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Rocking of a bell tower - investigation by non-contact video measurement

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Abstract

Church bells are rung in many historic towers, principally in United Kingdom, in a way which enables precise control of timing, and changes in the sequences of the bells – this is known as change ringing. The bells are heavy, many towers containing bells which weigh over a tonne. They are rung by swinging them on an axle through angles approaching 360°, applying substantial forces to the towers. These forces may cause structural damage, and movements of the tower that discomfort the ringers and can make ringing difficult or even dangerous. The movements of towers caused by ringing therefore concern both the ringers themselves, and the architects, engineers and authorities responsible for the historic buildings that house the bells. Movements are usually assumed to be caused by flexing of the bell towers. In this study video recording and imaging software has been used to show that a short tower moves by a combination of rocking on its foundations and bending of the masonry, associated with significantly different natural frequencies. It is shown that minor changes in the speed of the ringing can result in a forcing that works with or against these natural frequencies; the fundamental period of bending is much shorter than the fundamental period of the force from a bell, but the period of rocking is much longer. The significance of this longer period movement in short towers has not been previously recognised, as it has not been detected by the accelerometers normally used in investigations of tower movement. This investigation therefore advances understanding of bell tower movements, but also demonstrates the capability of the video recording and image processing used in the investigation.

1. Introduction

The objective of the work reported here is to explain bell ringers' occasional experience of strong movements during ringing of a short bell tower, which by conventional assessments should move very little, and with too high a frequency to be perceived.

1.1 Movement of a bell

It is easy to think of bells as being like a pendulum, but they are rung in different ways around the world (Ivorra et al [1]), and in no system is the amplitude small enough for the conventional simple mathematics to be applicable. In the English system bells are attached to wheels and swung through nearly 360°. Then there is a second pendulum, the clapper, inside the bell (Figure 1), which is also swinging, and strikes the bell to make a sound. Furthermore, the movement of the bell is controlled by a ringer, and even in "perfect" ringing, the interval between each successive strike is never uniform. Closed mathematical analysis can therefore never present an accurate model of how the loads applied by the bell vary over time. Consider then that a tower will usually have at least six bells, mounted in different positions in the tower with different orientations. The timing of their ringing varies in relation to each other, both deliberately through expert control, and accidentally through mistakes or imperfections. It can then be seen that any analysis which purports to give a complete picture is more likely to be misleading than useful.

The bell is controlled using a rope passing over a wheel attached to the headstock which holds the bell. The rope is arranged so that it can pull the bell in one direction only when it is hanging down, up to about 70° of rotation. The torque that can be applied by a ringer to the wheel is limited by their own weight or strength multiplied by the radius of the wheel. Most bells are many times heavier than the ringer, and this torque does not enable a significant rotation of the bell to be held statically. The ringer lets the bell swing back from this maximum position, and with a series of well-timed pulls, the angle the bell reaches gradually increases, the work done by the ringer being stored as potential energy at the limit of the swing, and kinetic energy as the bell swings increasingly quickly through the down position. Once the bell is swinging high enough, the rope can be used to pull the bell from the limit of its swing in both directions, until the bell reaches a fully inverted position. By imparting just the right amount of work, which in a modern installation is small even for a heavy bell, the bell can be taken to this inverted position repeatedly. This position is metastable, and the bell is described as being at 'balance'. A slightly harder pull will make the bell go past this balance point, and a little load applied to the rope restrains and controls the movement of the bell, so that is eased into the "stood" position. A "stay" attached to the headstock now rests against one side or the other of a slider, enabling the bell to be stopped; the bell is now "up". To reach this position requires the input of significant energy by the ringer, as the centre of mass of the bell has been raised by a metre or more. A comparatively light effort is required to pull the bell out of the "up" position, so that it swings down and then back up again. If the pull is just strong and long enough, the bell will swing through just over 360° to again rest on the stay on its other side. Normally bells are left ready for ringing "stood at handstroke" (see Figure 1), so it would now be "stood at backstroke". A pull which is too hard will either result in the stay bouncing off the slider, so that the bell swings right back again, or will break the stay so that the bell continues around completely out of control. However, the breaking of the stay acts as a fuse, preventing damage to the rest of the installation; the stay is easily replaced, and the ringer has little choice but to let go of the rope to avoid injury.

1.2 The sounding of the bell

As the bell swings, the clapper swings inside it; the clapper incorporates a flight, an extension of the shaft beyond the ball that strikes the bell (Figure 1), so that it behaves as a longer pendulum than the bell and so swings more slowly, catching up with the bell and striking it as the bell slows down approaching the top of its swing. This arrangement gives the ringers fine control of the timing. It is possible to produce a single stroke of the bell, but ringing is usually continuous. The speed can be adjusted, because a little resistance applied to the rope as the bell approaches the top of its swing will slow it down, allowing the clapper to catch up with it earlier. Conversely, if the rope is pulled a little harder, or less resistance applied as it swings towards the top, then the bell will swing closer to the top, and the clapper will take longer to catch up and strike it. In normal ringing the bells are not swung through the full 360°, so that they can be easily slowed down or sped up for a single strike, to change their place in the sequence of bells. "Rounds" is ringing the highest pitched bell (the treble) first, then each bell in sequence through to the lowest pitched (the tenor), with even time intervals between each strike. Then by using the techniques described to produce small changes in the timing, two bells can be made to swap their positions in the sequence. This enables what is known as "change ringing", in which the sequences are changed according to calls or repeated predefined patterns known as "methods" or "principles"; this is the classic sound of British church bells.

1.3 Tower movement

Bell towers are known to move while the bells are being rung. The bells may be heavy, with tenors weighing over a tonne being common; the tenor bell at Liverpool Cathedral weighs over four tonnes [2]. When they are rung, these bells impose considerable lateral forces on the towers holding them – over twice the weight of the bell, depending upon the speed of ringing. Large additional vertical forces are also generated. If the tower moves too far in response to these forces then control of the bells becomes more difficult – movements of just five millimetres are said to cause problems [3].

Tower movement can be particularly dangerous for relatively inexperienced ringers, as the swinging bell can exert a very large force on the rope; normally both bell and rope are under good control, but losing control can result in rope burns, bruising, broken fingers or limbs, or worse. This can also result in damage to the bell installation. There are therefore potentially serious hazards arising from tower movement, and the resulting challenges, especially to especially inexperienced ringers.

The bell towers that produce the most alarming movements are those which are tall and slender, and the movement is due to the flexing of the tower itself. Even comparatively light bells can result in movements which not only make control of the bells difficult, but also cause alarm to the ringers if the ringing chamber is part way up the tower. The height of the towers leads to significant displacements, but also lowers the resonant frequency of tower sway, to the point where it becomes more likely that the speed of ringing leads to resonance and large displacements, and more likely that the ringers will perceive them. Most awareness of tower movement is therefore for such towers. Mathematical modelling of tower movement has shown that including an open handstroke, in which the first bell to ring a handstroke introduces a pause equivalent to one more bell in the sequence, can disrupt the resonance [4], though the aim of doing this is to make the pattern of the ringing clearer. Not having an open handstroke can make the ringing sound frenetic, and produces an unvarying driving rhythm to the forcing of the tower; according to the mathematical model, it also doubles the number of possible frequencies that might produce resonance.

1.4 Investigation of tower movement

Brown et al [5] investigated a masonry bell tower in Bendigo, Victoria, Australia. A program of measurements with accelerometers and interferometric radar was supported by finite-element analysis, and showed that the stresses resulting from ringing would be small compared with accepted limits. Even so, they recommended alterations to the bell hanging which would be expensive and would have a detrimental effect on the ringing; the installation of a tuned mass damper system; and the implementation of a monitoring system to warn 'operators' of exceedances of specified safety limits. This apparent over-reaction was probably due to ringing having been stopped in the tower due to perceived movements, but it took into account that relatively small changes to the speed of ringing could result in a resonance that had not occurred during the tests. This study, and the over-cautious response to the uncertainty, makes clear that there is a need to find an observation-based approach to assessing tower behaviour.

Lund, Selby and Wilson [6], and Selby and Wilson [7], recognised that tower movements have three components of bending, shear, and rocking the foundations, but reported natural frequencies of free vibration in the range 1.3Hz to 3.9Hz, with shorter towers having higher frequencies. Most concerns about movement relate to tall towers, for which bending is the dominant behaviour, with frequencies that are long enough to cause problems.

1.5 The tower for this study

Observations were made of the bell tower at Saint Peter's and Saint Paul's church, Longbridge Deverill, Wiltshire. The height of this tower is 16m to the top of the parapet, the bells being hung at a height of about 13m. The plan dimension, not including the buttresses, is 6.3m (N-S) by about 6m (E-W). The walls are 1.35m thick at the base, reducing to 1.1m at 8.8m up (the level of the ringing chamber floor), and about 0.9m at the level of the belfry. The arrangement of the bells in the tower is shown in schematic plan in Figure 3. It can be seen that bells are orientated both North-South and East-West (Figure 3), and that the arrangement of ropes and pullies results in the bells moving in opposing directions were they all to be rung at exactly the same time. However, they are rung in sequences, which change, so this apparent attempt to reduce the imbalance of forces is not always effective. The heaviest bells, numbers 7 and 8, are orientated to ring on the E-W axis, in the expectation that the presence of the nave, the main body of the church, will stiffen the tower's response in that direction. Photographs of the outside and inside of the church, and of the bells, are given in Figure 4. The high and wide archway between the tower and the nave, seen in Figure 4b, introduces a weakness in the E-W direction, resulting in cracking at the top of the arch (Figure 4d). The width of the crack was observed in this study to change in correspondence with the forces imposed by the ringing of the bells.

This tower was of particular interest because ringers reported occasionally experiencing a 'lurching' movement of the tower which made them uncomfortable climbing the spiral staircase, particularly when the bells were being rung a little slowly. As a short tower, the natural frequency would be expected to be too high for significant movement to happen. Such movement was not observed during the experiments, but the characteristics which could result in this behaviour were observed.

2. Modelling of bell forces

Much of the work that has been done in the past (e.g. [8]) has been based upon mathematical approaches related to those used in analysing electrical circuits, and does not take into account the variations imposed by actual ringers of varying degrees of experience. The basic behaviour has been examined by a number of authors [9,10,11].

If a ringer provides no input to the movement of a bell, such as occasionally happens if a rope snaps, the bell continues to swing and sound for minutes; very little force is required to overcome air resistance and friction in the bearings. Figure 5 shows a plot of a simulation of the movement of a bell, using a time-stepping calculation in steps of two milliseconds. The bell parameters are those of the sixth bell in the Longbridge Deverill tower. This is in effect a numerical integration of the equations presented by Woodhouse et al [12], and models the clapper as well as the bell itself. The same method was used to produce Figures 6 to 8. These equations are:

$$[I_b + I_c + mr^2 + 2mrb\cos\emptyset]\ddot{\theta} + [I_c + mrb\cos\emptyset]\ddot{\emptyset} - mrb\dot{\emptyset}\left(2\dot{\theta} + \dot{\emptyset}\right)\sin\emptyset + Mga\sin\theta + mgr\sin\theta + mgb\sin(\theta + \emptyset) = Q_\theta$$

[1]

$$I_c\left(\ddot{\theta} + \ddot{\emptyset}\right) + mrb\ddot{\theta}\cos\emptyset + mrb\dot{\theta}^2\sin\emptyset + mgb\sin(\theta + \emptyset) = Q_\emptyset$$

[2]

In these equations, the bell and its wheel and headstock, with total mass *M*, have moment of inertia I_b about its bearing, and its centre of mass is distance *a* from its axis. Corresponding values for the clapper are *m*, I_c and *b*, while the axis of rotation of the clapper lies within the bell, a distance *r* below the axis of the bell. The angle between the bell and the downwards vertical is θ, and that between the clapper and the bell's axis is ø (Figure 6). Q_θ and $Q_ø$ are then 'generalised forces', the most important being that applied by the ringer via the rope – and hence not a continuous function.

A single bell produces a horizontal thrust in one direction followed immediately by a push in the opposite direction as it passes through a horizontal position (90°); it will then produce thrusts in the opposite sense as it swings back through the opposing horizontal to return to its starting point (Figure 5). The pattern of additional vertical forces is different, producing significant downwards thrust each time the bell swings through its 'down' position. Clapper movement relative to the bell is shown as a grey line; it swings through a small angle of about 55° from one side of the bell to the other, coming to a halt shortly after first contact, and then coming away from the side of the bell to catch up with the opposite side as the bell swings around. The thirteenth harmonic of the frequency of the swinging bell is also shown; this is approximately the natural frequency of flexing of the Longbridge tower. The time axis in Figure 5 begins at 10s to give the simulation time to settle down.

The most important variation from this simple even ringing is the open handstroke – this was addressed by Smith and Hunt [4]. Ringing without an open handstroke is called cartwheeling; it is deliberately practised in some places, but is unusual.

If six bells are rung without an open handstroke, then the interval between each successive strike is one twelfth of the time for a "whole pull" – a handstroke followed by a backstroke. Including the open handstroke means that the time from one handstroke sound to the next handstroke sound (T) will correspond to twelve strikes at intervals of T/13, followed by the open handstroke with no bell sounding before the sequence begins again. Hence the thirteenth harmonic, shown in figure 5, has the potential interest that the maximum forces from successive bells might align with the peaks of the tower's response. This would only be so if all of the bells were mounted on wheels of the same size – lighter bells are usually on smaller wheels, and come closer to the top to maintain the same speed of ringing as the heavier bells. It would only have a consequence if the layout of the bells led to these peaks of force reinforcing each other, and the effect was not being counteracted by opposing forces; even then, this might only happen with particular sequences of bells. Nevertheless, it may only be necessary for two or three of the heaviest bells to have the right timing to produce problems. It should be noted that the heaviest bells are usually installed to swing in the direction in which the tower is stiffest, from East to West for a typical tower placed on the East-West axis of the nave.

Figure 5 shows the bell swung up to 169^o, (0^o is the 'down' position, 180^o is 'up'); the interval between strikes corresponds approximately to that rung during the tests reported below, and during normal ringing. The thirteenth harmonic has a frequency of about that observed for the flexing of the tower, about 3.5Hz. There is some correspondence between the horizontal impulse and the natural frequency, in that each pulse corresponds to two peaks in the same direction but only one in the opposite direction. The forcing of the tower by this pattern is nevertheless weak. It can be seen that even a very small variation of the timing of the ringing will mean that the successive peak horizontal forces acting in opposite directions could switch from reinforcing the movement to counteracting it. Therefore no matter what might be said about the mathematical correspondence between an n^{th} harmonic of the bell swing and the resonant frequency of the tower, the actual movement of the bell, controlled by a ringer, with an open handstroke and changes of place, is very unlikely to continuously reinforce the flexing of a tower with such a high natural frequency. Typically towers are considered to have a fundamental frequency for flexing of about 46/H (for example [13]), where H is the height in metres and the frequency is in Hz; this yields 2.88 Hz for the 16m high tower at Longbridge Deverill. St Mary Magdalene in Taunton, Somerset, has a very tall tower of 50m (including the decorative pinnacles) which undergoes significant movement, and would be expected to have a natural frequency of about 0.92 Hz. Figure 7 shows the horizontal force from Figure 5 together with a harmonic at a frequency for a tower of about 44m (1.05Hz, approximately corresponding to the tower below the decoration). It can be seen that the simple harmonic curve corresponds with the more complex curve of the force from the bell in such a way that the tower will resonate: the peaks and troughs of horizontal force align with the peaks and troughs of the tower's resonance. See, for example, where the markers (a) and (b) have been placed in Figure 7 at about 15.1s and 15.6s.

The open handstroke means that the major impulses produced do not follow at even intervals, making resonance much less likely; it can be seen that the bell spends longer with a positive angle than with a negative angle, and the effect produced by one stroke could be countered by the effect of a subsequent stroke (Figure 8). The open handstroke is less significant if the natural frequency of the tower is lower (Figure 9), as it makes little difference to the correspondence between horizontal bell impulse and tower response.

Another disruptive effect is that individual bells are sped up or slowed down a little so that they change place, either in response to a call in called changes, or almost continuously in method ringing, hence producing varying sequences of stronger impulse. Smith and Hunt [4] examined the simplest "method" for a twelve bell tower, "Plain Hunt on 12", in which all twelve bells are alternately stepping up and down between first and twelfth place, with the even numbered bells moving in one direction and the odd-numbered in the opposite direction in the sequence in a weaving pattern taking nearly a minute to get back to rounds. If a bell is "hunting down", each strike occurs one position earlier in the sequence, requiring a shortening of its period by a factor of 11/13; the corresponding hunting up would involve a slowing down by a factor of 15/13. This means that during method ringing, when most bells are changing place continuously, the speed of individual bells is varying significantly, so potentially moving into and out of resonance with the tower. Thus it is possible that significant movements are only experienced when particular bells are hunting in the right direction – and that the direction of the resulting movement only causes problems for one of the other bells, which happens to be swinging in the same direction but not in time with the first bell. This phenomenon is known by very experienced ringers – in one tall Somerset tower, a particular method causes problems for one bell, to the surprise of the ringer who doesn't know about it, and to the amusement of those ringers who do.

3. Predictions of movements

The movements of bell towers are normally ascribed to their flexing. Expressions are given in many places for the estimation of their natural frequencies, for example Ivorra and Pallarés [14] give:

$$\omega_1 = \frac{\sqrt{L}}{0.06\,H\sqrt{\frac{H}{2L+H}}}$$

And then $\omega_2 = 3\,\omega_1$, $\omega_3 = 5\,\omega_1$, where L is the plan dimension of the building in the direction of the oscillation, and H is the building height – this is taken from the Spanish Standard NCSE 2002 [15]. For Longbridge this gives a frequency of $\omega_1 = 3.75$Hz.

They also offer a more sophisticated expression relating the elastic modulus to the natural frequency:

$$E = \frac{4ML^3}{b^4 - (b-e)^4}\,\omega^2\ (N/m^2)$$

where:

M = mass of the tower

L = crown height

b = width of the square tubular section

e = wall thickness

ω = single degree of freedom natural frequency

This gives frequencies over 10Hz, depending upon the modulus value chosen, which do not correspond to the frequency observed when a bell was stopped.

These expressions give relatively high natural frequencies, which result in a tower response which is easily disrupted by minor changes in speed of ringing, and in particular by the open handstroke. The motion occasionally experienced by ringers at Longbridge is of a much lower frequency which feels quite uncomfortable; hanging pictures can sometimes be observed moving. Therefore rocking of the tower on its foundations is considered, using a method given by Kelly [16]. This procedure requires the soil shear modulus G, Poisson's ratio ν, and a simplified Winkler soil model based on FEMA 356 [17]. The second moment of mass (the moment of inertia) must be determined. This is dependent upon the unit weight of the masonry. Whilst the stone itself probably has a unit weight of around 2400 kg/m^3, that of the masonry as a whole is likely to be lower. Even though the construction appears to be solid on the wall faces, it is likely that there is rubble infill between ashlar faces, and significant voids. Therefore values were also calculated for 2000 kg/m^3. From the measurements of the tower, this indicates a second moment of mass of between 49 and 59 x 10^6 kgm^2. The foundation soil is most likely to be the Cann Sand, at the base of the Upper Greensand, for which a Young's Modulus might be between 50MPa and 100MPa – a precise value is not needed to obtain an approximate period for the rocking, and no in-situ testing is available. Considering this range of values, Kelly's method indicates a period of between 4s and 6s. Therefore even accepting the uncertainty in the range of values, the period of the rocking response is considerable longer than that of the bending response, and is long enough that variations in ringing speed could lead to the impulses from the larger bells as they pass through +/- 90 reinforcing or counteracting the motion of the tower.

4. Observations of movements

Movements during ringing were observed on two separate occasions, with the aims of capturing the behaviour observed by the ringers, and of providing enough information to be able to explain it. A video displacement monitoring system by iMetrum [18] was used, in which pixel interpolation enables movement to be measured to a small fraction of a pixel, potentially at a high frequency. The camera used a telephoto lens to give a narrow field of view, and over the distances it was used was able to give a resolution of about one thousandth of a millimetre; however, the noise on the measurement is typically of the order of one hundredth of a millimetre. The system allows points to be chosen within the image frame, and the displacement of these points to be determined for each successive frame; the points need only show a clear pattern in the monochrome image used, they do not need to be conventional survey markers. For this study, the patterns and markings on the stonework were sufficient. The frequencies associated with the ringing of bells is well within the capabilities of the system. Fifty frames per second were observed, and noise reduced by taking rolling averages over five readings to arrive at the plots presented here.

Observations were made from outside the tower, and from inside the church underneath the arch between the tower and the nave, looking up at the top of the arch, where there was a crack over its full thickness (Figure 4c). In both cases the camera was mounted on a heavy wooden surveying tripod. Outside, the legs were spread at a good angle to provide optimum stability, but conditions were windy, and there was ground-transmitted vibration due to traffic on the adjacent road; this reduced the quality of the observations, even with the inclusion of a reference object in the distance.

For the inside observations of the top of the arch the tripod was placed on a solid tiled floor, and the camera was pointed up at the top of the arch. This meant that the tripod was sitting on ground which must have been affected at least in part by any movement of the tower foundations. Notwithstanding the opening of the arch, the tower was sufficiently massive and well buttressed that it would be expected to be stiff in bending in both E-W and N-S directions. The existence of the crack at the top of the arch indicates some outward flexing of the sections of wall to either side. Detailed processing of the images of observation points on either side of the crack indicated it to be opening and closing during the ringing of the bells, especially during the ringing of the heaviest bell, no.8, which swings in the E-W direction, thus applying vertical load to the top of the arch. Figure 5 shows that the pattern of vertical forces is different from the pattern of horizontal forces. When bell 6 is rung, which swings N-S, the pattern of crack opening is dominated by the 3.5Hz frequency of the bending oscillation, with only a slight effect from the horizontal forces. The extension of the crack is plotted in Figure 10 together with E-W and N-S movements for 20s during the ringing of bell 7. Bells 7 and 8 both swing E-W, and both produce a pattern of crack extension which coincides with the pattern of E-W movement, which corresponds to the pattern of horizontal force from the bell. This suggests that the crack may be flexing, opening up at its eastern end in response to the thrust from the bell. Detailed examination of the observations confirms that this is the case.

The major concern of this paper, however, is the overall pattern of movement, and the pursuit of an explanation for the behaviour experienced by the ringers. The N-S and E-W movement for the full sequence of observations is shown in Figure 11. The bells were rung in the sequence 6, 7, 8; the main movements correspond to the directions of swing. Both E-W and N-S show a drift in the readings, which is most likely to be due to a slow movement of the camera, probably due to expansion of the tripod legs as they had been outside prior to these observations – as little as a degree of temperature change could produce the movements observed. However, there are significant N-S movements when bells 7 and 8 are rung, and substantial E-W movements when bell 6 is rung. The latter may be due to the stiffening presence of the stair tower in the NE corner, so that there is some plan rotation about this corner as bell 6 is on the western side of the belfry, and so exerts a moment.

Figure 12 shows the response to the ringing of bell 6 in more detail. The bell started ringing at point (a). The high frequency bending response of the tower is seen clearly, continuing for over ten seconds after the bell stopped ringing at point (b), showing very little damping at these very small amplitudes of movement. A similar pattern of motion after the bell was stopped is seen in Figure 13 for bell 7. In both cases the lower frequency rocking response of the tower is seen most clearly after the bells stopped, with a period of about 6s. This is within the range of rocking frequencies that have been predicted. The pattern of motion throughout the ringing is complex, suggesting possible interferences between different modes of oscillation. It appears that the damping for these oscillations is no stronger than that for the bending - the amplitude of the oscillations seen is also small. It is possible that the movement detected before the start of the recording was produced by the immediately preceding ringing. The pattern of the longer wavelength response is not at all clear, and if anything is more sawtooth than sinusoidal. It is definitely complex and uneven, which may relate to the ‘lurching’ reported by the ringers.

There appears to be interference between the rocking and bending responses. This is seen in Figure 13. The bell is stopped at point (a), after which the oscillations reduce in magnitude, the amplitude halving over about eight cycles. At point (b), the amplitude increases again, as a significant movement occurs at the lower frequency.

Fourier Series analysis has been carried out on the observations, but the only frequency shown clearly is the bending response already noted. The complexity and unevenness of the motion prevents the technique from yielding more useful information

5. Discussion

The movements observed were all quite small, under half a millimetre; at no time during the experiment did anyone present feel the movement of the tower, though the movement is felt from time to time during normal ringing sessions. The observations show patterns of movement that correlate to the predicted forces; this is seen most clearly in comparing figures 5, 10 and 13. The observations also show that the tower is oscillating at the frequency expected for bending of a tower of this height. The damping on this frequency is slight. A lower frequency movement is also apparent in the observations, being particularly clear after the bells are stopped. The motion is far from sinusoidal, suggesting an interference between different modes of movement, but the frequency is within the range that is predicted for rocking on the foundations. The amplitude of the rocking movement appears to be about four or five times greater than that due to bending.

The longer period movements corresponding to rocking frequencies shows that a resonant response is possible. A movement at these frequencies is much more sensitive to changes in the speed of ringing. It is possible that slower than normal ringing is more likely to produce the strong movements reported; this correlates with the reports of the ringers. Particular sequences of bells could have stronger effects. When ringing in rounds, bells 7 and 8 swing in opposite directions almost together, almost cancelling out the forces they produce. In called changes, it is possible to be ringing the bells with a gap between them that could lead them towards reinforcing each other. It is also possible that the vertical component of force, which gives impulses which are always downwards, is more significant in its interaction with the movement of the tower because a thrust in one direction is not immediately cancelled by a thrust in the opposite direction. This introduces the possibility that the response of the tower is significantly influenced by the effect of the high archway through to the nave.

For this short tower, the rocking response is stronger than the bending response, and because it is at a lower frequency, it is more sensitive to variations in the ringing of the bells. Because the damping is slight, it is not difficult for the rocking to result in movements that are perceived by the ringers, which may even cause discomfort or difficulty in controlling the timing of the bells. In this tower, it is possible that slower than normal ringing was necessary to produce resonance, but in other towers this may happen at normal ringing speeds. The precise pattern of the ringing may also have an effect on whether a strong movement occurs. The forces produced by the bells appear to have caused cracking in the structure of this tower, and may do so in other towers. In general, it is necessary to understand the mode of deformation, and the cause of the deformation, if damage is to be correctly interpreted and controlled. These observations show that short towers may undergo significant movement due to rocking, at a low frequency which is unlikely to have been detected by accelerometers.

6. Conclusions

The response of the tower to the forcing by the swinging bells showed clear movements that corresponded directly to the predicted applied forces. There were additional responses which corresponded to the frequencies associated with different modes of oscillation of the tower. Movement of bell towers is normally treated as due to bending, and the corresponding natural frequency is usually assumed to be the most important. The observations reported here showed this response clearly, but it has also been shown that resonance at the bending frequency of a short tower is easily disrupted. A natural frequency for rocking on the foundations has been shown to be much sensitive to reinforcement by ringing; this is likely to be more important for most relatively short and wide towers, and is more likely than bending to explain perceived movement in such towers. The forces produced by these heavy bells could lead to structural damage, and a crack associated with the movement of the bells was observed here.

Through a better understanding of the resonant response of towers, better decisions may be made about what can and cannot be achieved through structural interventions. It is also possible to gain a better understanding of the patterns of ringing that might lead to problems controlling the bells, to ringer discomfort, or even to structural damage. These could lead to appropriate advice regarding patterns of ringing, in terms of methods and speeds, that could prevent such problems recurring. More broadly, this study points to the possibility that lower frequency responses associated with soil structure interaction, which may not have been detected using conventional accelerometers, could be important in understanding the dynamic response of structures.

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Figure 1 A church bell mounted for full-circle ringing. The wheel is on the left, to which is attached the headstock, to which the bell is attached. The shaft, ball and flight of the clapper are clearly seen inside the bell. A rope runs around the wheel rim to make the bell swing and control its movement.

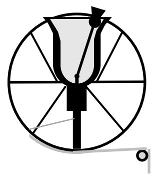
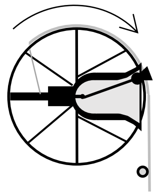
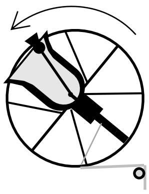
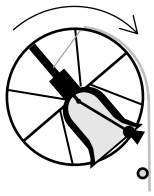
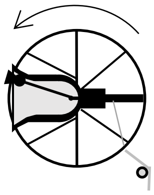
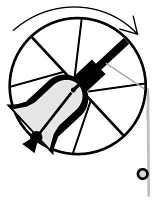
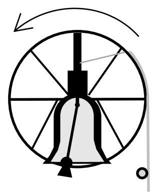
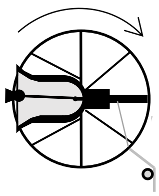
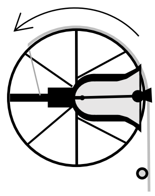
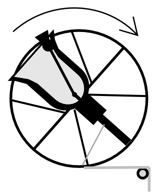
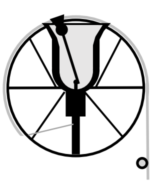
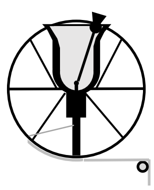
		θ, $\dot{\theta}$, position			θ, $\dot{\theta}$, position
1		180, start -ve Bell up at handstroke, just coming off balance	7		-90, +ve Backstroke still pulling bell down with full power
2		135, -ve Limit of maximum efficiency of handstroke	8		-45, +ve Effective limit of backstroke pull
3		90, -ve Sally at lowest point, limit of handstroke	9		45, +ve Sally is now coming down ready to be caught, tail is being swung down by the ringer
4		0, -ve Bell swinging fastest, sally going up, clapper swinging across	10		90, +ve Sally about to go up, will be caught on the way up, lifting ringer's arms
5		-90, -ve Bell slowing down, tail is lifting ringer's arms	11		135, +ve Bell slowing down, finely controlled by the ringer, holding the weight of the ringer's arms
6		-180, -ve to +ve Bell up at backstroke, about to be pulled down	12		180, +ve to -ve Up at handstroke, about to be pulled on handstroke

Figure 2 The sequence of rotation of a bell rung full circle

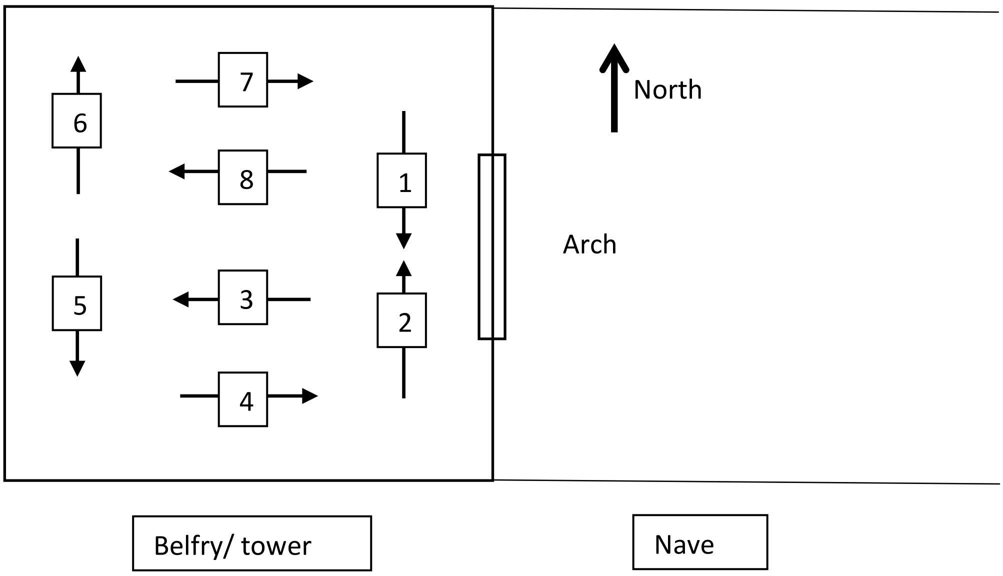


Figure 3 Schematic diagram showing the positions of the bells in the belfry, and the direction of first movement at the start of normal ringing

Figure 4. The church of Saint Peter and Saint Paul, Longbridge Deverill, Wiltshire

a) The outside of the tower. The ringing chamber is above the large west window, the belfry above that.
b) The arch between the nave and the tower.
c) The bells.
d) The crack at the top of the arch in a).

All photographs by the author.

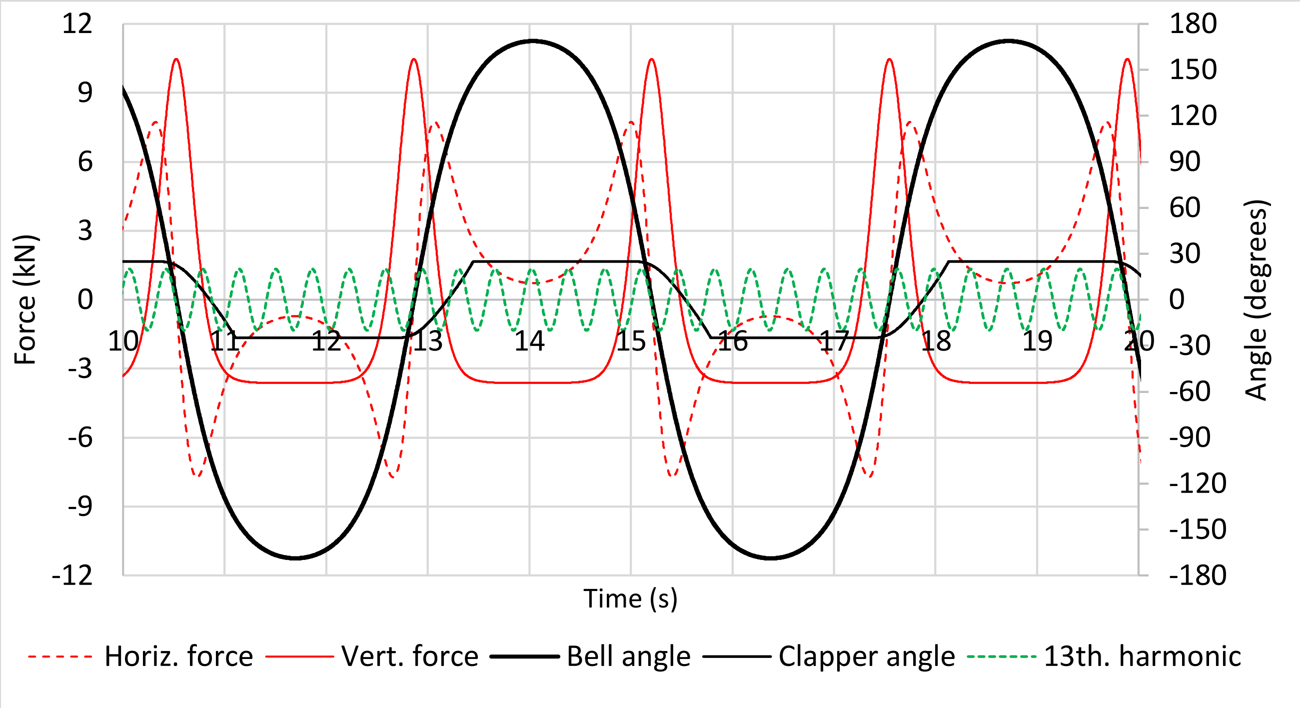


Figure 5. Simulation of bell motion and forces.

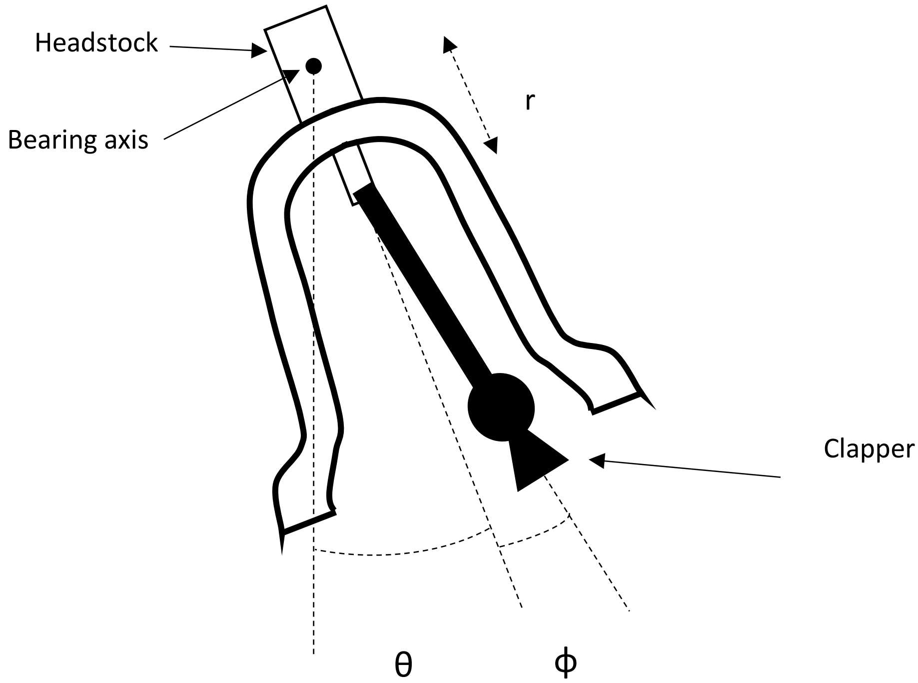


Figure 6 Schematic section through bell showing notation for equations

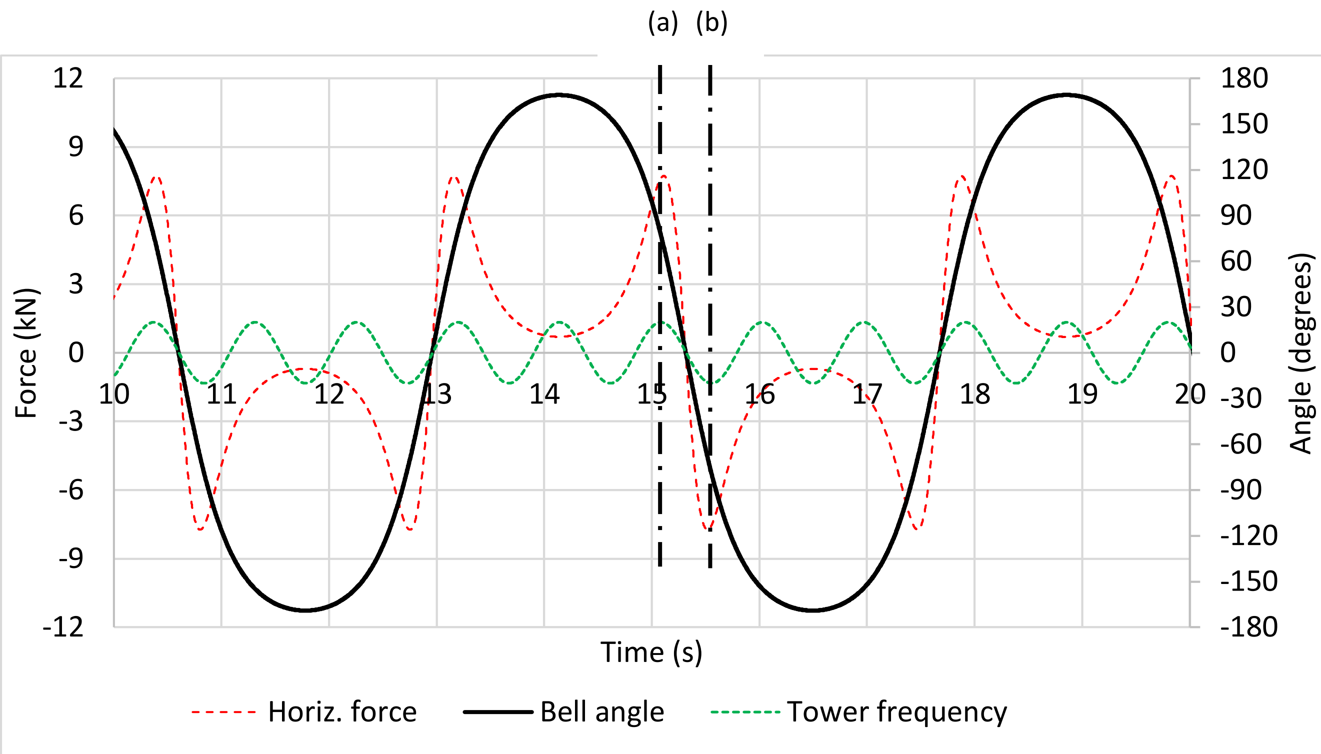


Figure 7. Relationship between horizontal bell force and the natural frequency of a tall tower

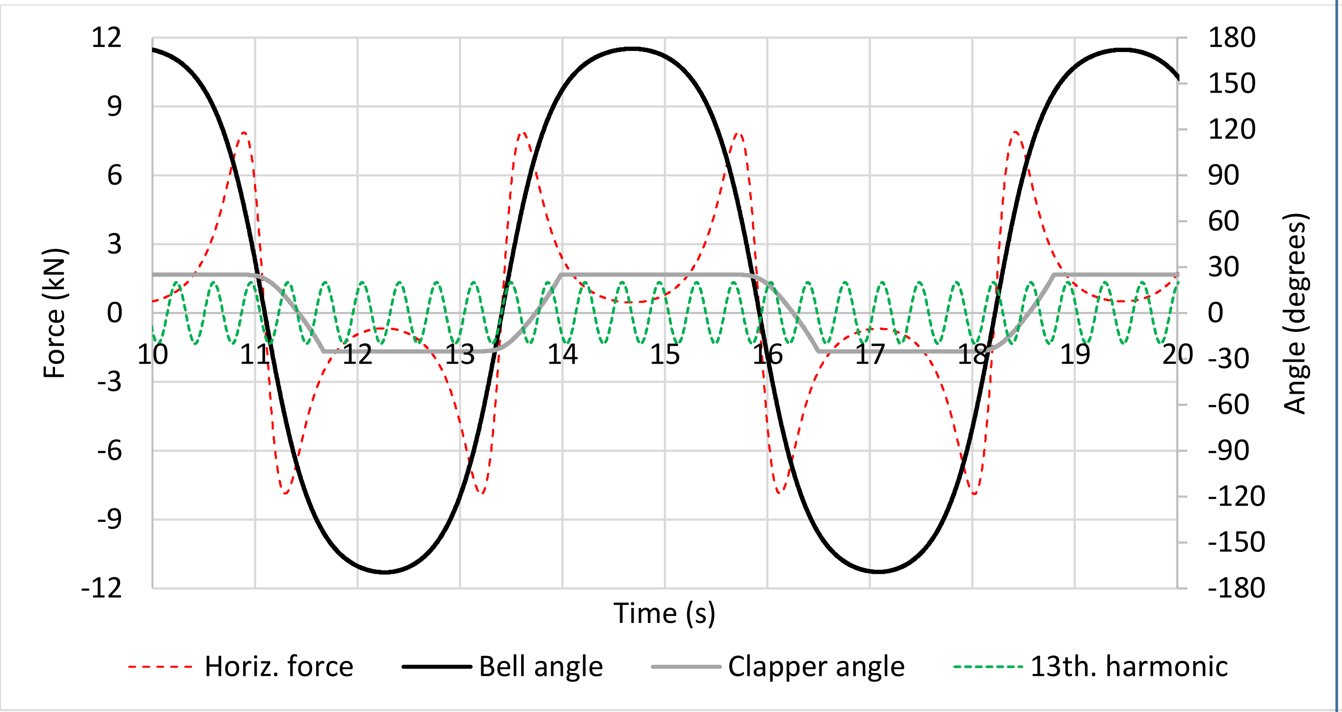


Figure 8. The effect of an open handstroke
The forces applied to the bell by the ringer have been adjusted to produce a typical ringing speed and the open handstroke. The dotted line is the thirteenth harmonic of the overall ringing speed, and is close to the natural frequency of the Longbridge tower.

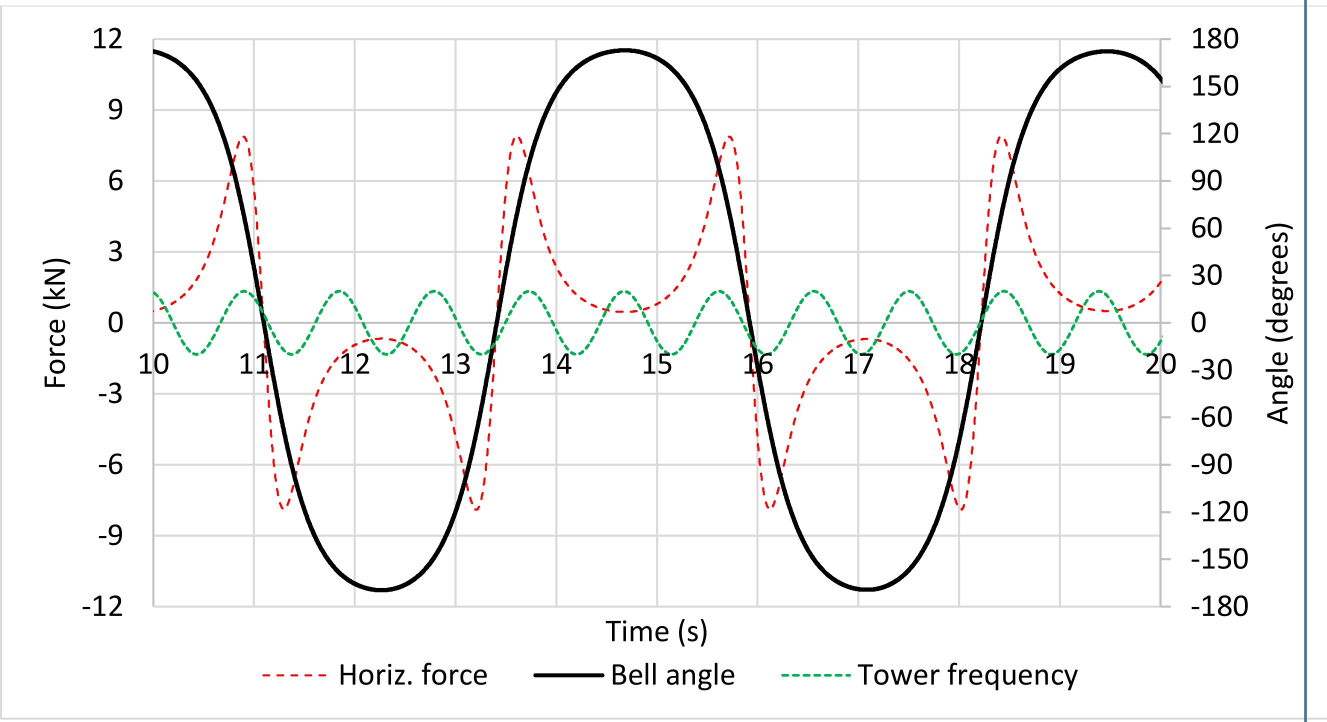


Figure 9. The effect of an open handstroke in a tall tower
This shows the equivalent of figure 7, but ringing with an open handstroke. Though the alignment between the horizontal force pattern and the tower frequency varies a little over time, it is still close enough that the force from the bell will produce a resonant response.

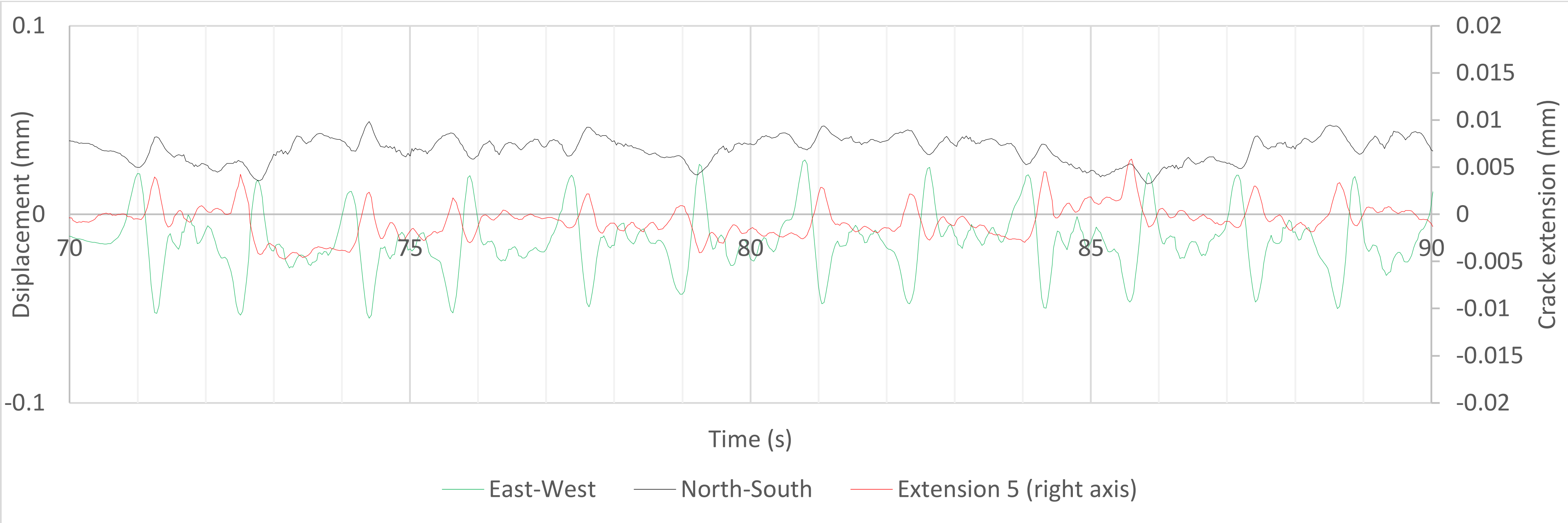


Figure 10. E-W and N-S movements of the arch, and extension of the crack, during the ringing of bell 7.

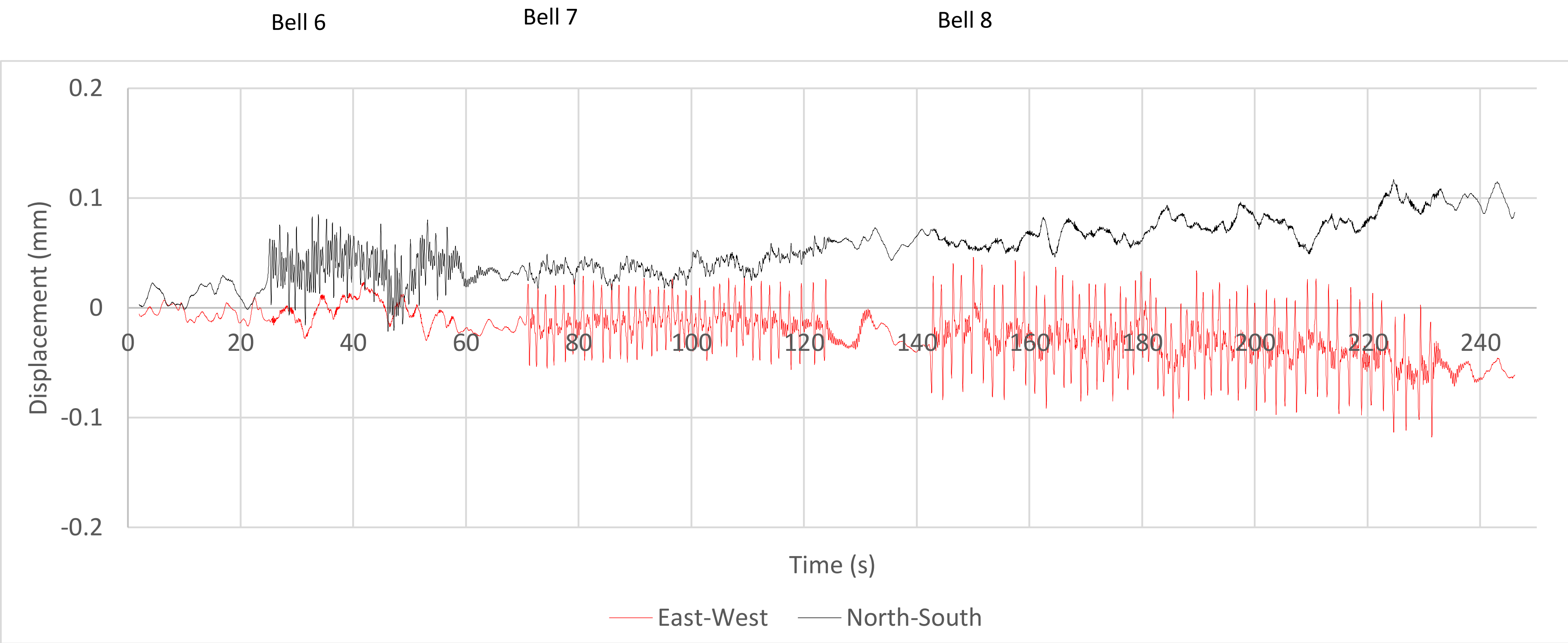


Figure 11. N-S and E-W movements of the top of the arch.

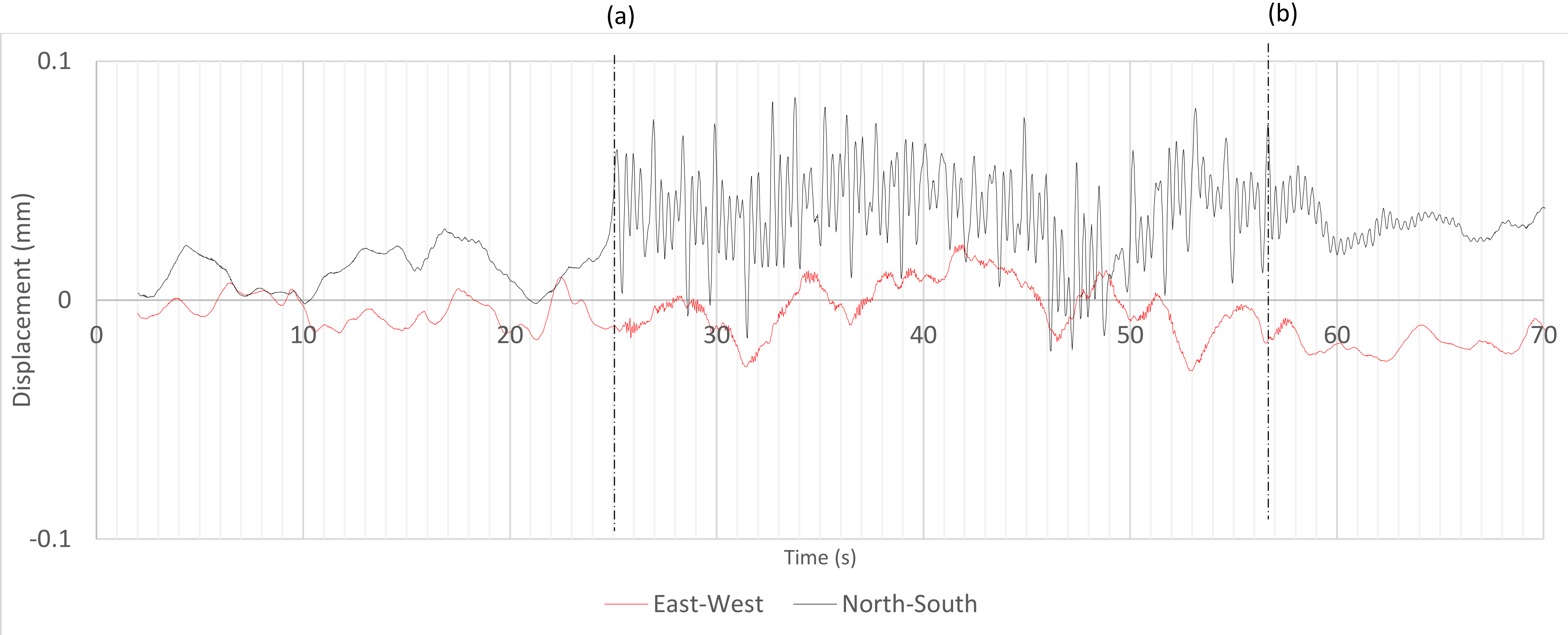


Figure 12. Observations of the movement of the top of the arch, including the ringing of bell 6.

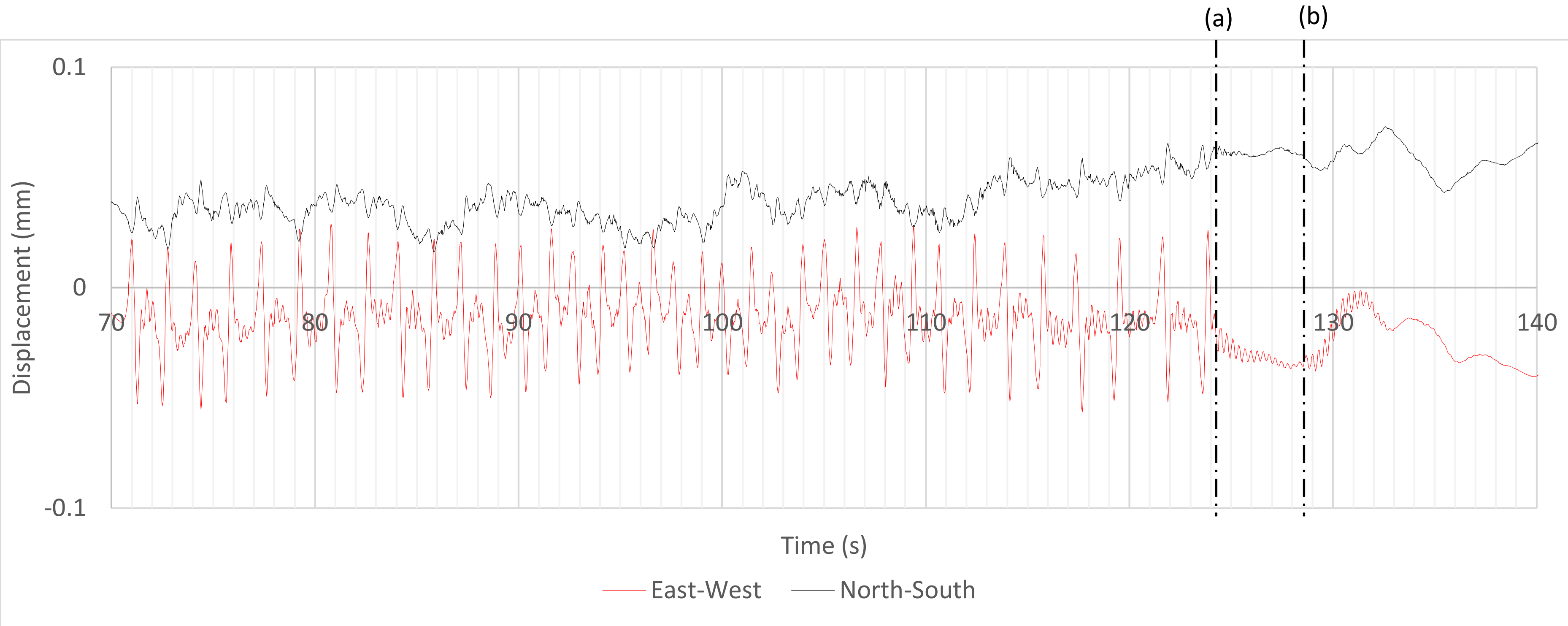


Figure 13. Observations during and immediately after the ringing of bell 7.