numerous efforts in the literature have adopted and applied grey numbers towards decision making problems. Among others are [17] in supply chain management model, forecasting [18], software effort estimation model [19], grey-TOPSIS in subcontractor selection [20] and contractor's selection [21]. Nevertheless, these applications have shortcomings and drawbacks because they conceptually utilised the aforementioned established concepts that are proven to be inconsistent with grey numbers.

As human preferences elicitation are non-homogeneous in nature, utilisation of grey numbers provides better representations for human related decision making. Thus, to complement both theoretical methodology and decision making application of grey numbers, this paper proposes a novel non-homogeneous preference elicitation based on grey numbers for risk analysis problem. This work also introduces a novel theoretical non-homogeneous consensus reaching method that resolves disagreement between risk analysts. A novel decision making approach that is developed based on the ranking concept, is then introduced to complement the consensus reaching method in solving decision making problems involving grey numbers. Later on, validations on both novelties are presented along with real world case study, as to demonstrate the novelty, validity and feasibility of the proposed methodology.

The rest of the paper is structured as follows. Section 7.2 provides brief overviews on theoretical preliminaries related to this study. Section 7.3 discusses the relevance of grey numbers in risk analysis management. Section 7.4 presents the research methodology of this study. Section 7.5 covers validation of results obtained throughout this study. Section 7.6 concerns with the application of the research methodology on real world case study and finally, the conclusion is given in Sect. 7.7.

7.2 Theoretical Preliminaries

7.2.1 Fuzzy Number

Definition 7.2.1 ([22]) A triangular type-1 fuzzy number A is represented by Eq. (7.1).

$$\mu_A(x) = (a_1, a_2, a_3; 1) = \begin{cases} \frac{(x-a_1)}{(a_2-a_1)} & \text{if } a_3 \leq x \leq a_4 \\ \frac{(x-a_4)}{(a_3-a_4)} & \text{if } a_3 \leq x \leq a_4 \\ 0 & \text{otherwise} \end{cases} \tag{7.1}$$

Definition 7.2.2 ([22]) A trapezoidal type-1 fuzzy number A is represented by Eq. (7.2).

$$\mu_A(x) = (a_1, a_2, a_3, a_4; 1) = \begin{cases} \frac{(x-a_4)}{(a_3-a_4)} & \text{if } a_2 \leq x \leq a_3 \\ 1 & \text{if } a_2 \leq x \leq a_3 \\ \frac{(x-a_4)}{(a_3-a_4)} & \text{if } a_3 \leq x \leq a_4 \\ 0 & \text{otherwise} \end{cases} \tag{7.2}$$

7.2.2 Grey Number

Definition 7.2.3 ([10]) A grey number, G_A, is a number with clear upper and lower boundaries but has an unknown position within the boundaries. Mathematically, a grey number for the system is expressed as:

$$G_A \in \lfloor g^-, g^+ \rfloor = \{g^- \leq t \leq g^+\} \tag{7.3}$$

where t is information about $g^\pm$ while g^- and g^+ are the upper and lower limits of information t respectively.

As mentioned in the introduction section, grey number is introduced in the literature as to clearly define the membership or characteristic function values of a set. Therefore, in this paper, the terms grey number is used interchangeably with characteristic function value and vice versa.

Definition 7.2.4 ([10]) For a set $A \subseteq U$, if its characteristic function value of each x with respect to A, $g_A^\pm(x)$, can be expressed with a grey number, $g_A^\pm(x) \in \bigcup_{i=1}^n [a_i^-, a_i^+] \in D[0,1]^\pm$, then A is a grey set, where $D[0,1]^\pm$ is the set of all grey numbers within the interval [0, 1].

In the literature on grey numbers, if the value of the characteristic function is completely known or completely unknown, then it is called as the white number or black number respectively. In other words, characteristic function value 1 refers to the element is a white numbers and 0 is a black number. Likewise, any values in [0, 1] are considered as the grey numbers. Consider the following definitions by [10].

Definition 7.2.5 (White Sets) For a set $A \subseteq U$, if its characteristic function value of each x with respect to A, $g_{A_i}^\pm, i = 1, 2, \ldots, n$, can be expressed with a white number, then A is a white set.

Definition 7.2.6 (Black Sets) For a set $A \subseteq U$, if its characteristic function value of each x with respect to A, $g_{A_i}^\pm, i = 1, 2, \ldots, n$, can be expressed with a white number, then A is a black set.

Definition 7.2.7 (Grey Sets) For a set $A \subseteq U$, if its characteristic function value of each x with respect to A, $g_{A_i}^\pm, i = 1, 2, \ldots, n$, can be expressed with a white number, then A is a grey set.

Definition 7.2.8 Let U be the finite universe of discourse, x be an element and $x \in U$. For a grey set $A \subseteq U$, the characteristic function value of x with respect to A is $g_A^\pm(x) \in D[0,1]^\pm$. The degree of greyness, $g_A^o(x)$, of element x for set A is expressed as

$$G_A \in \lfloor g^-, g^+ \rfloor = \{g^- \leq t \leq g^+\} \tag{7.4}$$

Definition 7.2.9 (Degree of Greyness of a Set[10]) Let U be the finite universe of discourse, A be a grey set and $A \subseteq U$. x_i is element relevant to A and $x_i \in U$ $i = 1, 2, \ldots, n$ and n is the cardinality of U. The degree of greyness of set A, g_A^*, is defined as

$$g_A^* = \frac{\sum_{i=1}^{n} g_A^o(x_i)}{n} \tag{7.5}$$

It is worth pointing out here that Eq. (7.5) can be expressed in term of fuzzy set expression [10], given by

$$A = g_A^{\pm}/x_1 + g_A^{\pm}/x_2 + \ldots + g_A^{\pm}/x_n \tag{7.6}$$

7.3 Relevance of Grey Numbers in Risk Analysis Management

In this section, a case study on risk analysis problem is carried out as to demonstrate the relevance of grey numbers towards non-homogeneous preference elicited by risk analyst. Information on the case study is summarised in Table 7.1, given as follows.

In Table 7.1, criteria B and C for each company under consideration are preferences elicited by risk analyst 1. It is also noted that risk level, D, which is defined based on criteria B and C is also in the form of preference elicitation. These preferences elicitation are expressed into characteristic functions defined as Eqs. (7.7) and (7.8) for B & D and C & D respectively.

$$f_{CD}(A_i) = \begin{cases} 1 & \text{if C= high} \\ [0,1] & \text{if C= medium} \\ 0 & \text{if C= low} \end{cases} \tag{7.7}$$

$$f_{BD}(A_i) = \begin{cases} 1 & \text{if B= high} \\ [0,1] & \text{if B= medium} \\ 0 & \text{if B= low} \end{cases} \tag{7.8}$$

Table 7.1 Information on risk level evaluation for companies in Malaysia by risk analyst 1

	Criteria		
Company, A	Probability of failure, B	Severity of loss, C	Risk level, D
A_1	B_{A_1} = Low	C_{A_1} = Low	D_{A_1} = Low
A_2	B_{A_2} = Medium	C_{A_2} = Low	D_{A_2} = Medium
A_3	B_{A_3} = Low	C_{A_3} = Medium	D_{A_3} = Low
A_4	B_{A_4} = High	C_{A_4} = High	D_{A_4} = High

From Eqs. (7.7) and (7.8), the following aggregated expressions are obtained.

$$\begin{aligned} B^* &= [0, 0]/A_1 + [0, 1]/A_2 + [0, 0]/A_3 + [1, 1]/A_4 \\ &= 0/A_1 + [0, 1]/A_2 + 0/A_3 + 1/A_4 \end{aligned} \tag{7.9}$$

$$\begin{aligned} C^* &= [0, 0]/A_1 + [0, 0]/A_2 + [0, 1]/A_3 + [1, 1]/A_4 \\ &= 0/A_1 + 0/A_2 + [0, 1]/A_3 + 1/A_4 \end{aligned} \tag{7.10}$$

where B^* and C^* are aggregated relationships for B & D and C & D respectively.

It is worth noting here that all preferences elicitation are now in the form of characteristic function values with 0 is the black number, 1 is the white number and is the grey numbers. In other words, the non-homogeneous preferences elicitation expressed by risk analyst 1 are in the form of grey numbers. Although, it is acknowledged based on Definition 7.2.4 that black numbers, white numbers and grey numbers are considered as grey numbers, all of them are still distinct in term of their value forms. Thus, this study describes grey numbers into two value form namely the numerical value and interval value forms. The following Table 7.2 presents details of these value forms of grey numbers.

Descriptions presented in Table 7.2 are important to be introduced here because they point out the non-homogeneous nature of a grey number. Unlike the established research concepts mentioned in the introduction, only one value form (homogeneous) is considered in their computation works that is either numerical value form or interval value form. The non-homogeneous value forms of grey numbers described here indicate that grey numbers are more relevant than established research concepts because both elements and the sets can simultaneously be non-homogeneous in certain decision making problems, for instance Eqs. (7.9) and (7.10). Even though, the significant nature of non-homogeneous value forms of grey numbers creates another level of complexity in terms of the computational methodology works, this challenge brings the motivation of this study.

Table 7.2 Descriptions of grey numbers value forms

Grey number	Value form	Example	
		Equation (7.9)	Equation (7.10)
0	Numerical	$0/A_1, 0/A_3$	$0/A_1, 0/A_2$
[0, 1]	Interval	$[0, 1]/A_2$	$[0, 1]/A_3$
1	Numerical	$1/A_4$	$1/A_4$

7.4 Research Methodology

In this section, novel theoretical methodology to deal with grey numbers is presented. It is worth mentioning here that this methodology consists of two layers namely the consensus reaching method as Layer 1 and the ranking approach as Layer 2. Details on both layers are explained as follows.

7.4.1 Layer 1: Consensus Reaching Method

As mentioned in Sect. 7.3, the value forms of grey number are non-homogeneous (i.e. numerical value form and interval value form). Due to this reason, a novel consensus reaching method which is the conversion of grey numbers into type-1 fuzzy numbers is proposed. The main purpose of the consensus reaching method is to ensure that both value forms of grey numbers are transformed into common value form for easier computation. Furthermore, type-1 fuzzy numbers are well established in decision making application [23–32]. This consensus reaching method is basically an extension of [10] research work on replacing the characteristic function on grey set with fuzzy membership function. Discussions on the aforementioned replacement are given in Sect. 7.5 while details on the consensus reaching method are as follows.

Numerical Value Form
If $g_A^{\pm} \in [0, 1]$ is a numerical value, then $g_A^{\pm}$ is converted into grey type-1 fuzzy numbers using conversion function, T_{1i}, given as follows.

Definition 7.4.1 A numerical value of $g_A^{\pm}$ is converted into grey triangular type-1 fuzzy numbers using conversion function, T_{1i} as

$$\begin{aligned} &T_{1i} : g_A^{\pm} \to G_A(x) \\ &T_{11} = G_A(x) = (g_{a_1}, g_{a_2}, g_{a_3}) \end{aligned} \tag{7.11}$$

and grey trapezoidal type-1 fuzzy number using conversion function, T_{12} as

$$T_{12} = G_A(x) = (g_{a_1}, g_{a_2}, g_{a_3}, g_{a_4}) \tag{7.12}$$

Interval Value Form
If $g_A^{\pm} \in [0, 1]$ is an interval value, then $g_A^{\pm}$ is converted into grey type-1 fuzzy numbers using conversion function, T_{2i}, given as follows.

Definition 7.4.2 An interval value of $g_A^{\pm}$ is converted into triangular grey type-1 fuzzy numbers using conversion function, T_{2i}:

$$\begin{aligned} &T_{21} : [a, b] \to G_A(x) \\ &T_{21}[a, b] = G_A(x) = (g_{a_1}, g_{a_2}, g_{a_3}) \end{aligned} \tag{7.13}$$

and grey trapezoidal type-1 fuzzy number using conversion function, T_{22} as

$$T_{22}[a, b] = G_A(x) = (g_{a_1}, g_{a_2}, g_{a_3}, g_{a_4}) \tag{7.14}$$

7.4.2 Layer 2: Ranking Approach

In this subsection, a ranking approach for grey type-1 fuzzy numbers is presented. The complete procedure is given as follows.

Let $G_A(x) = (g_{a_1}, g_{a_2}, g_{a_3}, g_{a_4})$ be a grey type-1 fuzzy number obtained from the conversion approach presented in Sect. 7.4.1. The complete theoretical procedure for ranking grey type-1 fuzzy number is as follows.

Step 1: Calculate the centroid $-x$ value for $G_A(x)$ based on [33] as

$$x_{G_A} = \frac{\int_{-\infty}^{\infty} x f(x) dx}{\int_{-\infty}^{\infty} f(x) dx}$$

and the centroid $-y$ value for $G_A(x)$ as

$$y_{G_A} = \frac{\int_0^{w_{G_A}} \alpha \mid G_A^\alpha \mid d\alpha}{\int_0^{w_{G_A}} \mid G_A^\alpha \mid d\alpha}$$

where $x_{g_A} \in [0, 1]$ and $y_{g_A} \in [0, 1]$.

Step 2: Compute the spread value for $G_A(x)$ based on [25] as

$$s_{G_A} = i_{G_A} \times ii_{G_A}$$

where $i_{G_A} =\mid g_{a_4} - g_{a_1}$ and $ii_{G_A} = y_{G_A}$

Step 3: Evaluate the ranking value for all grey type-1 fuzzy numbers under consideration as

$$\phi_{G_A} = x_{G_A} \times y_{G_A} \times (1 - s_{G_A}) \tag{7.15}$$

Ranking descriptions:

$$\text{If } \phi_{G_A} > \phi_{G_B}, \text{then } G_A(x) \succ G_B(x)$$
$$\text{If } \phi_{G_A} = \phi_{G_B}, \text{then } G_A(x) \approx G_B(x)$$
$$\text{If } \phi_{G_A} < \phi_{G_B}, \text{then } G_A(x) \prec G_B(x)$$

It is worth mentioning here that the ranking approach presented in this subsection is similar as in [25]. The distinction between [25] and this proposed work is the former is developed for type-1 fuzzy numbers while the latter is purposely made for grey type-1 fuzzy numbers.

7.5 Validation of Results

This section covers validation on the proposed methodology in the previous section. It is worth mentioning here that the relevant properties considered in this section justify the consistency of the proposed extension within the domain of grey numbers and these properties can be extended further.

7.5.1 Layer 1: Consensus Reaching Method

As mentioned in Sect. 7.4.1, the consensus reaching method developed is an extension of [10] work. The following Theorem 7.5.1 justifies the consistency on replacing the characteristic function of grey numbers with fuzzy membership function.

Theorem 7.5.1 *Let U be the finite universe of discourse, A be a grey set and $A \subseteq U$. x is an element and $x \in U$, $g_A^{\pm}(x)$ is the characteristic function value of x with respect to A, $g_A^{o}(x)$ is the degree of greyness of $g_A^{\pm}(x)$ and g_A^{*} is the degree of greyness for A.*

Proposition 3 *A is a type-1 fuzzy set if and only if $g_A^{*} = 0$ and $g_A^{\pm}(x)$ for any $x \in U$*

Proof 2 If A a type-1 fuzzy set, then $g_A^{*} = 0$ and $g_A^{\pm}(x) \in [0, 1]$ for any $x \in U$

Let A be a type-1 fuzzy set expressed as

$$A = \mu_A(x_1)/x_1 + \mu_A(x_2)/x_2 + \ldots + \mu_A(x_n)/x_n \tag{7.16}$$

where $\mu_A(x)$ is the membership degree for A with $\mu_A(x) \in [0, 1]$.

When $\mu_A(x) = g_A^{\pm}(x) \in [0, 1]$, then the following is obtained based on Eq. (7.5).

$$g_A^{*} = \frac{\mid (\mu_A(x_1) - \mu_A(x_1)) + (\mu_A(x_2) - \mu_A(x_2)) + \ldots + (\mu_A(x_n) - \mu_A(x_n)) \mid}{n} = 0 \tag{7.17}$$

where $\mu_A(x) = g_A^{\pm}(x) \in [0, 1]$ for any $x \in U$.

Proposition 4 *If $g_A^{*} = 0$ and $g_A^{\pm}(x) \in [0, 1]$ for any $x \in U$, then A is a type-1 fuzzy set.*

Let A be grey set expressed as

$$A = g_A^{\pm}(x_1)/x_1 + g_A^{\pm}(x_2)/x_2 + \ldots + g_A^{\pm}(x_n)/x_n$$

Based on Definition 7.2.8, $g_A^{\pm}(x_i) \in [0, 1]$ where $i = 1, 2, \ldots, n$, is a single grey number. Thus, the following is hold.

$$g_A^* = \frac{|\,(g_A^{\pm}(x_1) - g_A^{\pm}(x_1)) + (g_A^{\pm}(x_2) - g_A^{\pm}(x_2)) + \ldots + (g_A^{\pm}(x_i) - g_A^{\pm}(x_i))\,|}{n} = 0 \tag{7.18}$$

If $\mu(x) = g_A^{\pm}(x) \in [0, 1]$, then Eq. (7.6) is defined as

$$A = \mu_A(x_1)/x_1 + \mu_A(x_2)/x_2 + \ldots + \mu_A(x_n)/x_n$$

Theorem 7.5.1 holds.

With respect to the novel conversion methodology developed in Sect. 7.4, detail validation is as follows.

Let G_A and μ_A be the grey number and membership value for A respectively, where $G_A \in D[0, 1]^{\pm}$ and $\mu_A \in [0, 1]$.

Numerical Value

Property 1 If $G_A = \mu_A$, then $\mu_A : U \to D[0, 1]^{\pm}$.

Proof 3

$$\begin{aligned} &G_A = \mu_A, \textit{ implies that } G_A = \mu_A \in [0, 1]^{\pm} \\ &\text{hence, } \mu_A : U \to D[0, 1]^{\pm} \text{ (proven)} \end{aligned} \tag{7.19}$$

It is worth noting here that, Eq. (7.19) is consistent with Eqs. (7.16)–(7.18).

Interval Value

Property 2 If membership interval, $t = \lfloor g^-, g^+ \rfloor$, then $\mu : U \to D[0, 1]^{\pm}$.

Proof 4 $t = \lfloor g_A^-, g_A^+ \rfloor$ implies that $t \in D[0, 1]^{\pm}$

For continuous grey numbers, $G_A \in t$, any unknown value of G_A within t indicates that $G_A \in D[0, 1]^{\pm}$. Thus, when $G_A = \mu_A$ then $\mu_A : D \to D[0, 1]^{\pm}$ (proven).

7.5.2 Layer 2: Ranking Approach

Let G_A and G_B be any grey type-1 fuzzy numbers. All ranking properties presented here are based on [34, 35] on ranking fuzzy quantities.

Ranking Property 7.5.1 *If $G_A \succeq G_B$ and $G_B \succeq G_A$, then $G_A \approx G_B$.*

Proof 5 $G_A \succeq G_B$ implies that $\phi_{G_A} \geq \phi_{G_B}$ and $G_B \succeq G_A$ implies that $\phi_{G_B} \geq \phi_{G_A}$, thus $\phi_{G_A} = \phi_{G_B}$ which is $G_A \approx G_B$.

Ranking Property 7.5.2 *If $G_A \succeq G_B$ and $G_B \succeq G_C$, then $G_A \succeq G_C$.*

Proof 6 $G_A \succeq G_B$ implies that $\phi_{G_A} \geq \phi_{G_B}$ and $G_B \succeq G_C$ implies that $\phi_{G_B} \geq \phi_{G_C}$, thus $\phi_{G_A} = \phi_{G_C}$ which is $G_A \succeq G_C$.

Ranking Property 7.5.3 *If* $G_A \bigcap G_B = \phi$ *and* G_A *is on the right side of* G_B*, then* $G_A \succeq G_B$

Proof 7 $G_A \bigcap G_B = \phi$ and G_A is on the right side of G_B implies that $\phi_{G_A} \geq \phi_{G_B}$, thus $G_A \succeq G_B$

Ranking Property 7.5.4 *The order of* G_A *and* G_B *are not affected by other grey type-1 fuzzy numbers under comparison.*

Proof 8 The ordering of G_A and G_B are completely determined by ϕ_{G_A} and ϕ_{G_B} respectively, thus the ordering of G_A and G_B are not affected by other grey type-1 fuzzy numbers under comparison.

7.6 Case Study

In this section, assessments on the level of risk of three distinct companies in Malaysia are conducted. It is worth mentioning here that all companies under consideration are of same nature as they are producing the same product. Details on descriptions of severity of loss and probability of failure for each company under consideration in the form of grey numbers are summarised in Table 7.3.

As the methodology developed in Sect. 7.4 consists of two layers, the assessment of level of risk for each company under consideration follows the two layers developed.

Table 7.3 Descriptions of risk assessment of companies in the form of grey numbers

Company	Component	Severity of loss	Probability of failure
C_1	A_{11}	W_{11} = low	S_{11} = fairly-low
	A_{12}	W_{12} = fairly-high	S_{12} = medium
	A_{13}	W_{13} = very-low	S_{13} = fairly-high
C_2	A_{21}	W_{21} = low	S_{21} = very-high
	A_{22}	W_{22} = fairly-high	S_{22} = fairly-high
	A_{23}	W_{23} = very-low	S_{23} = medium
C_3	A_{31}	W_{31} = low	S_{31} = fairly-low
	A_{32}	W_{32} = fairly-high	S_{32} = high
	A_{33}	W_{33} = very-low	S_{33} = fairly-high

7.6.1 Layer 1: Consensus Reaching Method

Based on Table 7.2, it is acknowledged that grey numbers can exist in numerical and interval value forms. Thus, all grey numbers in Table 7.3 are converted into trapezoidal type-1 fuzzy numbers using Eqs. (7.12) and (7.14), as to ensure they are consistent in nature. The complete descriptions on the converted grey numbers into trapezoidal type-1 fuzzy numbers are presented in Table 7.4.

As to complete this layer, details in Table 7.4 are aggregated for consensus reaching purposes. Table 7.5 presents consensus reached for all companies under consideration after aggregating process.

7.6.2 Layer 2: Ranking Approach

Based on Sect. 7.4, details on centroid point, spread and ranking value for each company considered are evaluated and summarised in Table 7.6.

From Table 7.6, it can be concluded that the most risky company is C_2, followed by C_1 and C_3.

Table 7.4 Descriptions of risk assessment of companies in the form of type-1 fuzzy numbers

Company	Component	Severity of loss	Probability of failure
C_1	A_{11}	$W_{11} = (0.04, 0.10, 0.18, 0.23; 1.0)$	$S_{11} = (0.04, 0.10, 0.18, 0.23; 0.9)$
	A_{12}	$W_{12} = (0.58, 0.63, 0.80, 0.86; 1.0)$	$S_{12} = (0.32, 0.41, 0.58, 0.65; 0.7)$
	A_{13}	$W_{13} = (0.0, 0.0, 0.02, 0.07; 1.0)$	$S_{13} = (0.58, 0.63, 0.80, 0.86; 0.8)$
C_2	A_{21}	$W_{21} = (0.04, 0.10, 0.18, 0.23; 1.0)$	$S_{21} = (0.93, 0.98, 1.0, 1.0; 0.85)$
	A_{22}	$W_{22} = (0.58, 0.63, 0.80, 0.86; 1.0)$	$S_{22} = (0.58, 0.63, 0.80, 0.86; 0.95)$
	A_{23}	$W_{23} = (0.0, 0.0, 0.02, 0.07; 1.0)$	$S_{23} = (0.32, 0.41, 0.58, 0.65; 0.9)$
C_3	A_{31}	$W_{31} = (0.04, 0.10, 0.18, 0.23; 1.0)$	$S_{31} = (0.17, 0.22, 0.36, 0.42; 0.95)$
	A_{32}	$W_{32} = (0.58, 0.63, 0.80, 0.86; 1.0)$	$S_{32} = (0.72, 0.78, 0.92, 0.97; 0.8)$
	A_{33}	$W_{33} = (0.0, 0.0, 0.02, 0.07; 1.0)$	$S_{33} = (0.58, 0.63, 0.80, 0.86; 1.0)$

Table 7.5 Evaluation on risk assessment for each company after aggregation

Company	Aggregated level of risk evaluation
C_1	(0.10, 017, 0.46, 0.71; 0.7)
C_2	(0.20, 0.30, 0.70, 1.00; 0.85)
C_3	(0.22, 0.31, 0.68, 0.98; 0.8)

Table 7.6 Evaluation on risk assessment for each company after aggregation

Company	Centroid-x	Centroid-y	Spread	Ranking value
C_1	0.4836	0.3601	0.2905	0.1236
C_2	0.4793	0.3445	0.1917	0.1326
C_3	0.3899	0.3549	0.2108	0.1092

7.7 Conclusion

In this chapter, a novel decision making methodology for grey number has successfully developed. This study first discussed the relevance of grey numbers in risk analysis decision making to ensure consistency of grey numbers with real world application. A special notion of grey numbers which is the capability to represent non-homogeneous data sets is pointed out in this study where two novel definitions of grey numbers value forms are given as part of the proposed work. Then, a novel consensus reaching method and ranking approach are proposed for the first time where both novelties have been validated as to demonstrate the novelty, validity and feasibility of the proposed work. Later on, this study exemplified the usefulness of the proposed work by applying the methodology developed towards a real world case study on risk assessment.

Although, it is acknowledged that fuzzy sets has received tremendous attentions from the practitioners and decision makers on its capability to resolve various decision problems, they fell short when it comes to deal with non-homogeneous data sets. In this case, grey numbers outperform fuzzy numbers in terms of dealing with non-homogeneous data sets efficiently. For future research, further investigations on computation of grey numbers need to be carried out as these efforts will support in addressing the incomplete and vague real-world information in a more flexible and accurate way.

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