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Simulation of the Effect of Rainfall on Farm-level Cocoa Yield using a Delayed Differential Equation Model

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Abstract

Cocoa (*Theobroma cacao*) is an economically important crop grown by approximately six million of smallholder farmers throughout the tropics and sub-tropics. However, farm level yields are often very low, and sustainable intensification is urgently required. Assessing the impact of on-farm interventions of farm productivity and profitability requires an understanding of the contribution of inter-annual climate variability to cocoa yields. A Delayed Differential Equation model (DDE) was used to simulate the effect of rainfall on cocoa yields. A DDE model is an ordinary differential equation model that incorporates time lags, and is therefore able to incorporate the delay in yield response to rainfall due interactions with the cocoa flowering and the pod development processes. The DDE was constructed and based on regional rainfall and farm-level cocoa yield data from 96 farms across the main cocoa growing regions in Ghana. Model outputs indicate that a good likeness of seasonality in crop production was achieved. The potential to conduct a detailed parameterisation and extend this model to include other parameters such as agrochemical inputs and farm management practices are discussed. By further developing this model into a useful tool to predict and understand variability in cocoa yield, the sustainable intensification of small holder cocoa farming is supported.

Keywords: Climate variability, rainfall, crop model, cocoa, sustainable intensification, delayed differential equation.

1 Introduction

Cocoa (*Theobroma cacao*) is a mainstay cash crop for millions of small holders throughout the tropics and sub-tropics (Clay, 2013). Cocoa produced in West Africa accounts for approximately 74% of world output, with Côte d'Ivoire and Ghana being the main producing nations in the region (ICCO, 2017). However, despite its large share of global production, farm level yields in West Africa are often unsustainably low. In Ghana the average production is reported as approximately 400 kg/ha, which is among the lowest in the world (Aneani and Ofori-Frimpong, 2013). Low yields result in very low profitability in the sector and therefore undermines the economic viability of farms and hinders the sustainable development of communities (Kongor et al., 2017). To improve yields intensification of the production system is required.

When crop yields are low, investment in fertilisers and pesticides represents a large percentage of the total farm income and presents an excessively large risk to the financial status of the farm household. Additionally, farmers are often unable to access the credit required to purchase inputs such as fertiliser and pesticides. A better understanding of the effects that on-farm interventions have on cocoa yields would be advantageous and may make investment, by individual farmers or external lenders, more attractive. A major challenge in estimating the magnitude of the effect of interventions is large inter-annual and inter-farm variability as has been reported for cocoa production in West Africa (Daymond et al., 2017). Globally, climate is recognised as a major driver of inter-annual variability in crop yields (Ray et al., 2015). Such variability also discourages investment, making it more difficult to achieve sustainable intensification in the sector.

A modelling approach could be applied to better understand the effects of climate variability on farm-level cocoa yields, thus enabling a partitioning of climate effects. Whilst research is regularly undertaken into national cocoa production (ICCO, 2017), as well as modelling at the plant physiological level (Zuidema et al., 2005), we know relatively little about the effect of climate variability on cocoa production at the individual farm level. Modelling the potential variation in on-farm cocoa yields due to inter-annual climate variation could significantly aid sustainable development of farming communities. The aim of this study was to construct a mathematical model, to represent annual seasonality of cocoa yield in Ghana, and its dependence on rainfall patterns. The model construction was informed by actual rainfall and farm-level cocoa yield data from Ghana, and parameters were chosen to have values that broadly reflect the biological processes of cocoa production, with potential for more rigorous fitting at a later date.

An Ordinary Differential Equation (ODE) is a mathematical description of the relationship between a function, and its derivatives – which represent the rate of change of that function. A delayed differential equation (DDE) model is a time-delay system, made up of ODEs that incorporate additional time lags. This means the rate of change of a function can be related to its value in the past or the future, making DDEs a particularly useful tool for modelling systems where there is a delay in response, in this case the period of time that cocoa pods are ripening before harvest.

2 Materials and Methods

2.1 *Cocoa yield and rainfall data*

Farm-level cocoa yield data was collected as part of the 'Mapping Cocoa Productivity' project which ran in Ghana from 2012 – 2016 (Daymond et al., 2017). In this project 96 farms were monitored across four regions, in which cocoa farming is predominant: Ashanti, Brong-Ahafo, Eastern and Western (Figure 1). The farms were also characterised through the collection of detailed baseline data on cocoa planting material, agricultural practices and soil properties. However, in this modelling study we focus only on the yield results but acknowledge the importance the many other agronomic factors have on yield (see Wood and Lass, 1986). To monitor seasonality in cocoa yields, pod counts were conducted on (the same) 16 trees per farm every 6 weeks. This resulted in a data set of 17 time points over a two-year period (2012 – 2014). The number of cocoa pods harvested between each data collection point were calculated based on pod count data (See Appendix A 1.1 for further information). In this study we refer to the number of pods harvested per tree as yield. Rainfall data for the period 2012 - 2014 was obtained from the Ghanaian Meteorological Office from 18 stations across the four study regions (Figure 1 and see Appendix A 1.2 for further information). For the purposes of this study rainfall data was aggregated at a regional level, apart from the Western region which was sub-divided into north and south due to strong differences in climate within that region.

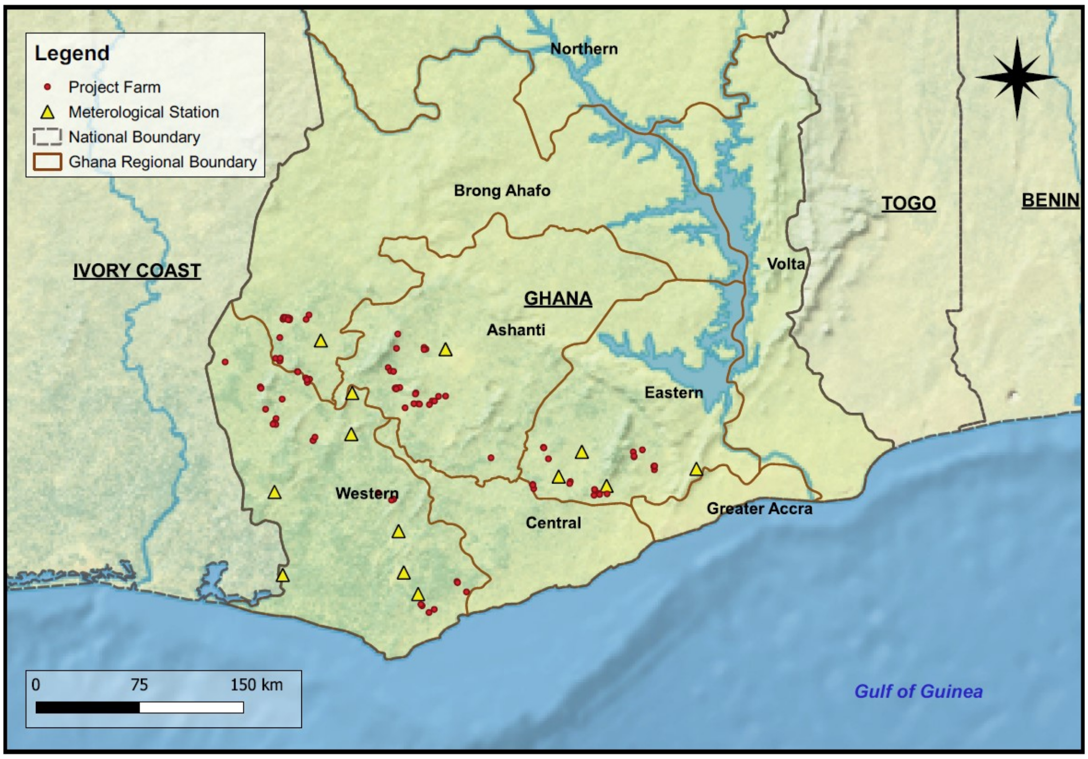

Figure 1. Map shows the locations of 96 cocoa farms from the Mapping Cocoa Productivity project, across the four main cocoa growing regions in Ghana (Ashanti, Brong-Ahafo, Eastern and Western Regions) and the locations of 13 meteorological stations. See Table A.1. for an analysis of mean farm to station distances. Note: data from a further 5 meteorological stations were included in modelling analysis, but coordinates for these stations were not available.

2.2 DDE model

Cocoa pod ripening (and hence, harvest) typically occurs 5 to 6 months after flowering (Wood and Lass, 1986; Daymond and Hadley, 2008), this time lag is incorporated into the model by using DDEs. A DDE model for the number of flowers $F(t)$ and pods $P(t)$ on a tree as well as the total number of harvested pods $H(t)$. The flowering process is given by:

$$\frac{dF}{dt} = \alpha(t)\big(1 - F(t)\big)F(t) - (\beta(t) + \gamma(t))\, F(t) \qquad \text{Eq. 1}$$

In Eq. 1, the first term represents the logistic development of cocoa flowers, whereby the growth rate starts off exponential, and gets smaller as population size approaches a theoretical maximum (France and Thornley, 1984). This growth is mitigated by the growth factor, $\alpha(t)$, which is a function representing rainfall; in this preliminary study other climatic variables, such as temperature, solar radiation, humidity and wind speed are excluded. Flower death occurs at rate$\beta(t)F(t)$, and conversion to pods occurs at rate $\gamma(t)F(t)$.

$$\frac{dP}{dt} = \gamma(t)F(t) - \lambda(t)\,\gamma(t-\tau)F(t-\tau) \qquad \text{Eq. 2}$$

$$\frac{dH}{dt} = \lambda(t)\,\gamma(t-\tau)F(t-\tau) \qquad \text{Eq. 3}$$

Hence, in Eq. 2, cocoa pods are formed at rate $\gamma(t)F(t)$. Pods are harvested at rate $\lambda(t)\gamma(t - \tau)F(t-\tau)$ where the lag τ represents the time taken for pods to ripen, from initial flowering. This harvest rate is given explicitly in Eq. 3.

The rate of flowering is taken to depend only on rainfall. To represent the general trend in rainfall, with two peaks over the year, Gaussian functions were used. These have the advantageous property of describing the height, centre, and spread of a symmetric peak. To capture two peaks over a year, the following form was chosen:

$$\alpha(t) = C_1 e^{-\left(\frac{t-m_1}{\sqrt{2}\sigma_1}\right)^2} + C_2 e^{-\left(\frac{t-m_2}{\sqrt{2}\sigma_2}\right)^2}. \quad \text{Eq. 4}$$

The parameters C_i (height), m_i (centre), and σ_i (spread) are determined by a least-squares fit to the rainfall data. For this preliminary analysis the lag τ from flowering to harvest is taken to be half a year (183 days). The rate functions; $\beta(t), \gamma(t)$, and $\lambda(t)$ are all taken to be time-independent constants, these values are assigned (see Table B.1), and the full model is solved numerically.

3 Results

3.1 Cocoa yield and rainfall results

Rainfall in Ghana is of a bi-modal nature, with two distinct peaks observed in every year. Over the full study period the highest accumulated rainfall recorded was in the Western-South region (2012: 1610mm, 2013: 1400mm, 2014: 2160mm), while the lowest was in the Eastern region (2012: 1310mm, 2013: 936mm, 2014: 1720mm), (Figure 2). The highest cumulative cocoa yields were recorded in the Western-South region of 541 harvested pods tree^{-1} (± 144 SD), while the lowest cocoa yields were for the Ashanti region at an average of 430 harvested pods tree^{-1} (± 153 SD; Figure 2). We acknowledge that the spread of the 18 meteorological weather stations between regions was not even, the highest number being for Eastern region and the lowest for Brong-Ahafo (Table A.1), and the exact coordinates of 5 stations unavailable. Furthermore, the rainfall records for some stations, and for the entire region of Ashanti, were incomplete. The results of this paper use the aggregated rainfall data for the whole country, shown in Figure 2. Future work would require rainfall data at more detailed locations, for example from satellite recordings.

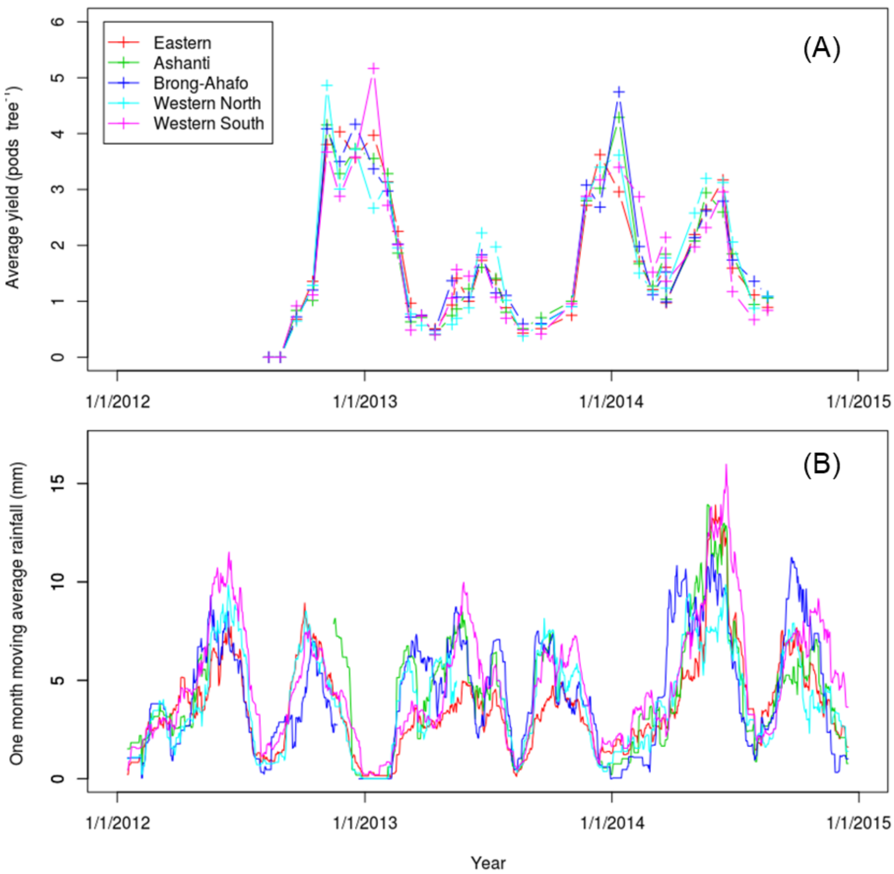

Figure 2. Average cocoa yield (harvested pods per tree) per tree (a) and regional rainfall patterns (b). See Appendix A for more information on calculation of yield and rainfall data.

3.2 Model parameterisation

The least squares fit (Eq. 4) to rainfall data from 2012, 2013, and 2014 is shown in Figure 3 along with and the corresponding model (Eq. 1)-(Eq. 3) output. The function and parameter values are shown in Table B.1. The output is scaled by setting the theoretical maximum number of flowers on the tree to 1, and the parameters and histories (which must be provided to enable numerical solution of the DDE, given it depends on past values) chosen so that the number of pods tends to fluctuate around 0-0.1, i.e. 10% of the maximum possible flowers. In reality the number of pods is an even smaller proportion of the flowers, and the parameters of the model could be adjusted to reflect this when realistic values are known. The lag τ from flowering to harvest is set at 183 days.

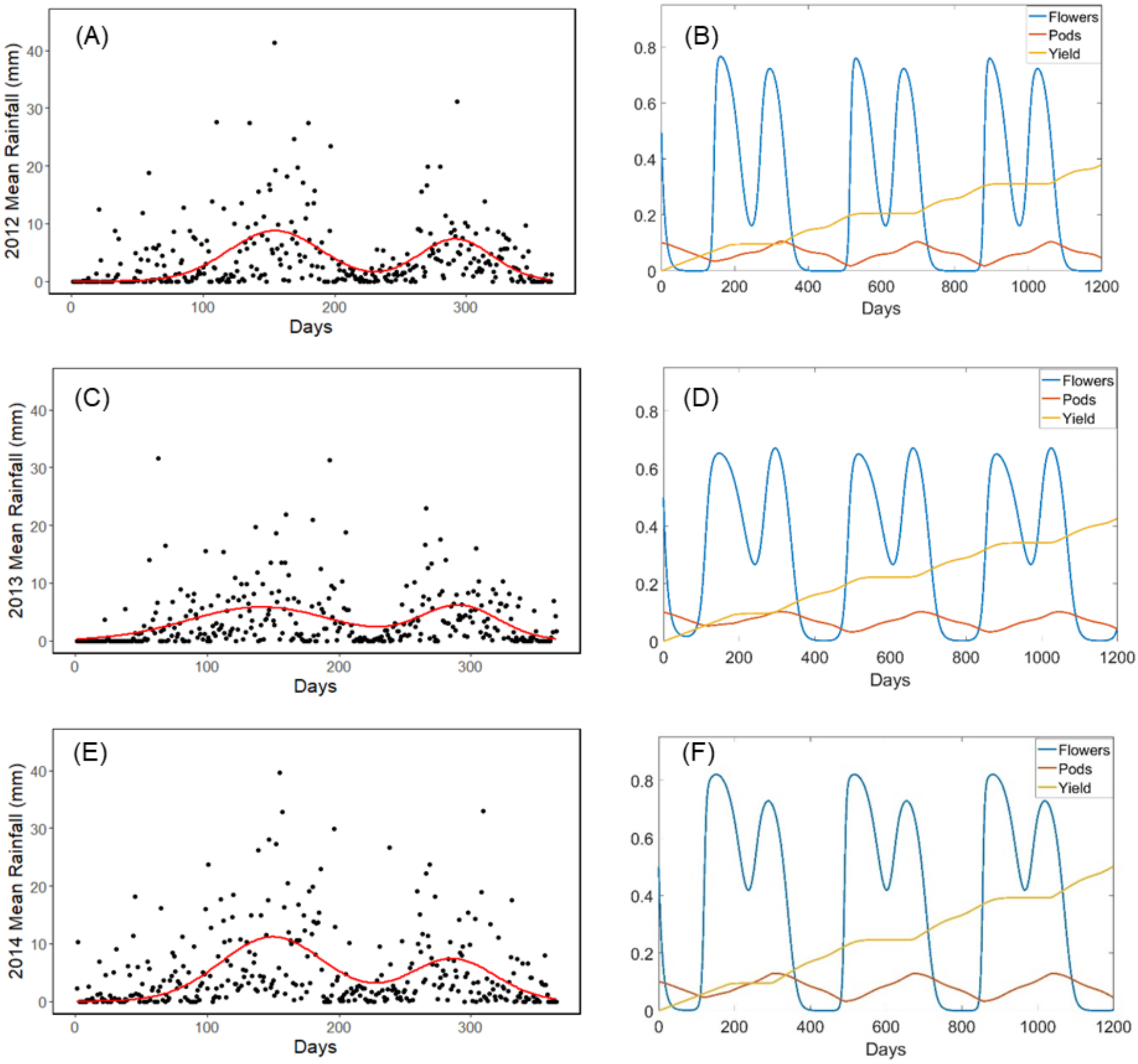

Figure 3. The least squares fit of Eq. 4 (solid line) to average daily rainfall data across all 18 weather stations (dots) is plotted for 2012, 2013, and 2014 in panels (A), (C), and (E), respectively. The corresponding model output taken from inputting the rainfall fit for 2012, 2013, and 2014 is plotted in panels (B), (D), and (F), this is the number of Flowers and Pods on a tree at any given point in time, and the cumulative Yield. The parameters are given in Table B.1.

3.3 Results of the DDE model

The model output in Figure 3 shows greatest yield resulting from 2014 rainfall patterns which were significantly higher than in previous years. The yield is greater for 2013 than 2012 suggesting prolonged lower rain levels result in more pod growth than shorter, more intense bursts of rain. The cumulative yield from the model for August 2012 to August 2014, with data input for rainfall fits from 2012-2013, is shown in Figure 4, alongside the actual cumulative yield data for this period. The first half of the cumulative yield data is driven by the 2012 rainfall where narrower peaks were observed (see Figure 3) the second half of the cumulative yield data is driven by the 2013 rainfall where much broader peaks were observed. The first half of the cumulative yield data has a more pronounced `step', where the harvest rate slows, this is also more pronounced in the first half of the model output. This suggests that the effect of flatter peaks in rainfall is to spread the harvest more evenly over the year, and that the DDEs are successfully able to propagate this effect through the model.

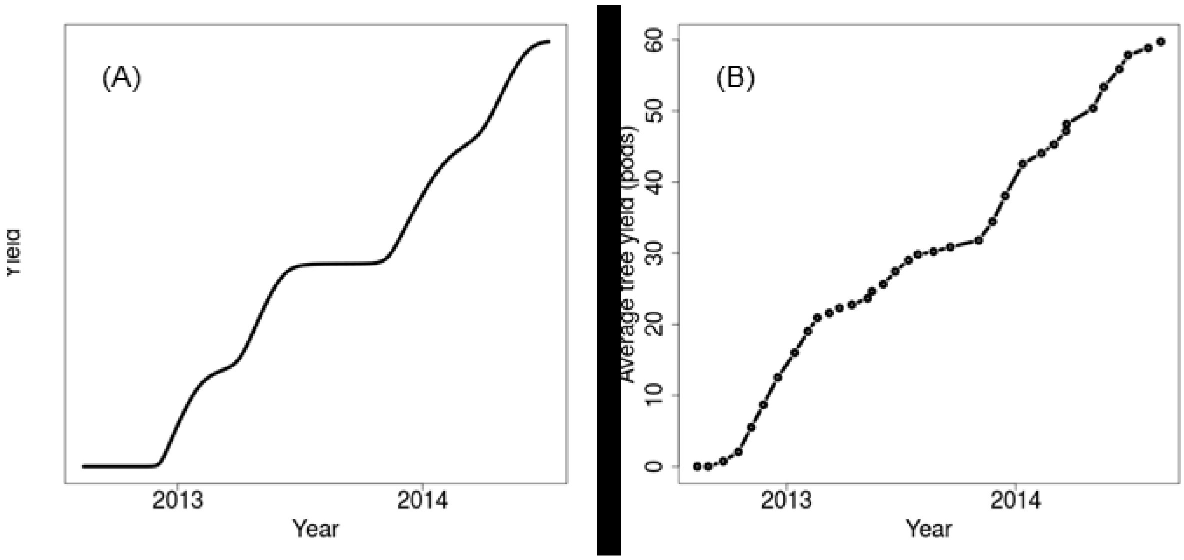

Figure 4. Cumulative cocoa yield from model (A) with 2012-2013 rainfall input (dimensionless), and the actual average yield per tree from the Harvest data (B).

4 Discussion

Using a DDE model, it was possible to successfully simulate the seasonality of cocoa yield based on rainfall pattern and captured the variation in shape of the cumulative yield curve due to different rainfall patterns. The DDE model was constructed with parameters chosen to reflect the biological processes and informed by actual rainfall and farm-level cocoa yield data from Ghana. There is clear potential to build upon this preliminary model through more detailed parametrisation and inclusion of other key factors which influence cocoa yield at the farm-level. This would allow estimation of the proportion of inter-annual yield variation that is attributable to climate, and therefore a much clearer understanding of the contribution of on-farm management. Consequently, this would be a stimulus to investment in small-holder farms which urgently require sustainable intensification to increase economic viability.

As has been mentioned, aside from climate, a myriad of other factors influence cocoa yields. It has been established elsewhere that the main limiting factors for increased cocoa yields on small holder farms are improved farm management practices including; control of pest and disease, the application of fertiliser and pruning of orchards (Kongor et al., 2017). While the presented model accounts for the effect of rainfall on cocoa yield it would be possible to add modules to the existing equations that represent other crop management factors. For example, the effect of applying fertiliser could be added to the model by including an additional multiplier in the growth rate, $\alpha(t)$. Location effects might be better represented in the model through more site-specific rainfall data, which would be possible by making use of satellite rainfall data as opposed to ground stations. Other climatic data such as solar radiation, temperature and relative humidity are known to affect cocoa physiologically, and many of these parameters could be added. However, as such parameters would require ground station data, the geographical coverage of meteorological stations could again be a constraint to making such additions. The availability of soil moisture for uptake by plants is a function of soil characteristics, such as texture and organic matter content and rainfall and soil depth. Due to its smoothing effect on the rainfall data, the double Gaussian curve used here to describe rainfall, could be considered a crude estimate of the soil moisture available to plants. However, given this is one of the main drivers of model output, this could be a fruitful area for further investigation. One method by which this could be achieved would be to incorporate the rainfall-soil interaction in more detail through the inclusion of, for example, soil textural class.

The DDE model can include time-dependent rates of flowers changing to pods, and pods being harvested. In the DDE model presented here, the multipliers on these rates were taken to be time-independent constants (see Table B.1). By restoring time dependence, the model could better reflect the plant biology, for example, if it is very dry, fewer flowers are likely to mature to harvestable pods. In this way the model could capture pod wilt due to environmental factors. Cherelle wilt due to late acting self-incompatibility is a specific genetically determined interaction which is outside of the scope of this modelling approach. Incorporating information from widely accepted physiological models for cocoa (Zuidema et al., 2005) could further aid this model refinement. Also worth consideration are the inclusion of biological constraints such as the dependence of the rate of flowering on the number of pods already on the tree, it is believed that when there are more pods on the tree the rate of flowering may be inhibited (Haberman et al., 2016; Andrew Daymond personal communication). Furthermore, if data were available the size of individual pods or total pod mass could be included as these vary throughout the season and between different cocoa cultivars.

Before any of this future work can be carried out, a range of realistic parameter values should be determined for the current model. This will require an estimation of realistic ranges, and then simulation of the model to determine the best fit to data for different parameter ensembles. Sensitivity analysis will assess how variability in cocoa yield results from altering each of the input parameters in this model. To achieve this, detailed data on rainfall and yield will be required to establish a model that can be implemented to help farmers and policy makers. In summary, there is huge scope to take the DDE model presented here to be developed into a useful tool to predict and understand variability in cocoa yield.

5 Acknowledgments

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6 Appendices

Appendix A. Cocoa Yield and Rainfall Data.

Appendix B. Model Parameterisation.

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