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Comparison Criteria for Argumentation Semantics

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Abstract. Argumentation reasoning is a way for agents to evaluate a situation. Given a framework made of conflicting arguments, a semantics allows to evaluate the acceptability of the arguments. It may happen that the semantics associated to the framework has to be changed. In order to perform the most suitable change, the current and a potential new semantics have to be compared. Notions of difference measures between semantics have already been proposed, and application cases where they have to be minimized when a change of semantics has to be performed, have been highlighted. This paper develops these notions, it proposes an additional kind of difference measure, and shows application cases where measures may have to be maximized, and combined.

1 Introduction

Argumentation is a reasoning model which has proved useful for agents in many contexts (*e.g.* decision making [3], negociation [2], persuasion [27]). Abstract argumentation frameworks (AFs) are classically associated with a semantics which allows to evaluate arguments' statuses, determining sets of jointly acceptable arguments called extensions [4,18].

In [7,8], a method to modify an AF in order to satisfy a constraint (a given set of arguments should be an extension, or at least included in an extension) is defined; this process is called extension enforcement. The authors distinguish between conservative enforcement when the semantics does not change (only the AF changes) and liberal enforcement when the semantics changes. A first study of semantic change in a situation of enforcement has recently been conducted in [17]: it shows how to minimize the changes to perform on an AF in order to enforce an extension, by changing the semantics, for a new one which is not too "different" from the current one.

A change of the semantics may be necessary for other reasons, for instance, for computational purposes: if a given semantics was appropriate at some point in a certain context for some AF, one may imagine that changes over time on the structure of the AF (number of arguments, of attacks, structure of cycles) may make this semantics too "costly" to compute. It may then be interesting to

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pick up another semantics to apply to the AF, possibly not too dissimilar to the former on its acceptability results, but quite dissimilar regarding computational complexity.

The other way round, in contexts like decision or deliberation, a given semantics may be interesting from a computational point of view, but the results that it returns may be found for instance too restrictive, in the sense that, if the agents agree on the extensions that it returns, they would like to have more options, as many as possible, including the ones which have been returned. It may then be interesting to change the semantics, for a new one which is not too dissimilar in complexity to the former, but which extends the set of extensions. *Difference measures* between semantics, to quantify how much a semantics is dissimilar to another one, allow to define different minimality and maximality *criteria*. Such criteria can be used and combined to select the new semantics among several options when a semantic change is required.

This paper recalls and presents several sensible ways to quantify the difference between two semantics, depending on:

– the computational complexity of semantics;
– the properties which characterize the semantics;
– the relations between semantics;
– the acceptance statuses of arguments the semantics lead to in a specific AF.

The first measure is new; the last three measures have been proposed in [16], and illustrated on a number of semantics; they are developed here, proofs of the properties that they satisfy (whether they are distances, semi-distances or pseudo-distances) are given, and additional semantics are considered.

2 Background Notions

An Argumentation Framework (AF) [18] is a directed graph $\langle A, R\rangle$ where the nodes in A represent abstract entities called *arguments* and the edges in R represent *attacks* between arguments. $(a_i, a_j) \in R$ means that a_i attacks a_j; a_i is called an *attacker* of a_j. Figure 1 gives an example of an argumentation framework.

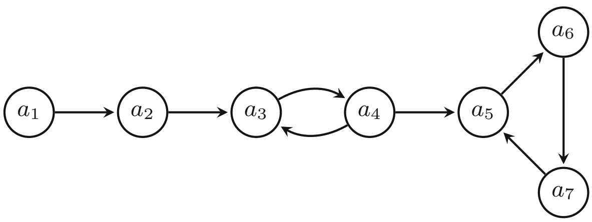

Fig. 1. The AF F_1

We say that an argument a_i (resp. a set of arguments S) defends the argument a_j against its attacker a_k if a_i (resp. any argument in S) attacks a_k. The *range* of a set of arguments S w.r.t. R, denoted S_R^+, is the subset of A which contains S and the arguments attacked by S; formally $S_R^+ = S \cup \{a_j \mid \exists a_i \in S \text{ s.t. } (a_i, a_j) \in R\}$. Different semantics allow to determine which sets of arguments can be collectively accepted [6,12,13,18–20,29].

Definition 1. *Let $F = \langle A, R \rangle$ be an AF. A set of arguments $S \subseteq A$ is*

- conflict-free *w.r.t. F if $\nexists a_i, a_j \in S$ s.t. $(a_i, a_j) \in R$;*
- admissible *w.r.t. F if S is conflict-free and S defends each of its arguments against all of their attackers;*
- *a* naive *extension of F if S is a maximal conflict-free set (w.r.t. $\subseteq$);*
- *a* complete *extension of F if S is admissible and S contains all the arguments that it defends;*
- *a* preferred *extension of F if S is a maximal complete extension (w.r.t. $\subseteq$);*
- *a* stable *extension of F if S is conflict-free and $S_R^+ = A$;*
- *a* grounded *extension of F if S is a minimal complete extension (w.r.t. $\subseteq$);*
- *a* stage *extension of F if S is conflict-free and there is no conflict-free T such that $S_R^+ \subset T_R^+$;*
- *a* semi-stable *extension of F if S is admissible and there is no admissible T such that $S_R^+ \subset T_R^+$;*
- *an* ideal set *of F if S is admissible and S is included in each preferred extension;*
- *an* ideal *extension of F if S is a maximal (w.r.t. $\subseteq$) ideal set of F;*
- *an* eager *extension of F if S is a maximal (w.r.t. $\subseteq$) admissible set that is a subset of each semi-stable extension.*

These semantics are denoted, respectively, $cf, adm, na, co, pr, st, gr, stg, sem, is, id, eg$. For each σ of them, $Ext_\sigma(F)$ denotes the set of σ-extensions of F.

Let us recall the definition of usual decision problems for argumentation.

Definition 2. *Let $F = \langle A, R \rangle$ be an AF and σ a semantics.*

$Cred_\sigma$ *An argument $a_i \in A$ is said to be* credulously accepted *by F w.r.t. σ if $\exists E \in Ext_\sigma(F)$ s.t. $a_i \in E$.*

$Skept_\sigma$ *An argument $a_i \in A$ is said to be* skeptically accepted *by F w.r.t. σ if $\forall E \in Ext_\sigma(F), a_i \in E$.*

$Exist_\sigma$ *F satisfies the* non-trivial existence *w.r.t. σ if F admits at least one non-empty σ extension.*

The set of credulously (resp. skeptically) accepted arguments in F w.r.t. σ is denoted $cr_\sigma(F)$ (resp. $sk_\sigma(F)$).

Example 1. *Let us consider the argumentation framework F_1 given at Fig. 1, and let us illustrate some of the semantics, and related decision problems.*

- $Ext_{adm}(F_1) = \{\emptyset, \{a_1\}, \{a_4\}, \{a_4, a_6\}, \{a_1, a_4, a_6\}, \{a_1, a_3\}, \{a_1, a_4\}\}$,
- $Ext_{st}(F_1) = \{\{a_1, a_4, a_6\}\}$,
- $Ext_{pr}(F_1) = \{\{a_1, a_4, a_6\}, \{a_1, a_3\}\}$,
- $Ext_{co}(F_1) = \{\{a_1, a_4, a_6\}, \{a_1, a_3\}, \{a_1\}\}$,
- $Ext_{gr}(F_1) = \{\{a_1\}\}$.

a_1 is skeptically accepted in F_1 w.r.t. the stable, preferred, complete and grounded semantics. a_4 is credulously accepted in F_1 w.r.t. the preferred and complete semantics, but it is not w.r.t. the grounded semantics.

Table 1 gives the complexity class of these decision problems[1]. Results come from [14,15,18,20–23,25]. We suppose that the reader is familiar with the basic notions of complexity. Otherwise, see [28] for instance. Computation of one extension and enumeration of all the extensions are not decision problems, so their complexity cannot be evaluated through the polynomial hierarchy as we do for credulous and skeptical acceptance. But the computational hardness of these functional problems can all the same be estimated. Indeed, the complexity of skeptical acceptance can be seen as a lower bound for the complexity of the enumeration of extensions, and the complexity of the non-trivial existence can be seen as a lower bound of the computation of an extension.

Table 1. Complexity of Inference Problems for the Usual Semantics. $\mathsf{C}{-}c$ (resp. $\mathsf{C}{-}h$) means that the considered decision problem is complete (resp. hard) for the complexity class C.

σ	$Cred_\sigma$	$Skept_\sigma$	$Exist_\sigma$
cf	Trivial	P	Trivial
adm	NP $-c$	Trivial	NP $-c$
na	P	P	L
co	NP $-c$	P	NP $-c$
pr	NP $-c$	$\Pi_2^P - c$	NP $-c$
st	NP $-c$	coNP$-c$	NP $-c$
gr	P	P	P
stg	$\Sigma_2^P - c$	$\Pi_2^P - c$	L
sem	$\Sigma_2^P - c$	$\Pi_2^P - c$	NP$-c$
id	coNP$-h$	coNP$-h$	P
eg	$\Pi_2^P - c$	$\Pi_2^P - c$	P

In order to compare, in the following section, the semantics, and propose measures of their differences, let us introduce a useful notation: given two sets X, Y, $X \Delta Y$ is the symmetric difference between X and Y. Let us recall also the definition of a distance and of an aggregation function.

[1] Up to our knowledge, the complexity class of $Cred_{is}$, $Skept_{is}$ and $Exist_{is}$ has not yet been determined.

Definition 3. *Given a set E, a mapping d from $E \times E$ to $\mathbb{R}^+$ is*

- *a* pseudo-distance *if it satisfies weak coincidence, symmetry and triangular inequality;*
- *a* semi-distance *if it satisfies coincidence and symmetry;*
- *a* distance *if it satisfies coincidence, symmetry and triangular inequality.*

weak coincidence $\forall x \in E, d(x, x) = 0$*;*
coincidence $\forall x, y \in E, d(x, y) = 0$ *iff* $x = y$*;*
symmetry $\forall x, y \in E, d(x, y) = d(y, x)$*;*
triangular inequality $\forall x, y, z \in E, d(x, y) + d(y, z) \geq d(x, z)$*.*

Definition 4. *An* aggregation function *is a function $\otimes$ which associates a non-negative real number to every finite tuple of non-negative numbers, and which satisfies:*

non-decreasingness *if* $y \leq z$ *then* $\otimes(x_1, \ldots, y, \ldots, x_n) \leq \otimes(x_1, \ldots, z, \ldots, x_n)$*;*
minimality $\otimes(x_1, \ldots, x_n) = 0$ *iff* $x_1 = \cdots = x_n = 0$*;*
identity $\forall x \in \mathbb{R}^+, \otimes(x) = x$*.*

For instance, we will use the sum $\sum$ as an aggregation function.

3 Complexity-Based Difference Measures

As mentioned in the introduction, the acceptability semantics may have to be changed because of the computational complexity of the reasoning tasks an agent is involved into. Indeed, depending on which kind of reasoning is actually used by the agent (computation of one extension, enumeration of all the extensions, credulous or skeptical acceptance), the use of a given semantics σ_1 may lead to a higher complexity than another semantics σ_2, as depicted in Table 1.

If the agent needs to change her semantics for practical purpose, it seems that she will choose the semantics which allows her to have the *lowest* possible *complexity* for her main reasoning task. For instance, if she uses skeptical acceptance frequently, and if she is currently using the preferred semantics, it is interesting to select a new semantics such that the complexity of skeptical acceptance is minimal. When we consider the set of semantics which select only complete extensions, the possible new semantics are $\{co, gr\}$. To choose among these two, the agent can use another criterion such as minimal change based on another of the measures defined here.

In some cases, the agent can be obliged to change her semantics to another one which has a *higher complexity*; it is not desirable in general, but it can be mandatory to satisfy a given constraint. In this case, if several options are possible, a notion of minimality can be used. It consists now in a minimal increase of the complexity. We formalize it by defining a difference measure between reasoning tasks, where such a task is parametrized by the semantics and the specific decision problem.

Definition 5. *Let $\mathcal{S} = \{\sigma_1, \ldots, \sigma_n\}$ be a set of semantics, and $T = \{\tau_1, \ldots, \tau_m\}$ be a set of reasoning tasks. We define $\mathcal{C} = \{C(\tau_\sigma) \mid \tau \in T, \sigma \in \mathcal{S}\}$, where $C(\tau_\sigma)$ denotes the complexity class which characterizes the decision problem τ_σ.*

The complexity graph on $\mathcal{S}$ and T *is $Comp(\mathcal{S}, T) = \langle \mathcal{C}, I \rangle$ with $I \subseteq \mathcal{C} \times \mathcal{C}$ defined by $\forall c_1, c_2 \in \mathcal{C}, (c_1, c_2) \in I$ iff $c_1 \subset c_2$ and $\nexists c_3 \in \mathcal{C}$ such that $c_3 \neq c_1, c_3 \neq c_2$ and $c_1 \subset c_3 \subset c_2$.*[2]

The difference measure $\delta_T^{\mathcal{S}}$ between decision problems τ_{σ_1} and τ'_{σ_2} is the non-negative integer $\delta_T^{\mathcal{S}}(\tau_{\sigma_1}, \tau'_{\sigma_2})$ which is the length of the shortest non-oriented path between $C(\tau_{\sigma_1})$ and $C(\tau'_{\sigma_2})$ in $Comp(\mathcal{S}, T)$.

In general, the complexity-based difference measure are not distances, they do not satisfy coincidence. They satisfy weak coincidence and symmetry.

Example 2. *Let us consider the classical Dung's semantics $\mathcal{S} = \{co, pr, st, gr\}$ and the reasoning tasks $T = \{Cred_\sigma, Skept_\sigma, Exist_\sigma\}$. As we see in Table 1, $\mathcal{C} = \{\mathsf{P}, \mathsf{NP}, \mathsf{coNP}, \Pi_2^P\}$. The corresponding graph $Comp(\mathcal{S}, T)$ is given in Fig. 2.*

$$\begin{tikzcd}
 & \Pi_2^P & \\
\mathsf{coNP} \arrow[ur] & & \mathsf{NP} \arrow[ul] \\
 & \mathsf{P} \arrow[ul] \arrow[ur] &
\end{tikzcd}$$

Fig. 2. Complexity Graph $Comp(\mathcal{S}, T)$

Then, for instance, $\delta_T^{\mathcal{S}}(Cred_{gr}, Cred_{co}) = 1$ and $\delta_T^{\mathcal{S}}(Skept_{gr}, Skept_{pr}) = 2$. As soon as two decision problems have the same complexity, the measure of their difference is 0 (for instance, $\delta_T^{\mathcal{S}}(Cred_{st}, Cred_{co}) = 0$); this explains why $\delta_T^{\mathcal{S}}$ is not a distance.

Minimality of this complexity difference measure can be used when all the alternatives have a higher complexity than the previous one. For instance, if the agent is forced to change her semantics from *gr* to another one because she needs to be able to consider several solutions to her problem (which means that she needs to obtain several extensions), then she can choose the complete semantics when skeptical acceptance is important for her, because $\delta_T^{\mathcal{S}}(skept_{gr}, skept_{co}) = 0$.

4 Property-Based Difference Measures

Semantics can be compared respectively to the set of properties that characterize them. Such a characterization can be defined as follows.

Definition 6 [16]**.** *A set of properties $\mathcal{P}$ characterizes a semantics σ if for each AF F,*

[2] Under the usual assumptions about inclusions between complexity classes.

1. *each σ-extension of F satisfies each property from $\mathcal{P}$,*
2. *each set of arguments which satisfies each property from $\mathcal{P}$ is a σ-extension of F,*
3. *$\mathcal{P}$ is a minimal set (w.r.t $\subseteq$) among those which satisfy* 1. *and* 2.

$Prop(\sigma)$ denotes the set of properties that characterizes a semantics σ.

[16] points out a set of properties, and shows how each semantics can be characterized given this set. Absolute properties, which concern only a set of arguments by itself (Definition 7) are distinguished from relative properties, which concern a set of arguments with respect to other sets of arguments (Definition 8).

Definition 7 [16]**.** *Given an AF $F = \langle A, R\rangle$, a set of arguments S satisfies*

- *conflict-freeness if S is conflict-free;*
- *acceptability if S defends itself against each attacker;*
- *reinstatement if S contains all the arguments that it defends;*
- *complement attack if each argument in $A \setminus S$ is attacked by S.*

Definition 8 [16]**.** *Given an AF $F = \langle A, R\rangle$ and a set of properties $\mathcal{P}$, a set of arguments S satisfies*

- *$\mathcal{P}$-maximality if S is maximal (w.r.t. $\subseteq$) among the sets of arguments satisfying $\mathcal{P}$;*
- *$\mathcal{P}$-minimality if S is minimal (w.r.t. $\subseteq$) among the sets of arguments satisfying $\mathcal{P}$;*
- *$\mathcal{P}$-inclusion if S is included in each set of arguments satisfying $\mathcal{P}$;*
- *$\mathcal{P}$-R-maximality if S has a maximal range (w.r.t. $\subseteq$) among the sets of arguments satisfying $\mathcal{P}$.*

It can be noticed that, by definition, if a set S satisfies $\mathcal{P}$-maximality (resp. $\mathcal{P}$-minimality, $\mathcal{P}$-R-maximality), then S satisfies $\mathcal{P}$.

A characterization of different semantics, that follows from the previous definitions, has been established in [16]; Proposition 1 recalls this characterization, and extends it to ideal sets, ideal and eager semantics.

Proposition 1. *The extension-based semantics considered in this paper can be characterized as follows:*

- $Prop(cf) = \{conflict\text{-}freeness\}$.
- $Prop(adm) = Prop(cf) \cup \{acceptability\}$.
- $Prop(na) = Prop(cf)$*-maximality.*
- $Prop(co) = Prop(adm) \cup \{reinstatement\}$.
- $Prop(gr) = Prop(co)$*-minimality.*
- $Prop(pr) = Prop(adm)$*-maximality.*
- $Prop(sem) = Prop(adm)$*-R-maximality.*
- $Prop(stg) = Prop(cf)$*-R-maximality.*
- $Prop(st) = Prop(cf) \cup \{complement\ attack\}$.
- $Prop(is) = Prop(adm) \cup \{Prop(pr)\text{-}inclusion\}$.

- $Prop(id) = Prop(is)$-*maximality*.
- $Prop(eg) = Prop(pr) \cup \{Prop(sem)$-*inclusion*$\}$.

Let us notice that we can consider other properties, and give alternative characterizations of the semantics (see [9,10] for contributions in this sense). Even if the value of the difference between two semantics (obviously) depends of the chosen characterizations, the general definition of property-based difference measures is the same whatever the characterizations.

The intuition which lead to define the characterization as the minimal set of properties is related to computational issues. Indeed, computing some reasoning tasks related to the semantics thanks to the semantics characterization can be done more efficiently with this definition. For instance, to determine whether a set of arguments is a stable extension of a given AF, checking the satisfaction of conflict-freeness and complement attack proves enough. For instance, $Prop(adm)$-maximality may be added in the characterization of the stable semantics, but computing the result of our problem would then be harder.

A weight can be associated to each property, depending on the importance of the property in a certain context.

Definition 9 [16]**.** *Let $\mathcal{P}$ be a set of properties. Let w be a function which maps each property $p \in \mathcal{P}$ to a strictly positive real number $w(p)$. Given σ_1, σ_2 two semantics such that $Prop(\sigma_1) \subseteq \mathcal{P}$ and $Prop(\sigma_2) \subseteq \mathcal{P}$, the* property-based difference measure *δ^w_{prop} between σ_1 and σ_2 is defined as:*

$$\delta^w_{prop}(\sigma_1, \sigma_2) = \sum_{p_i \in Prop(\sigma_1) \Delta Prop(\sigma_2)} w(p_i)$$

The specific property-based difference measure defined when all the properties have the same importance is as follows.

Definition 10 [16]**.** *Given two semantics σ_1, σ_2, the property-based difference measure δ_{prop} is defined by $\delta_{prop}(\sigma_1, \sigma_2) = |Prop(\sigma_1) \Delta Prop(\sigma_2)|$.*

Example 3. *Let us suppose that the initial semantics is the admissible one.*

- *When δ_{prop} is considered, naive and preferred semantics are "equivalent", since $\delta_{prop}(adm, na) = \delta_{prop}(adm, pr) = 3$.*
- *With a weighted measure δ^w_{prop} such that $w(Prop(cf)$-maximality$) = 1$ and $w(Prop(adm)$-maximality$) = 2$, the two semantics are no more equivalent, since $\delta_{prop}(adm, na) < \delta_{prop}(adm, pr)$.*

Proposition 2 [16]. *Given a set of semantics $\mathcal{S}$, the property-based measures defined on $\mathcal{S}$ are distances.*

5 Relation-Based Difference Measures

Most of the usual semantics are related according to some notions. For instance, it is well-known that each preferred extension of an AF is also a complete extension of it, and the grounded extension is also complete, but in general it is not a preferred extension. The preferred semantics may thus be seen closer to the complete semantics, than to the grounded semantics. This idea has been formalized with the notion of semantics relation graph.

Definition 11 [16]**.** *Let $\mathcal{S} = \{\sigma_1, \ldots, \sigma_n\}$ a set of semantics. A* semantics relation graph *on $\mathcal{S}$ is defined by $Rel(\mathcal{S}) = \langle \mathcal{S}, D \rangle$ with $D \subseteq \mathcal{S} \times \mathcal{S}$.*

This abstract notion of relation graph, where the nodes are semantics, can be instantiated with the inclusion relation between the extensions of an AF.

Definition 12 [16]**.** *Let $\mathcal{S} = \{\sigma_1, \ldots, \sigma_n\}$ a set of semantics. The* extension inclusion graph *of $\mathcal{S}$ is defined by $Inc(\mathcal{S}) = \langle \mathcal{S}, D \rangle$ with $D \subseteq \mathcal{S} \times \mathcal{S}$ such that $(\sigma_i, \sigma_j) \in D$ if and only if:*

- *for each AF F, $Ext_{\sigma_i}(F) \subseteq Ext_{\sigma_j}(F)$;*
- *there is no $\sigma_k \in \mathcal{S}$ $(k \neq i, k \neq j)$ such that for each AF F, $Ext_{\sigma_i}(F) \subseteq Ext_{\sigma_k}(F)$ and $Ext_{\sigma_k}(F) \subseteq Ext_{\sigma_j}(F)$.*

This idea has been discussed in [4], but the notion of relation between semantics had not been formalized before [16].

Example 4. *For instance, when $\mathcal{S} = \{co, pr, st, gr, stg, sem, is, id, eg, adm, cf, na\}$, $Inc(\mathcal{S})$ is the graph given at Fig. 3.*

$$\begin{tikzcd}
is \arrow[r] & adm \arrow[r] & cf & na \arrow[l] \\
id \arrow[u] \arrow[r] & co \arrow[u] & gr \arrow[l] & stg \arrow[u] \\
eg \arrow[ur, bend left] & pr \arrow[u] & sem \arrow[l] & st \arrow[l] \arrow[u]
\end{tikzcd}$$

Fig. 3. Extension Inclusion Graph $Inc(\mathcal{S})$

A family of difference measures between semantics which is based on the semantics relation graphs has been defined, to measure what it costs for an agent to change her semantics.

Definition 13 [16]**.** *Given $\mathcal{S}$ a set of semantics, a $\mathcal{S}$-* relation difference measure *is the mapping from two semantics $\sigma_1, \sigma_2 \in \mathcal{S}$ to the non-negative integer $\delta_{Rel,\mathcal{S}}(\sigma_1, \sigma_2)$ which is the length of the shortest non-oriented path between σ_1 and σ_2 in $Rel(\mathcal{S})$. In particular, the $\mathcal{S}$-inclusion measure is the length of the shortest non-oriented path between σ_1 and σ_2 in $Inc(\mathcal{S})$, denoted by $\delta_{Inc,\mathcal{S}}(\sigma_1, \sigma_2)$.*

Example 5. *Given two semantics σ_1 and σ_2 which are neighbours in the graph given at Fig. 3, the difference measure $\delta_{Inc,\mathcal{S}}(\sigma_1, \sigma_2)$ is obviously 1. Otherwise, if several paths allow to reach σ_2 from σ_1, then the difference is the length of* *the minimal one. For instance, $\delta_{Inc,\mathcal{S}}(st, cf) = 3$ since the minimal path is $st \rightarrow stg \rightarrow na \rightarrow cf$, but other paths exist (for instance, $st \rightarrow sem \rightarrow pr \rightarrow co \rightarrow adm \rightarrow cf$). Since here the question is to define the difference between semantics, the possibility to obtain several minimal paths (for instance, there are two minimal paths between the ideal and admissible semantics: $id \rightarrow is \rightarrow adm$ and $id \rightarrow co \rightarrow adm$) is not problematic.*

Proposition 3 [16]**.** *The $\mathcal{S}$-inclusion difference measure is a distance.*

The relation graph can be instantiated with other relations between semantics. The skepticism relation studied in [5] would be an appropriate candidate. The graph resulting from the intertranslatability relationship of semantics [24] may also be considered. Such instantiations would require a deeper investigation.

It can be noticed that, for any instantiation of the relation graph as defined above, which is absolute, that is, independent of any specific AF, a relative version can also be defined. In this case, the edges in the graph would depend on the relations for a given AF; the initial proposal considers the relations which are true for any AF. Such AF-based relation graph may also lead to interesting difference measures, which would require investigation as well.

6 Acceptance-Based Difference Measures

In line with the remarks at the end of the last section, regarding absoluteness (that is, independence of the measure from any specific situation or AF) and relativity (dependence on a given AF) of difference measures, a family of relative measures is presented in this section. Now, the difference between semantics depends on the acceptance status of arguments in a given AF, w.r.t. the different semantics in consideration.

The first acceptance-based measure quantifies the difference between the σ_1-extensions and the σ_2-extension of the AF to quantify the difference between σ_1 and σ_2.

Definition 14 [16]**.** *Let F be an AF, d be a distance between sets of arguments, and $\otimes$ be an aggregation function. The F-d-$\otimes$-*extension-based difference measure $\delta_F^{d,\otimes}$ *is defined by $\delta_F^{d,\otimes}(\sigma_1, \sigma_2) = \otimes_{\epsilon \in Ext_{\sigma_1}(F)} \min_{\epsilon' \in Ext_{\sigma_2}(F)} d(\epsilon, \epsilon')$.*

Proposition 4. *In general, the extension-based difference measures are not distances, they do not satisfy coincidence, symmetry.*

Example 6. *For instance, we consider the Hamming distance between sets of arguments, defined as $d_H(s_1, s_2) = |s_1 \Delta s_2|$. Now, we define the F_1-d_H-$\sum$-extension-based difference measure $\delta_F^{d_H,\sum}$ from d_H and the AF F_1 given at Fig. 1. Its set of stable extensions is $Ext_{st}(F_1) = \{\{a_1, a_4, a_6\}\}$.*

When measuring the difference between the stable semantics and the other classical Dung's semantics, we obtain:

- $\delta_{F_1}^{d_H,\sum}(st, gr) = 2$ *since* $Ext_{gr}(F_1) = \{\{a_1\}\}$*;*
- $\delta_{F_1}^{d_H,\sum}(st, pr) = 0$ *since* $Ext_{pr}(F_1) = \{\{a_1, a_3\}, \{a_1, a_4, a_6\}\}$*; on the opposite,* $\delta_{F_1}^{d_H}(pr, st) = 3$*;*
- $\delta_{F_1}^{d_H,\sum}(st, co) = 0$ *since* $Ext_{co}(F_1) = \{\{a_1\}, \{a_1, a_3\}, \{a_1, a_4, a_6\}\}$*.*

The following result shows that the restriction of the extension-based measure to some particular sets of semantics leads to satisfy the coincidence property.

Proposition 5. *For a given* F *and a given set of semantics* $\mathcal{S} = \{\sigma_1, \ldots, \sigma_n\}$*, if for all* $\sigma_i, \sigma_j \in \mathcal{S}$ *such that* $\sigma_i \neq \sigma_j$*,* $Ext_{\sigma_i}(F) \not\subseteq Ext_{\sigma_j}(F)$*, then the extension-based measure* $\delta_F^{d_H,\sum}$ *satisfies coincidence.*

Even in this case, the measure does no satisfy all the properties of distances. However, we can use the intuition behind this measure to define another one.

Definition 15 [16]**.** *Let* F *be an AF,* d *be a distance between sets of arguments, and* $\otimes$ *be an aggregation function. The* symmetric F-d-$\otimes$-extension-based difference measure $\delta_{F,sym}^{d,\otimes}$ *is defined by* $\delta_{F,sym}^{d,\otimes}(\sigma_1, \sigma_2) = \max(\delta_F^{d,\otimes}(\sigma_1, \sigma_2), \delta_F^{d,\otimes}(\sigma_2, \sigma_1))$*.*

This measure satisfies the distance properties under some conditions.

Proposition 6 [16]**.** *For a given* F *and a given set of semantics* $\mathcal{S} = \{\sigma_1, \ldots, \sigma_n\}$*, if for all* $\sigma_i, \sigma_j \in \mathcal{S}$ *such that* $\sigma_i \neq \sigma_j$*,* $Ext_{\sigma_i}(F) \neq Ext_{\sigma_j}(F)$*, then the symmetric extension-based measure* $\delta_{F,sym}^{d_H,\sum}$ *is a semi-distance.*

As suggested in [16], we can also use the set of skeptically (resp. credulously) accepted arguments instead of the whole set of extensions to define a difference measure between semantics. We propose here a definition of such measures.

Definition 16. *Given* F *an AF,* d *a distance between sets of arguments, and* $\mathcal{S}$ *a set of semantics, the* F-d-skeptical acceptance difference measure $\delta_{F,sk}^{d}$ *is defined, for any* $\sigma_1, \sigma_2 \in \mathcal{S}$*, by*

$$\delta_{F,sk}^{d}(\sigma_1, \sigma_2) = d(sk_{\sigma_1}(F), sk_{\sigma_2}(F))$$

The F-d-credulous acceptance difference measure $\delta_{F,sk}^{d}$ *is defined, for any* $\sigma_1, \sigma_2 \in \mathcal{S}$*, by*

$$\delta_{F,cr}^{d}(\sigma_1, \sigma_2) = d(cr_{\sigma_1}(F), cr_{\sigma_2}(F))$$

If two semantics lead to the same set of credulously (resp. skeptically) accepted arguments, then these measures cannot distinguish between these semantics. Other properties are satisfied.

Proposition 7. *Given* F *and AF and* d *a distance, the* F*-*d*-skeptical acceptance difference measure and the* F*-*d*-credulous acceptance difference measure are pseudo-distances.*

7 Obtaining Comparison Criteria

In the context of a semantic change, the difference measures can be used to define different minimality or maximality criteria. With σ the initial semantics, and $\mathcal{S}$ the set of options for the new semantics, the new semantics should be $\sigma' \in \mathcal{S}$ such that, given δ the chosen measure:

- $\forall \sigma'' \in \mathcal{S}$, $\delta(\sigma, \sigma') \leq \delta(\sigma, \sigma'')$ to define a minimality criteria denoted $\min_{\delta,\sigma}$,
- $\forall \sigma'' \in \mathcal{S}$, $\delta(\sigma, \sigma') \geq \delta(\sigma, \sigma'')$ to define a maximality criteria denoted $\max_{\delta,\sigma}$.

Given σ a semantics and $\mathcal{S}$ a set of semantics, $\min_{\delta,\sigma}(\mathcal{S}) = \{\sigma_i \in \mathcal{S} \mid \forall \sigma_j \in \mathcal{S}, \delta(\sigma, \sigma_i) \leq \delta(\sigma, \sigma_j)\}$ is the subset of $\mathcal{S}$ of semantics which minimize the criterion $\min_{\delta,\sigma}$; the counterpart for maximality criteria is $\max_{\delta,\sigma}(\mathcal{S}) = \{\sigma_i \in \mathcal{S} \mid \forall \sigma_j \in \mathcal{S}, \delta(\sigma, \sigma_i) \geq \delta(\sigma, \sigma_j)\}$.

A single criteria may not be enough to compare, or distinguish between, some semantics, as shown in the examples in the Introduction. Combining criteria may allow an agent to do so. It can be noticed that the order of application of the different criteria may then lead to different results.

Definition 17. *Let $X = \langle \chi_1, \ldots, \chi_n \rangle$ a vector of (minimality or maximality) criteria. Let σ be a semantics, and $\mathcal{S}$ a set of semantics. The X-*based semantic change selection function *is defined by $\chi_X(\sigma, \mathcal{S}) = \chi_X^n(\sigma, \mathcal{S})$*

with χ_X^n as follows:

$$\gamma_X^1(\sigma, \mathcal{S}) = \chi_1(\mathcal{S})$$
$$\gamma_X^k(\sigma, \mathcal{S}) = \chi_k(\gamma_X^{k-1}(\sigma, \mathcal{S}))$$

Let us notice that this definition is general enough to encompass any difference measure yet to be defined.

Example 7. *Let $\sigma = st$ be the current semantics. A change of semantics has to be done. The reasoning task to complete by the agent is credulous reasoning ($Cred_\sigma$). The candidate new semantics are $\mathcal{S} = \{pr, co, eg\}$. The new semantics must be as close as possible in terms of computational complexity to the current one, but it should contain not only the current results, but as many results as possible (the agent wants as many options as possible). Hence, the vector of criteria to be considered is $X = \langle min_{\delta^{\mathcal{S}}_{Cred}, \sigma}, max_{\delta_{Inc,\mathcal{S}}, \sigma} \rangle$. Then, by first minimizing the complexity-based difference measure, the only semantics to be considered are pr, co. By maximizing then the inclusion measure, the X-based semantic change selection function returns co.*

8 Conclusion

This paper presents several ways to quantify the difference between extension-based semantics, building on [16]. Some of them are absolute (they only depend on the semantics), while the other ones are relative (they depend on the considered AF). Let us mention the fact that there is no general relation between these difference measures; for instance we have seen on several examples that it may occur that $\delta_1(\sigma_1, \sigma_2) > \delta_1(\sigma_1, \sigma_3)$ while $\delta_2(\sigma_1, \sigma_2) < \delta_2(\sigma_1, \sigma_3)$. When a semantic change occurs, this permits the agent to use some very different criteria to select the new semantics, depending on which difference measures make sense in the context of her application. The minimization, or the maximization of these measures, and their combinations, permit to express many comparison criteria.

Let us notice that only the relation-based and property-based measures are distances, other methods failing in general to satisfy the distance properties, which seem to be desirable to quantify the difference between objects. However, the skeptical and credulous acceptance difference measures are pseudo-distances. Further study could lead to identify the necessary conditions that a set of semantics must satisfy to ensure that these are distances.

Table 2. Summary of properties satisfied by the measures

	$\delta_T^{\mathcal{S}}$	δ_{prop}^{w}	$\delta_{Inc,\mathcal{S}}$	$\delta_F^{d,\sum}$	$\delta_{F,sym}^{d,\sum}$	$\delta_{F,sk}^{d}$	$\delta_{F,cr}^{d}$
WC	✓	✓	✓	∘	✓	✓	✓
Co	×	✓	✓	×	∘	×	×
Sym	✓	✓	✓	×	✓	✓	✓
TI		✓	✓			✓	✓

Table 2 depicts the properties satisfied by our measures. WC, Co, Sym and TI stand respectively for weak coincidence, coincidence, symmetry and triangular inequality. A ✓ symbol means that the property is always satisfied, and × means that it is not satisfied in general. ∘ means that the property is satisfied under some additional assumption.

Several tracks can be considered for future works. We have noticed that we can order semantics, with respect to an initial semantics σ and a measure δ: $\sigma_1 \leq_{\sigma,\delta} \sigma_2$ if and only if $\delta(\sigma, \sigma_1) \leq \delta(\sigma, \sigma_2)$. In this case, we can investigate the relation of the orderings defined by different measures. For instance, if some pairs (σ, δ_1) and (σ, δ_2) lead to the same ordering, then we can choose to use the measure which is the least expensive one to compute among δ_1 and δ_2.

We also plan to define a similar notion of difference measures for labelling-based semantics [4], and for ranking-based semantics [1,11,26]. In this last context, we need to determine whether some relevant properties characterize the ranking which is used to evaluate arguments, or to determine meaningful notions of difference between the rankings.

Finally, we will investigate more in depth the question which is mentioned in the introduction: using (minimal) semantic change in argumentation dynamics scenarios. In particular, [17] has shown that semantic change can be used to guarantee minimal change on the attack relation when performing an extension enforcement. We will investigate this question in other scenarios.

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A Proofs

Proof (Proof of Proposition 2). From our definition of characterizations, the mapping that associates a semantics σ to a set of properties $Prop(\sigma)$ guarantees that a semantics cannot be associated with two different sets of properties, and a same set of properties cannot correspond to different semantics.

The weighted sum on sets of properties obviously defines a distance (in particular, when all weights are identical, we obtain the well-known Hamming distance; other weights just define generalization of Hamming distance). Since we can identify the semantics to the sets of properties, δ^{w}_{prop} is a distance.

Proof (Proof of Proposition 3). From the definition of the Σ-relation graph,

- the difference between σ_1 and σ_2 is 0 iff they are the same node of the graph (i.e. $\sigma_1 = \sigma_2$), so coincidence is satisfied;
- the shortest path between two semantics σ_1, σ_2 has the same length whatever the direction of the path (from σ_1 to σ_2, or vice-versa), since we do not consider the direction of arrows, so symmetry is satisfied;
- the shortest path between σ_1 and σ_3 is at worst the concatenation of the paths $(\sigma_1, \dots, \sigma_2)$ and $(\sigma_2, \dots, \sigma_3)$, or (if possible) a shorter one, so triangular inequality is satisfied.

Proof (Proof of Proposition 4). Example 6 gives the counter-examples for coincidence and symmetry.

Proof (Proof of Proposition 5). We consider a given AF F and a set of semantics $\Sigma = \{\sigma_1, \dots, \sigma_n\}$, such that for all $\sigma_i, \sigma_j \in \Sigma$ with $\sigma_i \neq \sigma_j$, $Ext_{\sigma_i}(F) \nsubseteq Ext_{\sigma_n}(F)$.

Obviously, for any semantics σ_i, $\delta_F^{d_H,\sum}(\sigma_i, \sigma_i) = 0$. Now, let us assume the existence of two semantics $\sigma_i, \sigma_j \in \Sigma$ such that $\delta_F^{d_H,\sum}(\sigma_i, \sigma_j) = 0$. We just rewrite this, following the definition of the measure: $\sum_{\epsilon \in Ext_{\sigma_i}(F)} \min_{\epsilon' \in Ext_{\sigma_j}(F)} d_H(\epsilon, \epsilon') = 0$. Since all distances are non-negative number, if the sum is equal to zero it means that $\forall \epsilon \in Ext_{\sigma_i}(F)$, $\min_{\epsilon' \in Ext_{\sigma_j}(F)} d_H(\epsilon, \epsilon') = 0$. Because of the properties of the Hamming distance, it means that $\epsilon \in Ext_{\sigma_j}$, and so $Ext_{\sigma_i} \subseteq Ext_{\sigma_j}$. From our starting assumption, we deduce that $\sigma_i = \sigma_j$.

Proof (Proof of Proposition 6). From the definition of the measure, $\delta_{F,sym}^{d_H,\sum}(\sigma_1, \sigma_2) = 0$ iff $Ext_{\sigma_1}(F) = Ext_{\sigma_2}(F)$. Under our assumptions, this is possible only if $\sigma_1 = \sigma_2$. The other direction is trivial, so coincidence is satisfied. Symmetry is obviously satisfied, since σ_1, σ_2 can be inverted in $\max(\delta_F^{d,\otimes}(\sigma_1, \sigma_2), \delta_F^{d,\otimes}(\sigma_2, \sigma_1))$.

Proof (Proof of Proposition 7). Weak coincidence and symmetry are trivial from the definition of the measures.

$$\begin{aligned}\delta^d_{F,sk}(\sigma_1,\sigma_2)+\delta^d_{F,sk}(\sigma_2,\sigma_3) &= d(sk_{\sigma_1}(F),sk_{\sigma_2}(F))+d(sk_{\sigma_2}(F),sk_{\sigma_3}(F))\\ &\geq d(sk_{\sigma_1}(F),sk_{\sigma_3}(F))=\delta^d_{F,sk}(\sigma_1,\sigma_3)\end{aligned}$$

The same reasoning apply for the credulous acceptance measure. So both satisfy the triangular inequality. Coincidence is not satisfied by the skeptical acceptance measure. For instance, for each AF F, $\emptyset \in Ext_{cf}(F)$ and $\emptyset \in Ext_{adm}(F)$, so $sk_{cf}(F) = sk_{adm}(F) = \emptyset$, and so $\delta^d_{F,skep}(cf, adm) = 0$. The same conclusion holds as soon as two semantics yield the same skeptically or credulously accepted arguments.

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